Matched Antenna Design based on Spherical Mode Expansion

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1. Introduction

In Multiple-Input Multiple-Output (MIMO) communications, receivers and transmitters have multiple antennas. Smaller size of array antenna is required in the MIMO system, while efficiency of MIMO transmission should be maintained. Generally, mutual coupling between the antenna elements increases if the size of array antenna decreases, and it results in inefficiency of the MIMO system. In a conventional approach to make smaller array antenna, a decoupling element to remove the effect of mutual coupling and a polarization antenna have been used. However, these approaches have been considered independently, and moreover an optimal array antenna structure to achieve best performance of MIMO communication has not been found yet. In this paper, we will propose a new array antenna design scheme to achieve best performance under a given propagation environment. For that purpose, Spherical Mode Expansion (SME) of antenna directivity is introduced. In SME, an electrical and magnetic field is expanded into orthogonal spherical wave functions with their weights of Spherical Mode Coefficients (SMCs)[1][2]. In a fixed size or volume of array antenna, the number of effective SMCs to be radiated from the volume is limited. Therefore, optimal antenna directivity in a given volume of array antenna can be achieved by designing limited SMCs. If the SMCs are given, current distribution on the antenna surface can be determined. It is completely a new approach of applying SME to antenna design compared with conventional schemes of [3] and [4] where SME is used for analysis of antenna directivity.

The system model for antenna design using SME is shown in Fig.1. In this model, a propagation environment such as angular profile, and volume of target, in which the array antenna is designed, are given. The optimal SMCs in this environment can be estimated by applying conjugate matching theory in the angular domain. It is called as matched antenna in this paper. In the following, we will explain about the details of antenna design based on SME for Single-Input Single-Output (SISO) system and Single-Input Multiple-Output (SIMO) system. Antenna design for MIMO system is our future work.
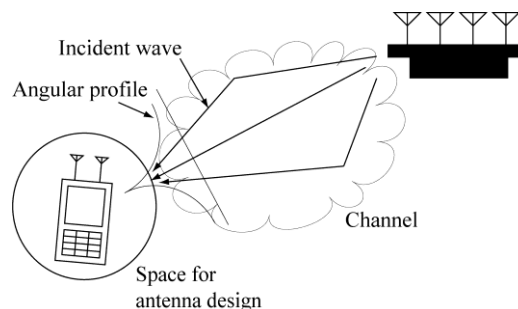

Figure 1: System model for matched antenna design using SME.

2. Spherical Mode Expansion (SME)

2.1 Far-field pattern functions

Spherical wave functions are canonical solutions of the Helmholtz equation in spherical coordinates. Spherical wave functions in far-field $\vec{K}_{smn}(\theta,\phi)$ $(s=1,2)$ can be expressed as follows

$$\vec{K}_{1mn}(\theta,\phi) = \sqrt{\frac{2}{n(n+1)}}\left(-\frac{m}{|m|}\right)^m e^{im\phi}(-i)^{n+1}\left\{\frac{im\overline{P}_n^{|m|}(\cos\theta)}{\sin\theta}\hat{\theta} - \frac{d\overline{P}_n^{|m|}(\cos\theta)}{d\theta}\hat{\phi}\right\}$$

$$\vec{K}_{2mn}(\theta,\phi) = \sqrt{\frac{2}{n(n+1)}}\left(-\frac{m}{|m|}\right)^m e^{im\phi}(-i)^n\left\{\frac{d\overline{P}_n^{|m|}(\cos\theta)}{d\theta}\hat{\theta} + \frac{im\overline{P}_n^{|m|}(\cos\theta)}{\sin\theta}\hat{\phi}\right\}, \qquad (1)$$

where θ and ϕ are angles of elevation and azimuth in spherical coordinates respectively, $\hat{\theta}$ and $\hat{\phi}$ are unit vectors for each direction. $\overline{P}_n^{|m|}(x)$ is the normalized associated Legendre function of $n(=1,2,3,...)$-th degree and $m(=-n,-n+1,...,0,...,n-1,n)$-th order. These functions are used for orthogonal basis for SME. The details of spherical wave function and far-field pattern function are given in [2].

2.2 Antenna directivity based on SME

Any antenna directivity $\vec{g}(\theta,\phi)$ can be expressed by using the far-field pattern functions of Eq.(1) as follows

$$\vec{g}(\theta,\phi) = \sum_{smn} Q_{smn}\vec{K}_{smn}(\theta,\phi), \qquad (2)$$

where Q_{smn} are the Spherical Mode Coefficients (SMCs). This expression is called as Spherical Mode Expansion (SME). In SME, an antenna directivity is expanded into orthogonal far-field pattern functions (orthogonal modes) with their weights of SMCs.

2.3 Truncation of SMC

For convenience, the indices of SME s, m, n are replaced by single index j. Since m is determined by n and $s=\{1,2\}$, the number of j depends on n, and is generally infinity. However, if the volume of the target, in which an array antenna is designed, is fixed, the number of n can be truncated at some $n=N$ with minimal error. A sufficient number of indices for convergence is given by

$$N = \lceil kr_0\rceil + n_1,$$

where k is the wave number, r_0 is the minimum radius of the volume for antenna design, and $\lceil kr_0\rceil$ indicates the largest integer smaller than or equal to kr_0. The integer n_1 controls the error of truncation, and approaches zero in the far-field pattern. Thus, the number of SMCs is limited if the volume of antenna design is fixed.

3. Optimal coefficients estimation

In the proposed approach, SMCs are designed to match the angular profile in a given propagation environment. The angular profile $P_h(\theta,\phi)$ is defined as $P_h(\theta,\phi) = E\left[|h(\theta,\phi,t)|^2\right]$, where $h(\theta,\phi,t)$ is a time variant channel response corresponds to the angles θ and ϕ. On this channel, uncorrelated scattering is assumed as $E\left[h(\theta_1,\phi_1,t)h^*(\theta_2,\phi_2,t)\right] = 0$ (if $\theta_1 \neq \theta_2$ or $\phi_1 \neq \phi_2$).

3.1 SISO model

As a first step, an optimal antenna for SISO system is designed by using SME. The optimal antenna for systems is defined as that to maximize the average Signal-to-Noise Ratio (SNR).

The received signal $y(t)$ of the SISO system is expressed as

$$y(t) = \int_\Omega g(\theta,\phi)h(\theta,\phi,t)s(t)d\Omega + n(t),$$

where $s(t)$ is the transmitted signal, $n(t)$ is the additive white Gaussian noise, and $g(\theta,\phi)$ is directivity of the receive antenna to be designed. The directivity is expanded by SME as in Eq.(2) and can be rewritten as $g(\theta,\phi) = \mathbf{k}^T(\theta,\phi)\mathbf{q}$, where $\mathbf{q} = [Q_1 Q_2 \cdots Q_J]^T$ is a SMCs vector and

$\mathbf{k} = \begin{bmatrix} \vec{K}_1 & \vec{K}_2 & \cdots & \vec{K}_J \end{bmatrix}^T$ is a far-field pattern functions vector. By using this expression, the channel gain G_h is expressed as follows

$$G_h = E\left[\left|\int_\Omega g(\theta,\phi)h(\theta,\phi,t)d\Omega\right|^2\right] = \int_\Omega \left\{\mathbf{q}^H \mathbf{k}^*(\theta,\phi)\mathbf{k}^T(\theta,\phi)\mathbf{q}\, E\left[|h(\theta,\phi,t)|^2\right]\right\}d\Omega = \mathbf{q}^H \mathbf{R}\, \mathbf{q},$$

where matrix $\mathbf{R}$, which corresponds to SMCs of the incoming propagation environment, is expressed as

$$\mathbf{R} = \int_\Omega \left\{\mathbf{k}^*(\theta,\phi)\mathbf{k}^T(\theta,\phi)P_h(\theta,\phi)\right\}d\Omega.$$

If the average noise power is defined as $P_n = E\left[|n(t)|^2\right]$, finally the received SNR is calculated as

$$\gamma_{siso} = G_h \frac{P_s}{P_n}.$$

where $P_s = E\left[|s(t)|^2\right]$ is the average transmitted signal power. Since it is quadratic form with respect to q, the maximum SNR can be solved easily as

$$\gamma_{siso}^{max} = \max(\gamma_{siso}) = \frac{\lambda_1 P_s}{P_n},$$

where λ_1 is the largest eigenvalue of $\mathbf{R}$. It means that the eigenvector corresponding to the largest eigenvalue contains optimal SMCs to maximize SNR of this environment. Hence, the optimal antenna for SISO system can be designed by using SME.
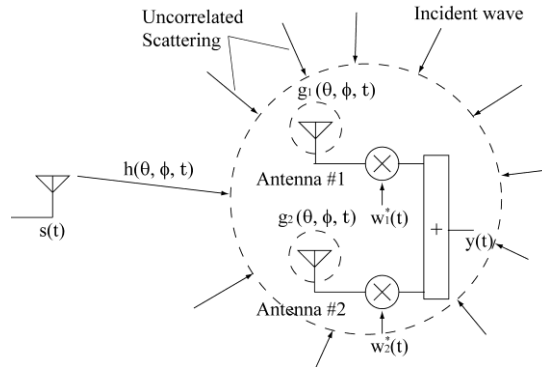
3.2 SIMO model

Figure 2: SIMO model

In this section, Single-Input-Multiple-Output (SIMO) model, especially 1-by-2 Maximum Ratio Combining (MRC) diversity shown in Fig.2 is considered. The received signal $y(t)$ of this model is expressed as follows

$$y(t) = \mathbf{w}^H(t)\left\{\int_\Omega \mathbf{g}(\theta,\phi)h(\theta,\phi,t)s(t)d\Omega + \mathbf{n}(t)\right\} = \mathbf{w}^H(t)\left\{\tilde{\mathbf{h}}(t)s(t) + \mathbf{n}(t)\right\},$$

where $\mathbf{w} = \begin{bmatrix} w_1(t) & w_2(t) \end{bmatrix}^T$ is an array antenna weight for MRC, $\mathbf{g}(\theta,\phi) = \begin{bmatrix} g_1(\theta,\phi) & g_2(\theta,\phi) \end{bmatrix}^T$ is a vector containing directivities for two antennas, and $\mathbf{n}(t) = \begin{bmatrix} n_1(t) & n_2(t) \end{bmatrix}^T$ is a vector of noise with identical variance of $P_n = E\left[|n_1(t)|^2\right] = E\left[|n_2(t)|^2\right]$. As in the conventional array processing, the instantaneous received SNR γ_{simo} can be calculated as

$$\gamma_{simo} = \mathbf{w}^H \tilde{\mathbf{h}}\tilde{\mathbf{h}}^H \mathbf{w} \frac{P_s}{P_n}.$$

Since it is quadratic form as well, the maximum instantaneous SNR with respect to w can be given by

$$\gamma_{\text{simo}}^{\max} = \sigma_1 \frac{P_s}{P_n},$$

where σ_1 is the largest eigenvalue of $\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H$. The eigenvector corresponding to the largest eigenvalue is selected as the MRC weight vector of $\mathbf{w}$. The role of antenna directivity in the SIMO system is to improve the statistical property of σ_1 to achieve better diversity gain. Therefore, the Cumulative Density Function (CDF) of the largest eigenvalue σ_1 can be used to design the SMCs in SIMO system. The CDF of σ_1 is fully described in [5]. The correlation matrix of SIMO system is given as

$$\Sigma = E\left[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H\right] = \begin{bmatrix} G & \rho\sqrt{\alpha}G \\ \rho^*\sqrt{\alpha}G & \alpha G \end{bmatrix},$$

where α $(0 < \alpha \le 1)$ and ρ $(0 < |\rho| < 1)$ are power difference and correlation coefficient between two branches respectively. By using this expression, the CDF of σ_1 $F(\sigma_1)$ at small channel gain is approximated as follows

$$\log(F(\sigma_1)) \cong 2\log\sigma_1 - \log\left[G^2\alpha\left(1-|\rho|^2\right)\right] - \log 2. \tag{3}$$

To achieve better performance, the second term of Eq.(3) should be large. When the determinant of Σ $\det(\Sigma) = G^2\alpha\left(1-|\rho|^2\right)$ is high, the 2nd term also becomes high and achieves better performance of diversity gain. Therefore, the determinant of the correlation matrix Σ is used as a metric to design the optimal SMCs. By using SME, the determinant of the correlation matrix Σ can be rewritten as $\det(\Sigma) = \det\left(\mathbf{Q}^T\mathbf{R}\mathbf{Q}^*\right)$, where $\mathbf{Q} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 \end{bmatrix}$ is SMCs of both antennas. Since the determinant of the correlation matrix is a quadratic form as well, the metric can be maximized with respect to $\mathbf{q}_1$ and $\mathbf{q}_2$ as

$$\lambda_1 \cdot \lambda_2 = \max_{\mathbf{q}_1,\mathbf{q}_2}\left[\det(\Sigma)\right] = \max_{\mathbf{q}_1,\mathbf{q}_2}\left[\det\left(\mathbf{Q}^T\mathbf{R}\mathbf{Q}^*\right)\right],$$

where λ_1 and λ_2 are largest and 2nd largest eigenvalues of $\mathbf{R}$. It means that the eigenvector corresponding to λ_1 contains optimal SMCs for 1st antenna, and the eigenvector corresponding to λ_2 contains optimal SMCs for 2nd antenna. Hence, the optimal antenna directivity for SIMO system is achieved.

Acknowledgments

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