Opportunistic Resource Allocation via Stochastic Network Optimization in Cognitive Radio Networks

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***Abstract*—In this paper, we develop an opportunistic scheduling policy for allocating spectrum in cognitive radio networks. We maximize the throughput utility of secondary users subject to maximum collision constraints with the primary users. Particularly, we consider a cognitive radio network with a subset of the secondary users desire to use the licensed channels of primary system in a stochastic environment. Based on Lyapunov technique, we formulate the above problem as a Lyapunov optimization problem on stability region of virtual and actual queues. Then, we propose an online flow control, scheduling and spectrum allocation algorithm that meets the desired objectives and provides explicit performance guarantees.**

Index Terms*—Cognitive Radio Network, Resource Allocation, Lyapunov Optimization.

I. Introduction

Cognitive radio network (CRN) emerged as a mean to improve the wireless spectrum utilization in future wireless networks [1]-[3]. In which, cognitive radio (CR) is a technique that allows a secondary user (SU) to access the licensed spectrum (allocated for primary users (PUs)). However the SU's access has a lower-priority than PU, thereby significantly improving overall spectrum utilization. In overlay CRN, the SU has to vacate the occupied channel immediately whenever a PUs transmission on the same channel is detected as the PU always takes precedence over SUs to access the licensed spectrum band [4]-[6].

In this paper, we consider an overlay cognitive radio network with multiple SUs which utilize licensed channels from a primary network with multiple PUs. Packets arrive randomly at the SUs and are queued for transmission. The PUs are the licensed owners of the channels, they transmit their data to receivers whenever they have data to send. The SUs do not have any licensed spectrum and seek to transmit opportunistically on the primary channels. Therefore, a SU cannot transmit its own data when the channel is busy. When a SU wants to transmit its data, initially it senses and detects an idle channel from a set of licensed channels, and then it selects one idle channel for transmitting data. But in fact, the sensing results of the SUs can be received without the ideal channel state information. Consequently, the SU's transmission can be conflicted with the PUs.

In this work, we use a novel alternative approach that overcomes these limitations. We first transform the problem [3] into a sequence of online unconstrained stochastic shortest path problems, using a ratio rule for Lyapunov optimization. After that, we use the techniques of adaptive queueing and Lyapunov Optimization [8] to design an online flow control, scheduling and spectrum allocation algorithm for a secondary network. Moreover, these algorithms maximizes the utility of SUs subject to maximum rate of collisions with the PUs. We develop a simple Lyapunov drift technique that achieves system stability and performance optimization simultaneously [8], [9] -[11].

The remaining of the paper is organized as follow: Section II introduces the network model and defines own problem. In Section III, we propose and analyze the dynamic algorithm based on the Lyapunov Optimization theory. In Section IV, we analyze the performance of the proposed algorithm. Numerical results are illustrated in Section V. Finally, we conclude our work in section VI.

II. Network model and problem definition

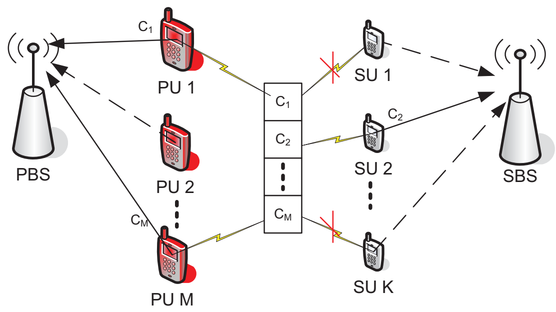

Fig. 1. System model

We consider an infrastructure based CRN consisting of three sets, let $\mathcal{K}$ be the set of SUs, $\mathcal{M}$ be the set of PUs, and both of them share a common set of M orthogonal channels as shown in figure 1. Let K and M respective be the size of these sets. Moreover, we assume that the CRN operates in slotted time with time slots $t \in \{0, 1, 2, ...\}$. Specifically, let $S_m(t)$ represent the state of the channel m on slot t. The availability of channels of PUs are characterized as two-state ergodic Markov Chain with idle probability π_m. We assume that π_m is obtained by secondary base station (SBS) through a knowledge of the traffic statistics and/or the channel sensing. Here $S_m(t) \in \{0, 1\}$ with the interpretation that $S_m(t) = 0$ if channel m is occupied by PU in timeslot t and $S_m(t) = 1$ if PU is idle in timeslot t. The steady state probability is represented by $\pi_m = Pr[S_m(t) = 1]$.

Define $x_k(t)$ as the total number of packets that SU k ($k \in \mathcal{K}$) transmits on slot t. Define an allocation process $\phi_{km}(t)$ as follows:

$$\phi_{km}(t) = \begin{cases} 1 & \text{if channel } m \text{ is allocated to SU } k \text{ at slot } t, \\ 0 & \text{otherwise.} \end{cases}$$

and the "collision" variable $C_m(t)$ for the PU can be devised as: $C_m(t) > 0$ if there is a collision with the PU in the channel m at slot t, $C_m(t) = 0$ otherwise.

For a given control algorithm, let $\bar{x}_k$, $\bar{\phi}_{km}$ and $\bar{c}_m$ represent the time average of the $x_k(t)$, $\phi_{km}(t)$ and $C_m(t)$ processes for all $k \in \mathcal{K}$ and $m \in \mathcal{M}$:

$$\bar{x}_k = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} x_k(\tau) \quad \bar{\phi}_{km} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \phi_{km}(\tau)$$

$$\bar{c}_m = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} C_m(\tau)$$

These limits are temporarily assumed to exist. The value $\bar{x}_k$ is the time average transmission rate of SU k, and $\bar{\phi}_{km}$ is the fraction of time that a given channel m is allocated to SU k, and $\bar{c}_m$ is the time average collision rate of PU m.

The collision variable $C_m(t)$ can be expressed in terms of the channel idle probability π_m and allocation process $\phi_{km}(t)$ as follows [3]:

$$C_m(t) = \sum_{k\in\mathcal{K}} c_m \phi_{km}(t)(1 - \pi_m), \tag{1}$$

where c_m is denoted to the fix link capacities of channel m.

The infinite horizon utility problem of interest is thus:

$$\text{Maximize:} \quad \sum_{k\in\mathcal{K}} U_k(\bar{x}_k) \tag{2}$$

Subject to:

$$\bar{x}_k \leq \sum_{m\in\mathcal{M}} c_k \pi_m \bar{\phi}_{km}, \quad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{k\in\mathcal{K}} \phi_{km}(t) \leq 1, \sum_{m\in\mathcal{M}} \phi_{km}(t) \leq 1, \quad \forall k, m \tag{4}$$

$$\bar{c}_m \leq \rho_m c_m, \quad \forall m \in \mathcal{M}. \tag{5}$$

where for each $k \in \mathcal{K}$, $U_k(x)$ are given concave functions and $\rho_m c_m$ is maximum packet collision rate that channel m can tolerate. The value $U_k(\bar{x}_k)$ represents the *utility* associated with SU k when using the channel of PU to transmit at rate $\bar{x}_k$. Fix link capacities of SUs and PUs (we use the terms capacity of PU and capacity of channel interchangeably) are denoted by c_k and c_m respectively. The constraint (3) ensures that the average transmission rate on PU channel cannot exceed the channel's capacity. The constraint (4) allows at most one SU to be allocated to one channel and at most one channel to be allocated to one SU at any time slot t, so the collision between two SUs is eliminated in our model. (5) ensures the collision maximum tolerance of all channels is guaranteed.

III. THE DYNAMIC ALGORITHM

The problem (2)-(5) can be solved via the stochastic network optimization theory [8]. This problem involves a function of a time average. It does not conform to the structure required for the Lyapunov drift-plus-penalty framework in [8]. We must transform the problem (2)-(5) to the problem that involves only time average (not function of time average), so that the Lyapunov framework can be applied.

Lemma 1: (Equivalent Transformation) The problem (2)-(5) is equivalent to the following transformed problem:

$$\text{Maximize:} \quad \sum_{k\in\mathcal{K}} \overline{U_k(\gamma_k)} \tag{6}$$

Subject to:

$$\bar{\gamma}_k \leq \bar{x}_k, \quad \forall k \in \mathcal{K} \tag{7}$$

$$\bar{x}_k \leq \sum_{m\in\mathcal{M}} c_k \pi_m \bar{\phi}_{km}, \quad \forall k \in \mathcal{K} \tag{8}$$

$$\sum_{k\in\mathcal{K}} \phi_{km}(t) \leq 1, \sum_{m\in\mathcal{M}} \phi_{km}(t) \leq 1, \quad \forall k, m \tag{9}$$

$$\bar{c}_m \leq \rho_m c_m, \quad \forall m \in \mathcal{M} \tag{10}$$

$$0 \leq \gamma_k(t) \leq x_k^{max}. \tag{11}$$

where $\gamma_k(t)$ is an auxiliary variable, and $\overline{U_k(\gamma_k)}$ is defined as the time average of the process $U_k(\gamma_k(t))$. The auxiliary variables $\gamma_k(t)$ act as proxies for the actual transmission rate variables $x_k(t)$.

A. Virtual Queues

To facilitate satisfaction of the constraint (7), for each $k \in \mathcal{K}$ define a *virtual queue* $Q_k(t)$ with dynamics:

$$Q_k(t+1) = \max[Q_k(t) + \gamma_k(t) - x_k(t), 0]. \tag{12}$$

The update (12) can be interpreted as queueing equation where $\gamma_k(t)$ is the arrival data of SU k on slot t, and $x_k(t)$ is the transmission data. Stabilizing $Q_k(t)$ ensures $\bar{\gamma}_k \leq \bar{x}_k$.

To satisfy the constraints (8), for each $k \in \mathcal{K}$ define a *virtual queue* $Z_k(t)$ with dynamics:

$$Z_k(t+1) = \max[Z_k(t) + x_k(t) - \sum_{m\in\mathcal{M}} c_k \pi_m \phi_{km}(t), 0]. \tag{13}$$

The intuition is that $x_k(t)$ can be viewed as the "arrivals" on slot t, and $\sum_{m\in\mathcal{M}} c_k \pi_m \phi_{km}(t)$ can be view as the "offered service" on slot t. Stabilizing virtual queue $Z_k(t)$ ensures the time average of the "arrivals" is less than or equal to the time average of the "service", which ensures constraints (8).

We define the collision queue $H_m(t)$ for each channel m as follows [11]

$$H_m(t+1) = \max[H_m(t) - \rho_m c_m + C_m(t), 0]. \tag{14}$$

where $C_m(t)$ is the collision variable for channel m defined in the previous section. Stabilizing collision queue $H_m(t)$ ensures the constraint of collision maximum tolerance (10).

B. The Drift-Plus-Penalty Algorithm

Define the following quadratic function $L(t)$:

$$L(t) \triangleq \frac{1}{2}\left[\sum_{k\in\mathcal{K}} Q_k(t)^2 + \sum_{k\in\mathcal{K}} Z_k(t)^2 + \sum_{m\in\mathcal{M}} H_m(t)^2\right]$$

Intuitively, taking actions to push $L(t)$ down tends to maintain stability of all queues. Define $\Delta(t)$ as the drift on slot t:

$$\Delta(t) \triangleq L(t+1) - L(t)$$

Let $\mathbf{\Theta}(t) = (Q_k(t), Z_k(t), H_m(t))|_{k\in\mathcal{K}, m\in\mathcal{M}}$ be the vector of all virtual queue values on slot t. The algorithm is designed to observe the queues and the current state of the channel on each slot t, and then choose $x_k(t)$ and $\gamma_k(t)$ subject to $0 \leq \gamma_k(t) \leq x_k^{max}$ to minimize a bound on the following *drift-plus-penalty expression* [8]:

$$\Delta(t) - V \sum_{k \in \mathcal{K}} U_k(\gamma_k(t))$$

where V is a non-negative weight that affects a performance bound. It is obvious that, the value of V affects the extent to which our control action on slot t emphasized utility optimization in comparison to drift minimization.

Lemma 2: Under any control algorithm, we have:

$$\begin{aligned} \Delta(t) - V \sum_{k \in \mathcal{K}} U_k(\gamma_k(t)) \leq B(t) - V \sum_{k \in \mathcal{K}} U_k(\gamma_k(t)) \\ + \sum_{k \in \mathcal{K}} Q_k(t)[\gamma_k(t) - x_k(t)] \\ + \sum_{k \in \mathcal{K}} Z_k(t)[x_k(t) \\ - \sum_{m \in \mathcal{M}} c_k \pi_m \phi_{km}(t)] \\ + \sum_{m \in \mathcal{M}} H_m(t)[C_m(t) - \rho_m c_m] \end{aligned} \quad (15)$$

where $B(t)$ is defined:

$$\begin{aligned} B(t) \triangleq & \frac{1}{2} \sum_{k \in \mathcal{K}} [\gamma_k(t) - x_k(t)]^2 \\ & + \frac{1}{2} \sum_{k \in \mathcal{K}} [x_k(t) - \sum_{k \in \mathcal{K}} c_k \pi_m \phi_{km}(t)]^2 \\ & + \frac{1}{2} \sum_{m \in \mathcal{M}} [C_m(t) - \rho_m c_m]^2 \end{aligned}$$

The value of $B(t)$ can be upper bounded by a finite constant B every slot, where B depends on the maximum possible values that $x_k(t)$, $\gamma_k(t)$ and $C_m(t)$ can take.

The algorithm is derived by identifying the factors that involve decision variables $\gamma_k(t)$, $x_k(t)$ and $\phi_k(t)$ in the last four terms on the right-hand-site of (15).

The algorithm below is defined by observing the queues states and channel state $S_m(t)$ every slot t, and choosing actions to minimize the last four terms on the right-hand-side of (15) (not including the first term $B(t)$), given these observed quantities. Using definitions of $C_m(t)$ in (1) leads to the following on each slot t:

Algorithm 1 The Drift-Plus-Penalty Algorithm

- (Auxiliary Variables) Every slot t, each SU $k \in \mathcal{K}$ observes $Q_k(t)$ and chooses $\gamma_k(t)$ as the solution to:

$$\begin{aligned} \text{Maximize} \quad & VU_k(\gamma_k(t)) - Q_k(t)\gamma_k(t) \\ \text{Subject to} \quad & 0 \leq \gamma_k(t) \leq x_k^{max} \end{aligned} \quad (16)$$

- (Flow Control) Every slot t, each SU $k \in \mathcal{K}$ observes $Q_k(t)$ and $Z_k(t)$ and choose $x_k(t)$ to maximize:

$$\begin{aligned} \text{Maximize} \quad & (Q_k(t) - Z_k(t))x_k(t) \\ \text{Subject to} \quad & x_k(t) \leq x_k^{max} \end{aligned} \quad (17)$$

- (Scheduling) The SBS observes all queues $(\boldsymbol{Q}(t), \boldsymbol{Z}(t), \boldsymbol{H}(t))$ and channel state $S(t)$ on slot t, and chooses vector $\boldsymbol{\phi}(t)$ to maximize:

$$\begin{aligned} \text{Maximize} \quad & \sum_{k \in \mathcal{K}} Z_k(t) \sum_{m \in \mathcal{M}} c_k \pi_m \phi_{km}(t) \\ & - \sum_{m \in \mathcal{M}} H_m(t) \sum_{k \in \mathcal{K}} c_m \phi_{km}(t)(1 - \pi_m) \\ \text{Subject to} \quad & \sum_{k \in \mathcal{K}} \phi_{km}(t) \leq 1, \sum_{m \in \mathcal{M}} \phi_{km}(t) \leq 1 \end{aligned} \quad (18)$$

- (Queue updates) Update virtual queues $Q_k(t)$, $Z_k(t)$ and $H_m(t)$ for all $k \in \mathcal{K}$ via (12), (13) and (14).

IV. SIMULATION

We consider a CRN that consists of 5 SUs, each of them has an opportunity to access to 9 orthogonal channels which are serving 9 PUs. Link capacities of all SUs and PUs are chosen randomly, from a uniform distribution on $[0, 1]$. We choose the utility function of SUs $U_m(x_m) = ln(1 + \nu_k x_m)$. The QoS constraint ρ_m is set to 0.2 for all channels. The Hungarian algorithm [12] is used to solve (21). We vary different values of V = 1, 5, 10, 15, 20 for the comparison. In order to show that our algorithm can adapt to the change of traffic statistics, we consider two cases: high and low channel-occupancy of PUs, where the channel-idle probability $\boldsymbol{\pi}$ is assumed to have a uniform distribution on $[0.1, 0.3]$ and $[0.7, 0.9]$ respectively.

Fig.2 represents the resulting average throughput of 5 SUs

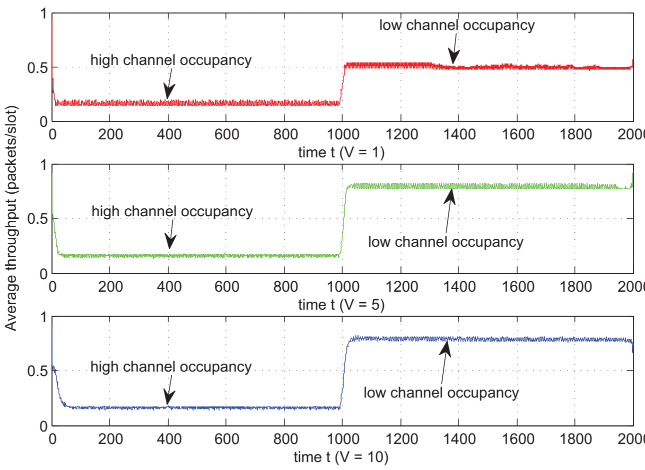

Fig. 2. Average throughput per-secondary user versus time (5 users, 9 channels) in three cases of V. Each SU has a maximum transmission rate of 1 packet/slot.

in three cases of $V = 1, 5, 10$ and $x_k^{max} = 1$. At the beginning, we assume that the network is under high channel-occupancy condition, after 1000th timeslot, the network state changes to the low channel-occupancy condition leading to the increase of SUs throughput. In case of $V = 1$ the average throughput is lower than in case of $V = 5$ and $V = 10$ which illustrates the proportion of utility function with V values, and it converges to an optimal value when V's value gets large enough.

Fig.3 shows that the average values of $Q_k(t)$ and $Z_k(t)$ never exceed the worst-case guarantee. Intuitively, this figure also shows that when the channels are under the low occupancy condition the queue backlogs will decrease in comparison with the high channel-occupancy condition.

In Fig 4, we plot the average throughput achieved by the secondary users. It can be seen that the average throughput increases with V and converges to the optimal with the difference exhibiting a $O(1/V)$ behavior [8]. In Fig. 5, we plot the average queue backlog of the secondary user over this period. It can be seen that the average queue backlog grows linearly in V.

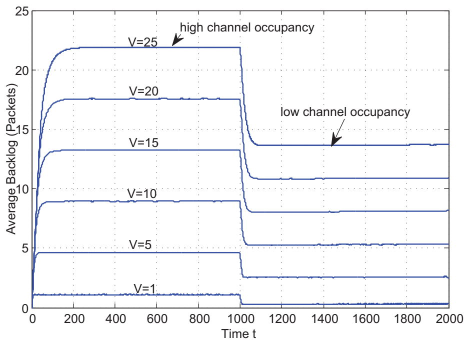

Fig. 3. Average queue backlog versus time with different V values. The values of queues never exceed the deterministic bound for all time by constant that is proportional to V.

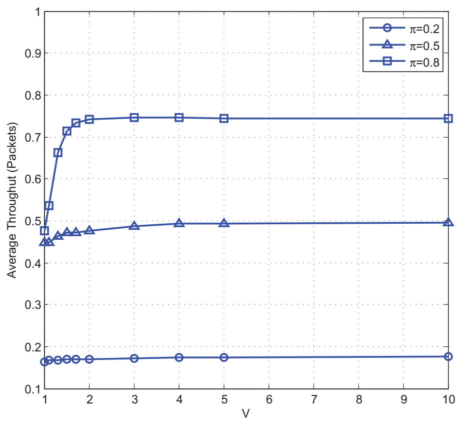

Fig. 4. Average throughput versus V for channel idle probability $\pi = 0.2$, $\pi = 0.5$ and $\pi = 0.8$.

V. Conclusion

This paper considered the resource allocation problem for secondary network in the CRN. An algorithm is developed for choosing policies and scheduling on each timeslot to maximize a concave functions of the time average transmission rate vector of SUs, subject to capacity of channel and maximum collision tolerance of PUs. The proposed algorithm is based on Lyapunov optimization concepts and involves minimizing a *drift-plus-penalty* over each timeslot. Our results reveal that using this technique we achieve optimal throughput values along with network stability.

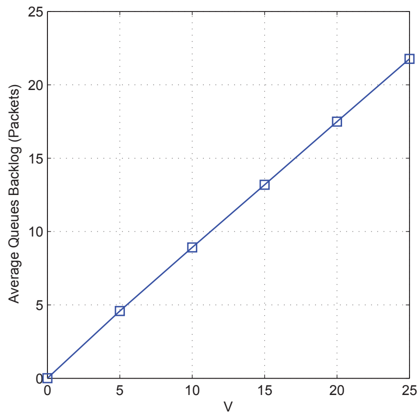

Fig. 5. Queue backlog versus V, demonstrating the $O(V)$ behavior.

VI. Acknowledgement

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