Minimum Q Electrically Small Spherical Magnetic Dipole Antenna - Theory

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1. Introduction

Electrically small antennas have been a subject of research for many years but recent years' escalation of miniaturized wireless technology has caused a renewed interest in developing solutions for maintaining acceptable matching, bandwidth, and radiation efficiency of small antennas. In particular, it is of great interest to investigate how close a small antenna can have its quality factor Q approach the so-called Chu lower bound Q_{Chu}. For a resonant antenna radiating either a TM electric-dipole field or a TE magnetic-dipole field, operating at a frequency with free-space wave number k, and with the smallest circumscribing sphere of radius a, this bound is [1], [2]

$$Q_{Chu} = \frac{1}{(ka)^3} + \frac{1}{ka}, \quad ka << 1. \tag{1}$$

The Chu lower bound is based on the assumption that the internal volume of the circumscribing sphere does not store any electric or magnetic energy – except what is needed to make the antenna resonant. Any internally stored energy obviously increases the quality factor. For spherical electric surface current densities in free space, that will produce an internal field possessing a stored energy, Wheeler [3] and Thal [4] showed that the quality factor for TE magnetic-dipole radiation becomes 3 times the Chu lower bound; thus,

$$Q_{TE} = 3Q_{Chu}, \quad ka << 1. \tag{2}$$

Recently reported electrically small magnetic-dipole antennas by Kim [5], consisting of wire structures on spherical surfaces in free space, exhibit quality factors in agreement with (2).

In order to approach the Chu lower bound (1) an antenna design with vanishing internally stored energy is required. More precisely, for an electrically small magnetic-dipole antenna, where the stored magnetic energy dominates the stored electric energy, the internally stored magnetic energy must vanish. Wheeler [3] and Thal [4] reported that this can be accomplished with a magnetic core of relative permeability μ_r for which the quality factor Q will be

$$Q = \left(1 + \frac{2}{\mu_r}\right) Q_{Chu}, \quad ka << 1. \tag{3}$$

The purpose of this work is to investigate the stored energies, the radiated power, and the quality factor of a magnetic-dipole antenna, consisting of a spherical electric surface current density enclosing a magnetic core, for varying electrical size; in particular to study its behaviour as the electrical size becomes small. This is accomplished by expressing the radiated field, externally and internally, in terms of spherical vector wave functions and determining the stored energies and radiated power from direct spatial integrations of these functions. This paper is a companion of Kim and Breinbjerg [6].

2. Configuration

The magnetic-dipole antenna configuration consists of a time-harmonic spherical electric surface current density $\boldsymbol{J}$ of radius a and amplitude J_0 enclosing a magnetic core with relative permeability μ_r and wave number k_s. Introducing a spherical $r\theta\phi$-coordinate system with its origin at the centre of the magnetic core, and employing phasor notation with suppressed time factor $\exp(-i\omega t)$, the surface current density can be expressed as ($\boldsymbol{a}_\phi$ is the azimuthal unit vector)

$$\boldsymbol{J} = \boldsymbol{a}_\phi J_0 \sin\theta \,. \tag{4}$$

3. Radiated fields, stored energies and radiated power

The radiated field is obtained by expressing the internal field for $r < a$ as a spherical vector wave of the standing wave type, the external field for $r > a$ as a spherical vector wave of the outward propagating type, and then enforcing the appropriate boundary conditions at $r = a$ to determine the complex amplitudes C^- and C^+ of the internal and external fields, respectively Since the surface current density is azimuthally constant and have a first-order polar variation, this will also be the case for the radiated fields; thus, only the magnetic-dipole spherical vector wave with azimuthal index $m = 0$ and polar index $n = 1$ will exist. Due to paper length limitations, the explicit field expressions are not given here.

The stored energies and the radiated power are determined through direct spatial integration of the radiated fields. In the calculation of the externally stored energy the contribution of the propagating field is subtracted from that of the total field to obtain the energy density of the non-propagating field only [2]. With $W_E^-, W_H^-, W_E^+,$ and W_H^+ denoting the internal electric, internal magnetic, external electric, and external magnetic stored energy, respectively, it is found that

$$W_E^- = \frac{1}{4\omega}\left|C^-\right|^2\left\{-\frac{\sin^2 k_s a}{k_s a} + \frac{k_s a}{2} + \frac{\sin 2k_s a}{4}\right\}, \tag{5a}$$

$$W_H^- = \frac{1}{4\omega}\left|C^-\right|^2\left\{-\frac{\sin^2 k_s a}{(k_s a)^3} + \frac{\sin 2k_s a}{(k_s a)^2} - \frac{\cos^2 k_s a}{k_s a} + \frac{k_s a}{2} - \frac{\sin 2k_s a}{4}\right\}, \tag{5b}$$

$$W_H^+ = \frac{1}{4\omega}\left|C^+\right|^2\left\{\frac{1}{(ka)^3} + \frac{1}{ka}\right\}, \qquad W_E^+ = \frac{1}{4\omega}\left|C^+\right|^2\frac{1}{ka}. \tag{5c, 5d}$$

where the internal and external complex amplitudes C^- and C^+ are

$$C^- = J_0 ia\frac{\sqrt{\eta_s}}{\eta}\frac{4\sqrt{\pi}}{\sqrt{6}}\left(1+\frac{i}{ka}\right) I \,, \qquad C^+ = -J_0 ia\frac{4\sqrt{\pi}}{\sqrt{6}}\frac{e^{-ika}}{\sqrt{\eta}}\left\{-\cos k_s a + \frac{\sin k_s a}{k_s a}\right\} I \tag{6a, 6b}$$

with η and η_s being intrinsic admittance of free space and the magnetic core, respectively, and I is

$$I = \left\{\left(-i + \frac{1}{ka} + \frac{i}{(ka)^2}\right)\left(-\cos k_s a + \frac{\sin k_s a}{k_s a}\right) + \frac{\eta_s}{\eta}\left(1+\frac{i}{ka}\right)\left(\sin k_s a + \frac{\cos k_s a}{k_s a} - \frac{\sin k_s a}{(k_s a)^2}\right)\right\}^{-1}. \tag{7}$$

Finally, the radiated power P_{rad} is

$$P_{rad} = \frac{1}{2}\left|C^+\right|^2. \tag{8}$$

Note that the expressions (5) – (8) hold for arbitrary free-space electrical radius *ka.*

Figure 1 shows the ratio of the internally stored and externally stored magnetic energy vs. free-space electrical radius *ka* of the magnetic sphere for different relative permeabilities μ_r. For $\mu_r = 1$ (non-magnetic core) and $ka << 1$ the internal energy is twice the external energy; this is the reason for the factor of 3 in the quality factor (2) that includes also internally stored energy. For small *ka*, the energy ratio decreases with increasing μ_r and becomes equal to 0.02 for $\mu_r = 100$. However, for larger values of *ka* it is seen that the energy ratio can increase with increasing μ_r and even become infinite when the resonance condition, $C^+ = 0$, is satisfied and the external field thus is zero.

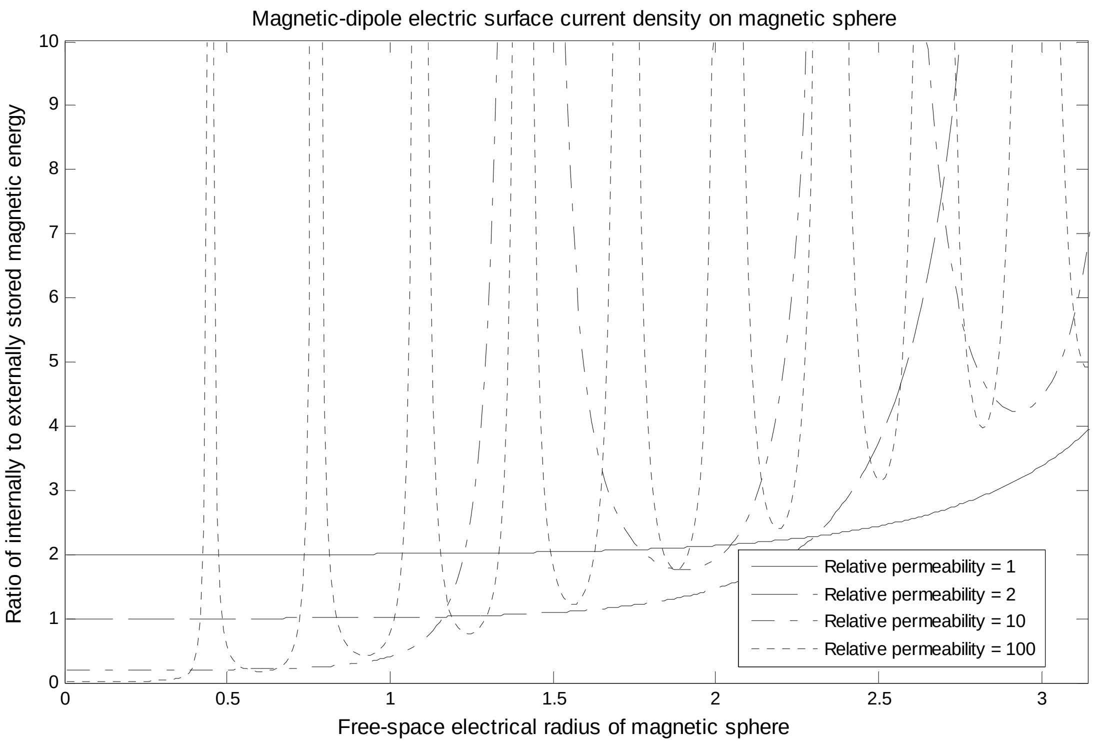

Figure 1: Ratio of internally and externally stored magnetic energy for the magnetic-dipole antenna vs. free-space electrical radius of magnetic sphere.

4. Quality factor

Generally, the quality factor is defined as 2π times the ratio of the stored, electric and magnetic, energy to the radiated power per period. However, in order to establish a resonant system (equal electric and magnetic stored energy) the stored energy is set to twice the maximum of the electric and magnetic energies. It is understood that the lesser of the two has been increased to equal the larger by use of e.g. a lumped-component tuning circuit. For the magnetic-dipole antenna, the magnetic energy dominates for small electrical radius but the electric energy may dominate at larger radii. Thus, the quality factor *Q* is determined from (5)-(8) as

$$Q = 2\omega \frac{\max\left(W_H^- + W_H^+, W_E^- + W_E^+\right)}{P_{rad}}. \quad (9)$$

Figure 2 shows *Q*, normalized by Q_{Chu}, vs. free-space electrical radius *ka* of the magnetic sphere for different relative permeabilities μ_r. For $\mu_r = 1$ (non-magnetic core) the normalized *Q* is equal to 3 for vanishing *ka* and it remains fairly constant for $ka < 1$ (this agrees with Thal [4; table I]) where it starts increasing towards infinity due to a resonance. For $\mu_r = 2$, 10, and 100 the normalized *Q* equals 2, 1.2, and 1.02, respectively, for small *ka* in agreement with (3). Hence, the Chu lower bound can be approached with modest values of μ_r; e.g. within 20% for $\mu_r = 10$ and $ka < 0.5$. However, for larger values of *ka* it is seen that (3) does not apply and that the smallest *Q* is not necessarily obtained with the largest μ_r. Indeed, for any given free-space electrical radius *ka* an optimal relative permeability μ_r can be determined that gives the lowest quality factor *Q*.

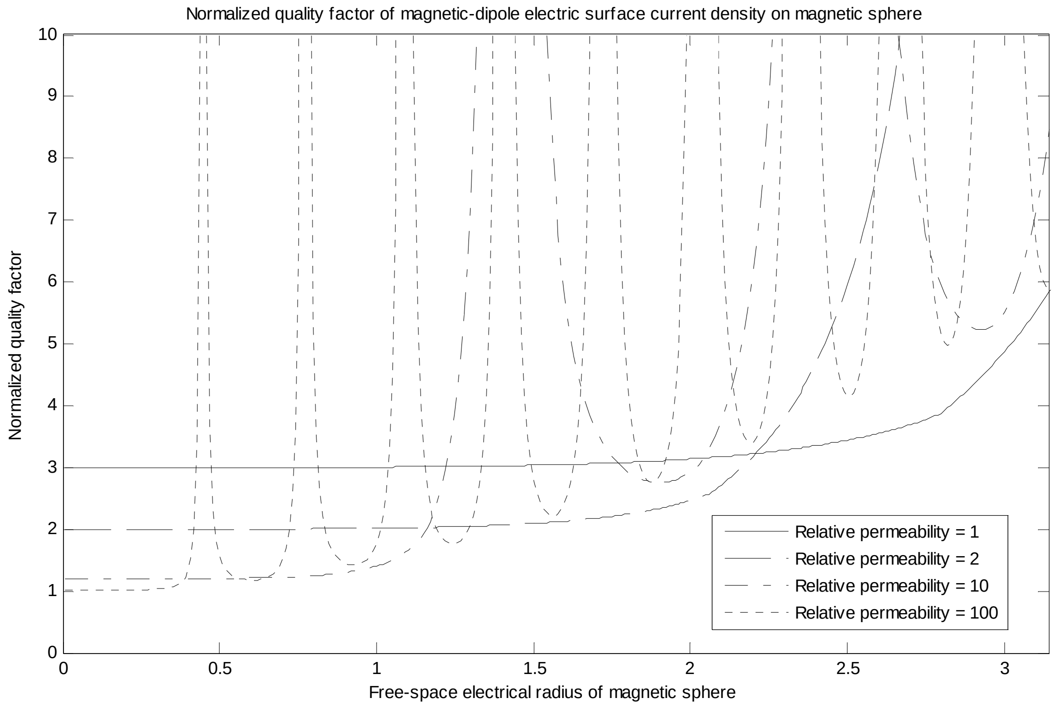

Figure 2: Quality factor (normalized by Chu lower bound) of the magnetic-dipole antenna vs. free-space electrical radius of magnetic sphere.

5. Conclusions

The stored energies, radiated power, and quality factor of a magnetic-dipole antenna with a magnetic core have been obtained through direct spatial integration of the internally and externally radiated field expressed in terms of spherical vector waves. The quality factor (9) agrees with that of Wheeler and Thal (2) for vanishing free-space electric radius but holds also for larger radii and facilitates the optimal choice of permeability in the presence of the resonances.

Acknowledgments

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