Design of Dual-band E-shaped Patch Antennas by Using the Multi-Conductor Transmission Line Mode Theory

#Keisuke Noguchi [1], Harish Rajagopalan [2], Yahya Rahmat-Samii [2]
[1] Department of Information and Communication Engineering, Kanazawa Institute of Technology
7-1 Ohgigaoka, Nonoichi, Ishikawa, 921-8501, Japan, noguchi@neptune.kanazawa-it.ac.jp
[2] Department of Electrical Engineering, University of California, Los Angeles
420 Westwood Plaza, Los Angeles, CA 90095-1594, USA, harish@ee.ucla.edu

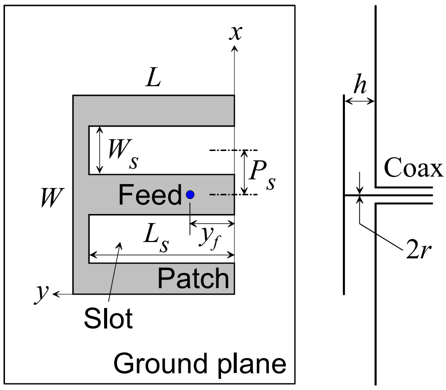
1. Introduction

Microstrip patch antennas are extremely popular in wireless systems due to their low-profile structure. They form attractive solutions for various applications like handheld devices and satellite communications. Some of the inherent advantages of these antennas are light weight, low profile nature, low fabrication cost, mechanically robust, conformality to host surface and easily integration with microwave integrated circuits (MICs) [1]. Some of the disadvantages of these antennas include narrow bandwidth, low efficiency and surface wave excitation [1]. Conventional approaches to increase the bandwidth of a microstrip patch include use of stacked patches or using parasitic elements.

Due to the size constraints and the need for single-patch design solutions in wireless systems, the E-shaped patch antenna presents an attractive single-patch dual-band design [2]. Theoretical analysis gives great insight into the design of these E-shaped patch antennas. This paper proposes a new circuit model for an E-shaped patch antenna for dual-band operation using the multi-conductor transmission line mode theory [3]. From mode theory, it can be assumed that the E-shaped patch antenna forms three-conductor transmission line system and thus has three transmission line modes. Based on these three transmission line modes, an equivalent circuit model is developed. The E-shaped patch antenna is analyzed for dual-band operation. Finally, theoretical impedance characteristics obtained from circuit model are compared with those obtained from full wave 3D electromagnetic analysis.

2. Mode theory for three-conductor transmission line system

The E-shaped patch antenna consists of a microstrip patch with two slots along the length of patch as shown in Fig. 1. A coaxial feed probe connects to the center of the patch. The parameters designating the patch length, patch width, slot length, slot width and thickness of the substrate are given by L, W, L_s, W_s and h respectively. P_s is the distance between patch and slot centers and y_f is the distance of the feed probe from the edge. Air substrate is assumed along with metallic copper conductor for the patch and the ground plane. Due to the presence of the slots, this antenna can be considered as three parallel strips on a ground plane. This forms a multi-conductor transmission line system which can be analyzed using mode theory.



Top view Side view

Figure 1: Geometry of the E-shaped patch antenna

Three-conductor transmission line on a ground plane can be analyzed with the following matrix equation

$$[Q]=[Z][Y] \qquad (1)$$

In the above equation, $[Z]$ and $[Y]$ represent impedance and admittance matrices, and their matrix elements Z_{mn} and Y_{mn} (m, n = 1, 2, 3) are self or coupling impedance and admittance, respectively. Once matrix $[Q]$ is obtained, an eigenvalue problem can be solved with numerical method and eigenvector matrix $[A]$ can be generated. Then, eigenvalue matrix $[P]$ corresponding to the propagation constant matrix $[\gamma^2]$ in each mode is obtained by the following operation.

$$[A]^{-1}[Q][A]=[P]=[\gamma^2] \qquad (2)$$

Using matrix operations, characteristic impedance in modes $[z_0]$ and $[y_0]$ can be also obtained.

Due to the mode theory, an E-shaped patch antenna works in three modes as shown in Fig. 2. Mode 1 corresponds to typical patch antenna mode because currents flow on the three strips in phase and anti-phase currents flow on the ground plane as shown in Fig. 2 and this mode contributes to radiation. Modes 2 and 3 are transmission line modes which work on the patch surface. Since the feed point is located at the center conductor of the antenna, mode 2 does not operate (currents cancel each other due to symmetric structure). In mode 3, current flows on all conductors and it works like a coplanar waveguide. This interpretation creates an equivalent circuit as shown in Fig. 3. A typical patch antenna equivalent circuit consists of radiation impedance and probe reactance, however, the equivalent circuit for E-shaped patch antenna includes transmission line impedance in series to radiation impedance. Therefore, we can use the transmission line impedance in mode 3 to design dual band E-shaped patch antenna.

3. Design of dual band E-shaped patch antennas with equivalent circuit

In order to design dual band E-shaped patch antennas, equations with respect to resonant dual frequencies are derived using the equivalent circuit. Input impedance of an E-shaped patch antenna Z_{in} can be expressed as

$$Z_{in}=j(X_p-X_s)+\frac{R}{1+jQ\left(\frac{f}{f_0}-\frac{f_0}{f}\right)} \qquad (3)$$

where, X_p and X_s represent reactance of the probe and impedance Z_s in mode 3 respectively. R, Q, and f_0 are radiation resistance, quality factor, and resonant frequency [1]. Frequency variation can be expressed with the following first order approximation.

$$\frac{f}{f_0}-\frac{f_0}{f}\approx 2x\,. \qquad (4)$$

After estimating Z_{in} as a function of x, conditions for dual band resonance are

(i) if $x=0$, then $f=f_0$, VSWR $S=S_m$,

$$R=S_mZ_0 \qquad (5)$$

(ii) if $x=\pm x'$, $S=1$,

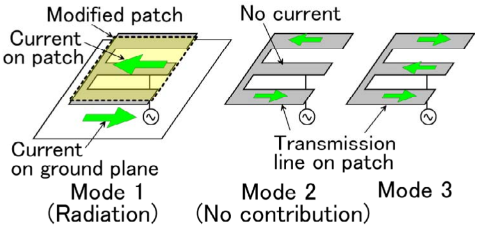
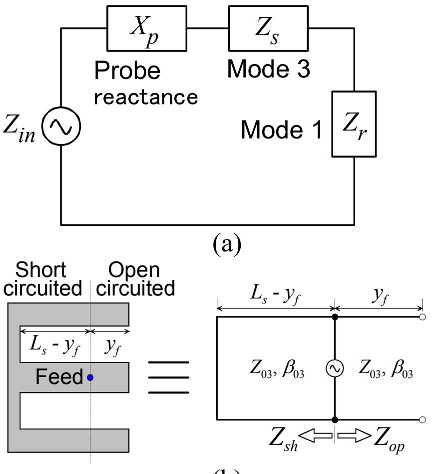


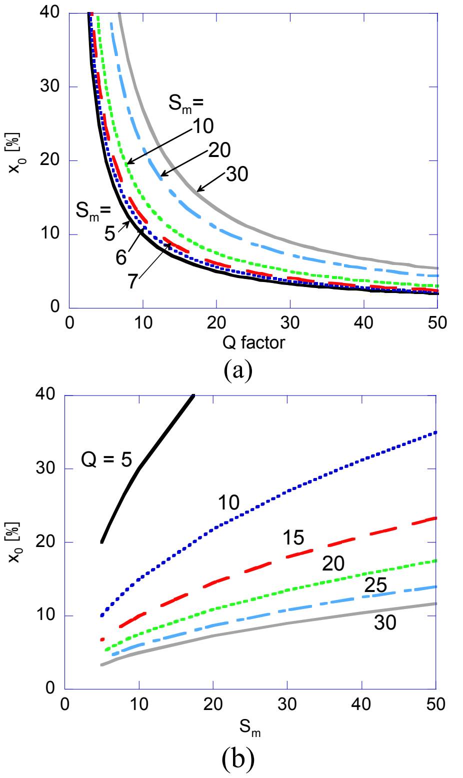
Figure 2: Each mode of the E-shaped patch antenna



(a)

(b)

Figure 3: An equivalent circuit of the E-shaped patch antenna, (a) whole circuit, (b) circuit in mode 3



(a)

(b)

Figure 4: Frequency variation ratio, (a) x_0 vs. Q, (b) x_0 vs. S_m

$$\frac{R}{1+x'^2Q^2}=Z_0. \qquad (6)$$

By using the above equations, frequency variation ratio is given by

$$x'=\pm\frac{\sqrt{S_m-1}}{2Q} \qquad (7)$$

then, we can have lower and upper resonant frequencies rather than f_0. That is, let $x_0=|x'|$,

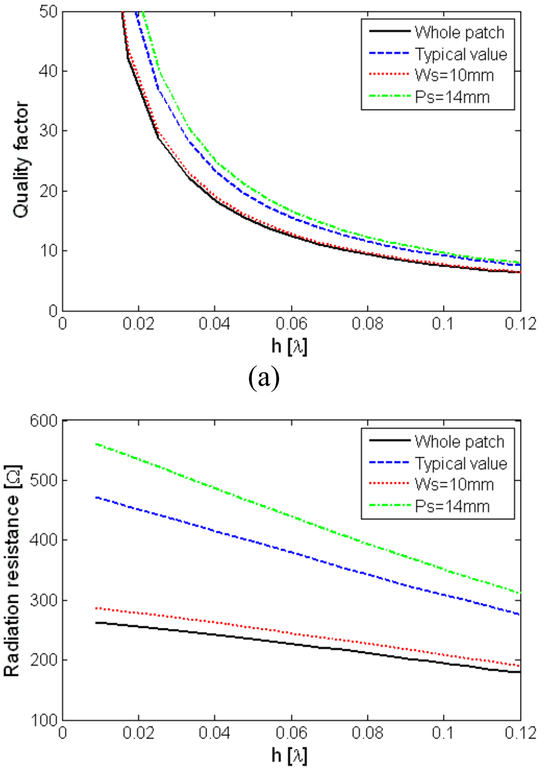
$$f_{01}=f_0(1-x_0),\ f_{02}=f_0(1+x_0). \qquad (8)$$

The frequency variation ratio versus Q and S_m is shown in Fig. 4. Large value of x_0 can be obtained for smaller Q factor and large S_m as shown in Fig. 4 (a). Once S_m is decided as a certain value, Q factor gives a limitation for maximum possible x_0 variation. For example, when values of S_m and Q are equal to both of 10, possible x_0 can be 15% which means for f_0 = 2GHz, f_{01} = 1.7GHz and f_{02} = 2.3GHz can be achieved.

An E-shaped patch antenna has three strips instead of a whole surface patch. Hence, we need to investigate the Q factor and radiation resistance relation in mode 1. Since three coupled strips can be treated as one strip using the equivalent radius principle [4], equations to calculate Q factor and radiation resistance of an equivalent whole patch (modified patch in Fig. 2) are available [1]. Now, we consider a patch antenna that has a resonance at 2GHz, we assume typical values of parameters for the E-shaped patch antenna as shown in Table 1. Based on the table, variations of Q factor and radiation resistance versus height h for different types of the patch were calculated. As shown in Fig. 5, Q factor for the different types of the patch is slightly larger than that of a whole patch, while radiation resistance becomes larger for E-shaped patch than whole patch. For radiation resistance, we can adjust feed point and change the resistance.

Table1: Typical values of the dual band E-shaped patch antenna parameters

Parameter	Value [mm]
L	55
W	50
h	15
L_s	45
W_s	20
P_s	12
y_f	10
r	0.5



(a)

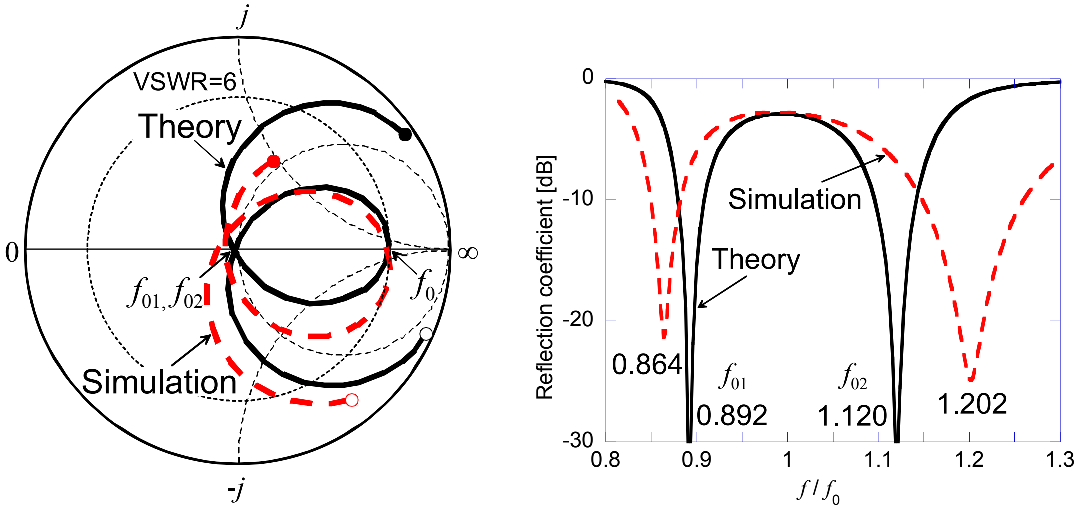
(b)

Figure 5: Variation of (a) Q factor and (b) radiation resistance for different patches.

4. Results

As discussed above, S_m and Q factor limit frequency variation ratio x_0. For x_0 nearly equal to 10%, we assume S_m = 6 (half power reduction, reflection coefficient corresponds to -2.92dB) and Q = 10. Parameters of radiation impedance (mode 1) in the equivalent circuit are assumed as $R = S_mZ_0$ = 300Ω and Q = 10. Probe reactance X_p was computed as 170Ω at f_0. With respect to the transmission line mode (mode 3), characteristic impedance Z_{03} = 200Ω, transmission line lengths L_s-y_f = 0.39λ_0 (λ_0 represents wavelength at f_0), and y_f = 0.01λ_0 were computed based on the E-shaped antenna's parameters. Impedance characteristics calculated using the equivalent circuit are shown in Fig. 6. Simulations were implemented with a full wave 3D electromagnetic simulator, Ansoft HFSS [5], and simulated results which considered the parameters of the equivalent circuit are also plotted in Fig. 6. In the simulation, same values of L, W, and h as shown in Table 1 were used, while different values were used for L_s = 51mm, W_s = 23mm, P_s = 12.5mm, y_f = 13.5mm, and r = 0.2mm.

As shown in Fig. 6 (a), the contour of the theoretical impedance passes through the center point at lower and higher frequency f_{01} and f_{02}. Reflection coefficient as shown in Fig. 6 (b) has complete dual band characteristics with $f_{01} = 0.892f_0$ and $f_{02} = 1.120f_0$ obtained from circuit model, while $f_{01} = 0.864f_0$ and $f_{02} = 1.202f_0$ (f_0 is 2GHz in theory and simulation) using full wave electromagnetic simulation. The difference comes mainly from deviation of impedance corresponding to VSWR = 1 for f_{01} and f_{02} in the simulation results. Additionally, radiation resistance and Q factor have slight difference between theory and simulation results.



(a) (b)

Figure 6: Impedance characteristics of the equivalent circuit, (a) impedance (start frequency = $0.8f_0$ = 1.6GHz, stop frequency = $1.3f_0$ = 2.6GHz), (b) reflection coefficient vs. normalized frequency

5. Conclusions

This paper proposed a new circuit model for an E-shaped patch antenna for dual-band operation using the multi-conductor transmission line mode theory. An equivalent circuit was developed for the E-shaped patch antenna, and equations with relation to resonant frequency were derived. Based on the equations, possible frequency variation was discussed. Finally, a dual-band E-shaped patch antenna was designed with the equivalent circuit, and the results were compared with full-wave simulations.

Acknowledgments

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