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Cesaro means of Fourier series for designing interleaver filters with planar lightwave circuit-type lattice structure

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Abstract
A simple and effective algorithm, based on the Cesaro means of Fourier series, is proposed for designing Fourier flat-top interleavers. Structural parameters for a PLC-type lattice-form interleaver can be easily obtained with a required spectral response.

1. Introduction

Lattice-form interleavers are based on the interference principle of Mach-Zehnder structure. In practical applications, several lattices are often cascaded to obtain better performance. In terms of cost, reliability and integration, the most attractive configurations of lattice-form interleaver are those based on planar lightwave circuit (PLC) designs.

To get optimized structural parameters is a key point in R&D of PLC-type lattice-form interleavers. It can be accomplished by a synthesis algorithm [1]. However, this algorithm is very complex and the waveform flatness is not good enough [2].

In this paper, A simple and effective method, based on Cesaro means of Fourier series, is proposed. All the structural parameters can be obtained by a one-step simple analysis, solving a simple equations-set and correspondence of structure parameters. As an example, a 25GHz 3-lattice-cascaded PLC-type interleaver is designed using this method.

2. Theoretical analysis

2.1 Fourier series of periodic rectangle function

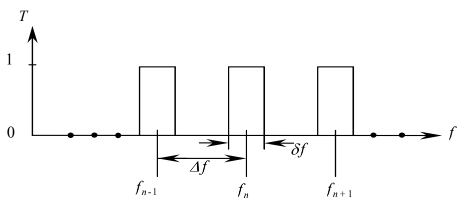

Fig. 1 The ideal periodical rectangle spectral transmittance

An ideal periodical rectangle function can be expressed by infinite Fourier series. For the ideal periodical rectangle spectral transmittance (as shown in figure 1) with δf/Δf of 1/2, it can be expressed as

$$T(f)=\frac{1}{2}+\sin c(\frac{\delta f}{\Delta f})\cos(2\pi\frac{f}{\Delta f})+\sin c(\frac{3\delta f}{\Delta f})\cos(2\pi\frac{3f}{\Delta f})+....$$
$$+\sin c[\frac{(2N-1)\delta f}{\Delta f}]\cos[2\pi\frac{(2N-1)f}{\Delta f}]+.... \quad (1)$$

The more the Fourier series, the closer the waveform to the ideal periodical rectangle function. In general, the term N needs up to be hundred.

2.2 Cesaro means of Fourier series

Cesaro means of Fourier series is a method that only a few Fourier series is needed for obtaining a relatively ideal waveform [3]. For the periodical rectangle spectral transmittance with δf/Δf of 1/2, it can be expressed by the Cesaro means of Fourier series with 5 Fourier series as

$$T(f)=\frac{1}{2}+\frac{2}{\pi}\cdot\frac{15}{16}\cdot\cos(2\pi\frac{f}{\Delta f})+\frac{2}{3\pi}\cdot\frac{9}{16}\cdot\cos(2\pi\frac{3f}{\Delta f})+\frac{2}{5\pi}\cdot\frac{3}{16}\cdot\cos(2\pi\frac{5f}{\Delta f}) \quad (2)$$

which is denoted as the solid line in Fig. 2. The dash line in Fig. 2 is the waveform with 7 Fourier series considered.

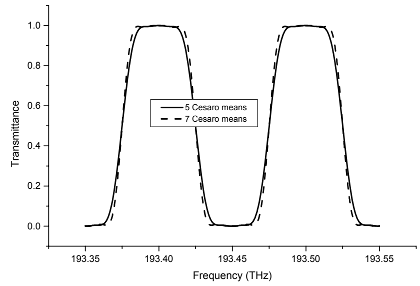

Fig. 2 The waveform corresponding to Cesaro means with five and seven Fourier Series

2.3 Flattening of spectral transmittance

The spectral transmittance of a birefringent interleaver can be described by Jones Matrix method:

$$T(f)=T_0+T_1\cdot\cos(2\pi\gamma_1 f)+T_2\cdot\cos(2\pi\gamma_2 f)+...$$
$$+T_n\cdot\cos(2\pi\gamma_n f)+... \quad (3)$$

The coefficient value of each cosine series and the constant term in Eq. (3) can be obtained by comparing with the expression of Cesaro means of Fourier series of the rectangle function. The relationship of structural parameters and a structural parameter can be determined through analyzing the expression, the value of the constant term and even term coefficients of cosine series in Eq. (3). Other structural parameters can be directly determined through solving an equations-set, which is composed of the odd term coefficients of cosine series. On the basis of these structural parameters, the structural parameters of lattice-form interleaver can be derived by the mathematical equivalence relationship between structural parameters of the lattice-form and birefringent interleavers.

2.4 Equivalent structural parameters of PLC-type lattice-form interleaver

This mathematical equivalence relationship can be expressed as [4]:

$$\begin{cases} \varphi_1 = \theta_1 \\ \varphi_2 = \theta_2 - \theta_1 \\ \quad \vdots \\ \varphi_i = \theta_i - \theta_{i-1} \\ \varphi_{i+1} = \theta_p - \theta_i \\ \Phi_i = t_i \end{cases} \tag{4}$$

Where φ_i is the phase factor correspond to the coupling ratio of the ith coupler, Φ_i is the phase delay of the ith phase delay line. θ_i and θ_P are, respectively, the azimuth angle of the ith crystal and polarizer at the output port. t_i is the phase delay of the ith crystal. It can be seen that when the structural parameters of one of the two types of interleaver are obtained, the structural parameters of the other can be figured out using Eq. (4) directly.

3. Design example

A 25GHz 3-lattice-cascaded PLC-type interleaver is designed as an example. According to the Section 2, we should solve the structural parameters of 3-crystal-cascaded birefringent interleaver firstly, and then the structural parameters of the 3-lattice-cascaded PLC-type interleaver can be obtained using Eq. (4).

The spectral transmittance of the birefringent interleaver calculated by Jones Matrix is expressed as

$$\begin{aligned} T(f) = {} & a_0 + a_1\cos(t_1) + a_2\cos(t_2) + a_3\cos(t_3) + a_4\cos(t_1+t_2) + a_5\cos(t_1+t_3) \\ & + a_6\cos(t_2+t_3) + a_7\cos(t_3-t_2) + a_8\cos(t_3-t_1) + a_9\cos(t_2-t_1) \\ & + a_{10}\cos(t_1+t_2+t_3) + a_{11}\cos(t_3+t_2-t_1) + a_{12}\cos(t_3-t_2+t_1) \\ & + a_{13}\cos(t_1+t_2-t_3) \end{aligned} \tag{5}$$

In order to obtain a desired spectral transmittance, the Eq. (5) and (2) should be equalized or approximated. Comparing Eq. (5) (detailed expression of parameters a_0-a_{13} are not given here) with Eq. (2), we obtain that the coefficient of the first, the third, the fifth Fourier series and the constant term in the Eq. (5) should be 15/8π, 3/8π, 3/40π and 1/2 respectively. And the coefficient of the second, the fourth Fourier series in Eq. (5) equal to zero. In order to satisfy the requirement of the constant term and each even Fourier series term, the azimuth angle of the first crystal can be 45° and the ratio of the three crystal thickness can be 1:2:2. The azimuth angle of the second, the third crystal and the polarizer at the output port can be obtained by solving the equations-set:

$$\begin{cases} a_1 + a_8 + a_9 + a_{12} + a_{13} = \dfrac{15}{8\pi} \\ a_4 + a_5 + a_{11} = \dfrac{3}{8\pi} \\ a_{10} = \dfrac{3}{40\pi} \end{cases} \tag{6}$$

Solving the Eq. (6), we obtain θ_1=45°, θ_2=103.49°, θ_3= -95.59°, θ_p=0.03°, and it is denoted as the curve C in Fig. 3. Curve D is the waveform with θ_1=45°, θ_2= -69°, θ_3=81°, θ_p=0° obtained in Ref. [5]. Curve B is the waveform with θ_1=45°, θ_2= -14.5°, θ_3=9.93°, θ_p=0° obtained in Ref. [6]. Curve A is the waveform with C_1=50%, C_2=70%, C_3=10% obtained in Ref. [2].

It can be seen that the isolation in curve C is greater than curve A and B. The flat-top of the curve C is accord with the curve D well. Only isolation is a little lower than the curve D. It shows that the Curve C obtained by Cesaro means of Fourier series can satisfy design requirement well. The results above are obtained by Cesaro means of Fourier series with 5 Fourier series considered. The waveform can be optimized further when 7 Fourier series are considered. At last, the structural parameters of the 3-lattice-cascaded PLC-type interleaver can be obtained using Eq. (4): φ_1＝π/4，φ_2＝0.324π，φ_3＝ -96.165π，φ4＝0.531π.

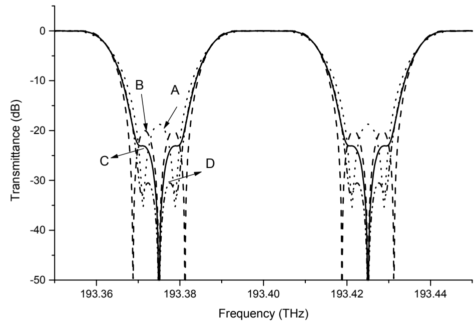

Fig. 5 the waveform corresponding to structural parameters obtained by different methods (A) C_1=50%, C_2=70%, C_3=10%; (B) θ_1=45°, θ_2= -14.5°, θ_3=9.93°, θ_p=0°; (C) φ_1＝π/4, φ_2＝0.324π, φ_3＝−96.165π, φ_4＝0.531π; (D) θ_1=45°, θ_2= -69°, θ_3=81°, θ_p=0°

4. Conclusion

Based on the Cesaro means of Fourier series and the mathematical equivalence relationship between structural parameters of the lattice-form and the birefringent interleavers, a new method for designing planar lightwave circuit (PLC)-type lattice-form interleavers was proposed. All the structural parameters can be obtained by a one-step simple analysis, solving a simple equations-set and correspondence of structure parameters. As an example, a 25GHz 3-lattice-cascaded PLC-type interleaver is designed using this method. The parameters obtained can satisfy design requirement well.

Acknowledgements

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