Numerical Analysis of Electromagnetic Scattering Using Constrained Interpolation Profile Method

Jerdvisanop Chakarothai, Qiang Chen, Kunio Sawaya
Department of Electrical and Communication Engineering, Tohoku University
6-6-05 Aoba-aza, Aoba-ku, Sendai, Miyagi, Japan 980-8579
{jerd, chenq, sawaya}@ecei.tohoku.ac.jp

1. Introduction

The field of computational electromagnetics (CEM) was significantly broadened by the pioneer work of Yee, when the time-dependent Maxwell's equations are solved numerically in an isotropic medium [1]. This algorithm is called finite-difference time-domain (FDTD) Method, and it has second-order accuracy in temporal and spatial domain [2]. Since FDTD method is relatively simple to implement, efficient and robust for many types of problems, it has been widely used nowadays. However, the conventional Yee's FDTD method suffers from the numerical dispersion or the anisotropy: the numerical velocity of propagation is dependent on the mesh size, time step size, and the direction of propagation. This anisotropy causes the accumulative phase error which reduces the global accuracy when analysis domain is relatively large.

Recently, the application of the characteristic-based constrained interpolation profile (CIP) method to the field of CEM was proposed by Yabe et al. [3]. CIP method can accurately solve hyperbolic equations with third-order accuracy in space. Okubo [4] showed that CIP method provides higher accuracy comparing with FDTD method under the condition of the same cell size. He also concluded that CIP method requires less memory and less calculation time for the same accuracy. However, to the best of authors' knowledge, the three-dimensional scattering analysis using CIP method has not been reported so far. This paper proposes an approach utilizing CIP method to solve the EM scattering problem and shows the algorithms that enable the analysis of three-dimensional problems including perfect conducting (PEC) objects and dielectric objects. The radar cross sections (RCS) of PEC sphere and dielectric sphere are calculated to demonstrate the possibility to use CIP method as alternative CEM tools.

2. Formulation of Maxwell's Equations

The Maxwell's equations in free space are expressed in the conservative form in the Cartesian coordinate as

$$\frac{\partial W}{\partial t}+\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}=0\,, \qquad \ldots (1)$$

where $W=\begin{pmatrix}E_x & E_y & E_z & H_x & H_y & H_z\end{pmatrix}^T$, $F_x = AW$, $F_y = BW$, and $F_z = CW$.

The coefficient matrix *A*, *B*, *C* represents the property of the medium and their expressions involve permittivity ε and permeability μ of medium as matrix elements [5].

The eigenvalues of coefficient matrices are found to be zero and $\pm c$, where *c* denotes the velocity of the electromagnetic wave in medium. The eigenvalues have multiplicities and hence the corresponding eigenvectors are not unique. However, in the Cartesian coordinate, linearly independent equations of the system still have been found by the orthogonalization of coefficient matrices. When the similarity matrices of diagonalization are known, the coefficient matrices can be diagonalized individually, and by a straight-forward matrix multiplication, the diagonalized matrices are expressible as

$$D_x = S_x^{-1} A S_x\,,\ D_y = S_y^{-1} B S_y\,,\ D_z = S_z^{-1} C S_z \qquad \ldots (2)$$

where the diagonal elements of D_x, D_y and D_z are equal to the eigenvalues of A, B and C, respectively, i.e.

$$\mathrm{Diag}(D_m) = \{c, c, -c, -c, 0, 0\}, \quad m = x, y, z \qquad \dots (3)$$

S_m in (2) denotes a non-singular similar matrix composed of the eigenvectors as the column vectors and S_m^{-1} is the inverse matrix. Similar matrices associated with the coefficient matrices A, B and C are found in [5]. Since each of the coefficient matrices can be diagonalized in x, y, and z axis, the three-dimensional problem is split into three one-dimensional systems as

$$\frac{\partial\left(S_m^{-1}W\right)}{\partial t} + D_m \frac{\partial\left(S_m^{-1}W\right)}{\partial m} = 0, \quad m = x, y, z \qquad \dots (4)$$

The Riemann invariants or the characteristic variables $S_m^{-1}W$ hold a constant value along a trajectory in time and space, with a slope defined by the each eigenvalue. Since every equation is completely uncoupled each other, the system of equations can be solved individually as one-dimensional problem, and the CIP method is applied to solve the system of equations. The details of CIP method are described in [4] and will be omitted here. It should be noted that the calculation procedure here is different from previous studies which used the finite-difference or finite-volume method to solve the system of equations [6]. The characteristic variables are advected forward or backward depending on the associated sign of their eigenvalues by CIP method. The update equations for electric and magnetic fields at a point x_i are consequently expressed in an explicit form as

$$E_y^{n+1} = \frac{1}{2}\left\{ E_{y+}^n + E_{y-}^n + \eta H_{z+}^n - \eta H_{z-}^n \right\} \qquad \dots (5)$$

$$H_z^{n+1} = \frac{1}{2}\left\{ \frac{E_{y+}^n}{\eta} - \frac{E_{y-}^n}{\eta} + H_{z+}^n + H_{z-}^n \right\} \qquad \dots (6)$$

where E_{y+}^n, H_{z+}^n are the field components which is located at x_i+$c\Delta t$ and propagating toward a point at x_i. Similarly, E_{y-}^n, H_{z-}^n are the field components which is located at x_i–$c\Delta t$ and propagating toward a point x_i, and η is the intrinsic wave impedance. Other field components can be found to be the similar form as above equations.

A. PEC boundary condition in CIP method

Figure 1 shows boundary between free space and PEC boundary in one-dimensional space. The incident plane wave is coming from the right region with velocity c. PEC boundary is placed at a middle plane between x_i and x_{i-1}. Then, the electric field at the boundary satisfies the following boundary condition:

$$\hat{\mathbf{n}} \times \mathbf{E} = 0, \qquad \dots (7)$$

where $\hat{\mathbf{n}}$ is the unit normal vector. Therefore the fields nearby the PEC boundary can be determined by creating an image of the electromagnetic field at location x_{i-1}, i.e.

$$E_t^n(x_{i-1}) = -E_t^n(x_i) \ \text{ and } \ H_t^n(x_{i-1}) = H_t^n(x_i). \qquad \dots (8)$$

Then, CIP method is applied to determine the field values at next time step n+1 according to equations (5) and (6).

B. Dielectric boundary condition in CIP method

Figure 2 shows boundary between two different media in one-dimensional space. The update equations for electromagnetic field can be easily derived and given by

$$E_y^{n+1} = \frac{\eta_1\eta_2}{\eta_1+\eta_2}\left\{ \frac{E_{y+}^n}{\eta_1} + \frac{E_{y-}^n}{\eta_2} + H_{z+}^n - H_{z-}^n \right\}, \qquad \dots (9)$$

$$H_z^{n+1} = \frac{1}{\eta_1+\eta_2}\left\{ E_{y+}^n - E_{y-}^n + \eta_1 H_{z+}^n + \eta_2 H_{z-}^n \right\}, \qquad \dots (10)$$

where η_1, η_2 are the intrinsic impedance in media #1 and #2, respectively.

3. Analysis Model

In order to confirm the validity of the proposed algorithms, the RCS of PEC sphere and dielectric sphere with 5 cm radius are calculated by using CIP method and compared with the results by FDTD method. The dielectric sphere has the relative permittivity ε_r of 2. Figure 2 shows the analysis model. The total size of analysis region is 30 cm× 30 cm×30 cm and cell size is 1 mm in each axis. In order to illuminate the plane wave on spheres, the total-field scattered-field (TF/SF) boundary [2] for FDTD model is applied. The perfect matched layer (PML) is also placed outside of the scattered field region. The Gaussian waveform with pulse width of τ_0=133 ns and attenuation constant of α=16/τ_0^2 as the incident plane wave. While an 8-layer PML is utilized to truncate the analysis region in FDTD model, direction of propagation is used instead of ABC to suppress the spuriously reflected waves at the truncated region in CIP model. The Courant-Friedrich-Levy (CFL) number is defined as $\mathrm{CFL} = c\Delta t / \Delta$, where Δ denotes the cell size and Δt is the time step. The maximum time step is restricted by Courant stability condition [2]. The maximum CFL number for FDTD model is about $(1/3)^{1/2}$= 0.577 but CFL number is unity for CIP model, which means that CIP method requires less time steps compared with FDTD method.

To calculate the RCS evaluated by the far-zone field, equivalent electric surface current $\mathbf{J}_s$ and magnetic surface current $\mathbf{M}_s$ located outside a cubic surface are used as shown in Figure 2. Since the outermost boundary of CIP model behaves like the first-order Mur's ABC in FDTD method, the reflected wave will appear and affect the accuracy of analysis. To avoid this problem, the equivalent current surface is placed far enough from the outermost boundary, i.e. 80 cells from the outermost boundary. Finally, the RCS is obtained by using these equivalent surface currents and free-space Green's function.

4. Numerical Results

Figures 3 and 4 show the RCS of PEC sphere and dielectric sphere, respectively, calculated by the proposed CIP method and FDTD method. Exact RCS obtained by Mie series is also plotted in Figure 3 and 4. Both results show good agreements with the Mie series. As can be seen in Figure 3, although the equivalent current surface is far from the outermost boundary in the CIP model, reflected wave is still observed and affect the RCS amplitude around $\theta \simeq 30^\circ - 60^\circ$. The RCS results of dielectric sphere are more accurate than PEC sphere because the reflected wave from the outermost boundary of CIP model is less than that in the case of PEC sphere.

5. Conclusion

The characteristic-based CIP method for solving three-dimensional Maxwell's equations has been successfully developed. The RCS results using proposed CIP method and FDTD method have been presented to show the validity of the proposed method. Thus, the possibility to utilize CIP method as analysis tool for solving electromagnetic problem has been demonstrated, which would broaden the field of CEM.

Acknowledgments

This work was partly supported by Tohoku University 21st Century Center of Excellent (COE) program. Parts of the numerical results in this research were obtained by using supercomputing resources at Information Synergy Center, Tohoku University.

References

[1] K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," IEEE Trans. Antennas Propagat., vol.14, no.4, pp.302-307, 1996
[2] A. Taflove, *Computational Electrodynamics, The Finite-Difference Time-Domain Method* Artech House Publisher, 1995

[3] Y. Ogata, T. Yabe, K. Odagaki, "An Accurate Numerical Scheme for Maxwell Equation with CIP-Method of Characteristics," Comm. Comput. Phys., vol.1, no.2, pp.311-335, April 2006

[4] K. Okubo, N. Takeuchi,"Analysis of an Electromagnetic Field Created by Line Current Using Constrained Interpolation Profile Method," IEEE Trans. Antennas and Propagat., Vol. 55, No. 1, Jan. 2007

[5] J. S. Shang, "A Fractional-Step Method for Solving 3D, Time-Domain Maxwell Equations", J. Comput. Phys., vol.118, pp.109-119, 1995

[6] J. S. Shang, "Characteristic-Based Algorithms for Solving the Maxwell Equations in the Time Domain", IEEE Antennas and Propagation Magazine, vol.37, no.3, June 1995

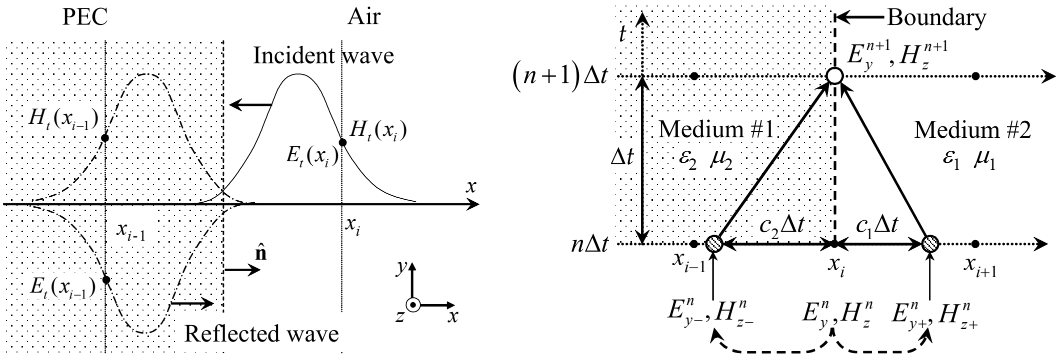

Figure 1: Boundary condition for PEC object (left) and dielectric object (right).

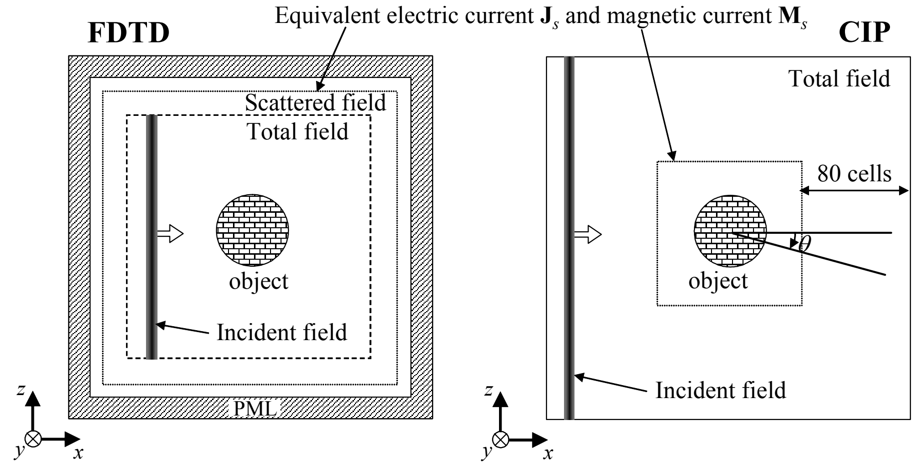

Figure 2: Analysis model for FDTD method and CIP method.

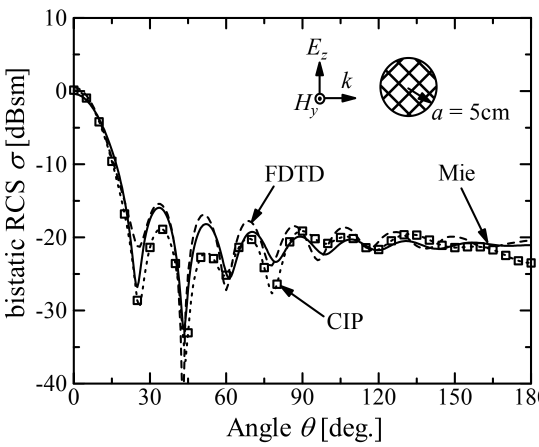

Figure 3: RCS of PEC sphere
with 5 cm radius at 10 GHz.

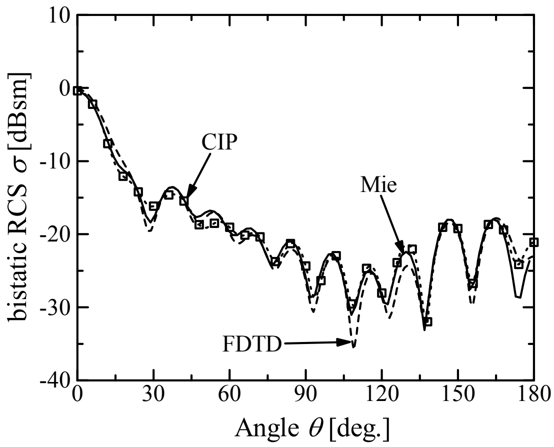

Figure 4: RCS of dielectric sphere
with 5 cm radius at 10 GHz.