On the Accuracy of DOA Estimation via Blind Calibration

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1. Introduction

Direction-Of-Arrival (DOA) estimation is one of significant techniques for high-speed mobile communication to know wave propagation environment [1]. "Superresolution" DOA estimation methods such as MUSIC [2], and ESPRIT [3] methods are paid so much attention because of their high accuracy in estimating DOAs of incident signals. The accuracy of those DOA estimation methods is often deteriorated by the mutual coupling among array elements. Therefore we have to calibrate array elements in order to compensate the mutual coupling in advance but it often takes time and is difficult to accurately calibrate antenna elements.

To resolve this problem, the MUSIC-based DOA estimation method that compensating the influence of mutual coupling without actual calibration – called "Blind Calibration" – has been proposed [4]. This method can hide mutual coupling coefficients by installing some virtual elements around Uniform Linear Array (ULA).

In this paper, we expand the formulation of this method to correspond to URA configuration, and confirm that the blind calibration can work for URA. Also we verify the DOA estimation accuracy of the blind calibration for ULA through the experiment in anechoic chamber.

2. Signal and Array Model

Assume that L uncorrelated narrowband incident waves $s_\ell(t)$, $(\ell = 1, 2, \cdots, L)$ arrive at N-element array antenna from the direction of θ_ℓ, respectively. We assume AWGN process and the incident waves are uncorrelated to each other and also to noise signals. The complex input signal vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_K(t)]^T$, where x_k is the input signal at n-th array element, is written as

$$\boldsymbol{x}(t) = \boldsymbol{C}\sum_{\ell=1}^{L} s_\ell(t)\boldsymbol{a}(\theta_\ell) + \boldsymbol{n}(t) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{a}(\theta_\ell) = [1, \beta(\theta_\ell), \ldots, \beta(\theta_\ell)^{(N-1)}]^T$ denotes the array response vector for ℓ-th wave with $\beta(\theta_\ell) = e^{(j2\pi d \sin\theta_\ell/\lambda)}$, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_L)]$ is the array response matrix, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T$ is the incident signal vector, and $\boldsymbol{n}(t) = [n_1(t), n_2(t), \cdots, n_N(t)]^T$ is the additive noise vector. Note that λ and $d(= 0.5\lambda)$ denote the wavelength and array interval, respectively. The $N \times N$ matrix $\boldsymbol{C}$ in (1) is given by

$$\boldsymbol{C} = \begin{bmatrix} 1 & c_1 & \cdots & c_{P-1} & \cdots & 0 \\ c_1 & 1 & c_1 & \cdots & \ddots & 0 \\ \vdots & c_1 & 1 & \ddots & \cdots & c_{P-1} \\ c_{P-1} & \cdots & \ddots & \ddots & c_1 & \vdots \\ 0 & \ddots & \cdots & c_1 & 1 & c_1 \\ 0 & \cdots & c_{P-1} & \cdots & c_1 & 1 \end{bmatrix} \tag{2}$$

where P is the number of nonzero mutual coupling coefficients c_p.

3. Blind Calibration

This section briefly reviews the conventional blind calibration approach for ULA [4] and develop it to correspond to URA configuration.

3.1 Blind Calibration for ULA [4]

Consider N-element ULA that has mutual coupling as in (2). Mutual coupling coefficients decrease as array interval becomes longer, and the coupling between two far elements can be approximated as zero. We add $(P-1)$ virtual elements at each side of the array to form a new ULA within the same element space in order to compensate the effect of mutual coupling in an N-element ULA. Then the output signal of the N-element ULA in the middle of the extended $(N+2P-2)$-element ULA can be expressed as

$$\boldsymbol{x}(t) = \tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{3}$$

where $\tilde{\boldsymbol{A}} = [\tilde{\boldsymbol{a}}(\theta_1), \tilde{\boldsymbol{a}}(\theta_2), \cdots, \tilde{\boldsymbol{a}}(\theta_L)]$ and $\tilde{\boldsymbol{a}}(\theta_\ell) = [\beta(\theta_\ell)^{1-P}, \cdots, \beta(\theta_\ell)^{-1}, 1, \cdots, \beta(\theta_\ell)^{N+P-1}]^T$. In (3), $\tilde{\boldsymbol{C}}$ is the mutual couplig matrix for the N-element ULA. Here we calculate $\tilde{\boldsymbol{C}}\tilde{\boldsymbol{a}}(\theta_\ell)$ as follows.

$$\begin{aligned}\tilde{\boldsymbol{C}}\tilde{\boldsymbol{a}}(\theta_\ell) &= \begin{bmatrix} c_1 & 1 & c_1 & \cdots & \ddots & 0 \\ \vdots & c_1 & 1 & \ddots & \cdots & c_{P-1} \\ c_{P-1} & \cdots & \ddots & \ddots & c_1 & \vdots \\ 0 & \ddots & \cdots & c_1 & 1 & c_1 \end{bmatrix} \begin{bmatrix} 1 \\ \beta(\theta_\ell) \\ \vdots \\ \beta(\theta_\ell)^{N-1} \end{bmatrix} \\ &= \left(2\sum_{n=1}^{P-1} c_n \cos(2n\pi \sin(\theta_\ell) d/\lambda) + 1 \right) \boldsymbol{a}(\theta_\ell) = c(\theta_\ell)\boldsymbol{a}(\theta_\ell), \end{aligned} \tag{4}$$

where $c(\theta_\ell)$ is obviously a scalar and dose not affect to the orthogonality between noise subspace vector $\tilde{\boldsymbol{e}}_n$ and array responce vector $\tilde{\boldsymbol{a}}$. Hence we can estimate DOA by MUSIC method by

$$P(\theta) = \frac{1}{\left|\left|\tilde{\boldsymbol{e}}_n^H \tilde{\boldsymbol{C}}\boldsymbol{a}(\theta)\right|\right|} = \frac{1}{\left|\left|c(\theta)\tilde{\boldsymbol{e}}_n^H \boldsymbol{a}(\theta)\right|\right|} = \frac{1}{\left|\left|\tilde{\boldsymbol{e}}_n^H \boldsymbol{a}(\theta)\right|\right|}. \tag{5}$$

3.2 Blind Calibration for URA

We develop the blind calibration approach to correspond to URA configuration. Suppose a (2×2)-element URA whose mutual coupling coefficients are given by c_1 and c_2, where c_1 and c_2 are coeffisients between the closest and the diagonal elements, respectively, Similarly to the virtual elements for ULA, we install virtual 12 elements to surround (2×2)-element URA and form (4×4)-element URA. Then the mutual coupling matrix for the (2×2)-element URA is written as

$$\boldsymbol{C}_2 = \begin{bmatrix} 1 & c_1 & c_1 & c_2 \\ c_1 & 1 & c_2 & c_1 \\ c_1 & c_2 & 1 & c_1 \\ c_2 & c_1 & c_1 & 1 \end{bmatrix} \tag{6}$$

Similarly to (4), $\tilde{\boldsymbol{C}}_2\tilde{\boldsymbol{a}}(\phi_\ell, \theta_\ell)$ can be arranged as

$$\begin{aligned} &\tilde{\boldsymbol{C}}_2\tilde{\boldsymbol{a}}(\phi_\ell, \theta_\ell) \\ &= \left[(e^{0x+1y} + e^{1x+0y} + e^{1x+2y} + e^{2x+1y})c_1 + (e^{0x+2y} + e^{2x+0y} + e^{2x+2y} + 1)c_2 \right] \begin{bmatrix} 1 \\ e^{0x+1y} \\ e^{1x+0y} \\ e^{1x+1y} \end{bmatrix} \\ &= c(\phi_\ell, \theta_\ell)\hat{\boldsymbol{a}}(\phi_\ell, \theta_\ell) \end{aligned} \tag{7}$$

where $\hat{\boldsymbol{a}}(\phi_\ell, \theta_\ell) = [1, e^{0x+1y}, e^{0x+2y}, \cdots, e^{3x+3y}]$ is the array response vector for URA with $e^{mx+ny} = e^{j2\pi\{(m-1)d \sin\theta_\ell \cos\phi_\ell + (n-1)d \sin\theta_\ell \sin\phi_\ell\}/\lambda}$. The mutual cupling matrix is reduced into a scalar as in (7), the blind calibration approach is applicable also to the (2×2)-element URA.

4. Simulation

DOA estimation accuracy is evaluated through some simulation in this section. Specifications of the simulation are summarized in Table 1.

4.1 Case of ULA

Consider a 8-element ULA extended from 4-element ULA. Figure 1 shows the SNR dependency of the RMSE for the blind calibration method where the mutual coupling coefficients is given as $c_1 = 0.43301 - 0.25j$ and $c_2 = 0.14142 - 0.14142j$. The number of nonzero mutual coupling coefficients is $P = 3$. From Fig. 1, we can see that the DOA estimation accuracy of the blind calibration method is better than the conventional MUSIC method even for high SNRs.

4.2 Case of URA

Consider a 4×4-element URA extended from 2×2-element URA. Figure 2 shows the SNR dependency of the RMSE for the blind calibration method where the mutual coupling coefficients is again given as $c_1 = 0.43301 - 0.25j$ and $c_2 = 0.14142 - 0.14142j$. From Fig. 2, the DOA estimation accuracy of the blind calibration method with 2×2-element in the middle of 4×4-element is better than that of MUSIC method of 2×2-element and also MUSIC method of 4×4-element. Similarly to the result of ULA, we confirm that the blind calibration approach works effectively also in the case of URA.

5. Experiment

The DOA estimation accuracy of the blind calibration approach is also evaluated through experiment in anechoic chamber. Table 2 shows the specifications of the experiment, and Figure 3 illustrates the angular dependency of RMSE for ULA. The averaged RMSEs are 0.92 for MUSIC method with 8 elements, 4.62 for MUSIC method with 4 elements, and 0.49 for the blind calibration method with 4 actual plus 4 virtual elements. This result confirms the superiority of the blind calibration approach in actual environment. The RMSE in experiment becomes larger than that of simulation as seen from Figs. 1 and 3. The reason would be due to the error of array element interval, and also due to the angular dependency of the coupling coefficients. The coefficients may depend on DOAs in actual environments and it does not lead Toeplitz matrix in (2), and finally it makes larger RMSE in experiment. That remains as one of future studies.

6. Concluding Remarks

In this paper, we developed the MUSIC-based blind calibration approach to correspond to URA. The accuracy of the developed approach was evaluated through simulation and confirmed that it was superior than that of the conventional MUSIC method. also we verified the DOA estimation accuracy of the blind calibration for ULA through the experiment in anechoic chamber. We again confirmed that it was superior than that of the conventional MUSIC method also through experiment. Further improving DOA estimation accuracy is one of future studies.

References

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Table 1: Specifications of Simulations

array configuration	ULA	URA
# of original elements	4	2×2
# of total elements	8	4×4
array element interval	0.5λ	
DOA in azimuth	15°	
DOA in elevation	30°	
carrier frequency	2.0GHz	
# of snapshots	100	
# of trials	100	

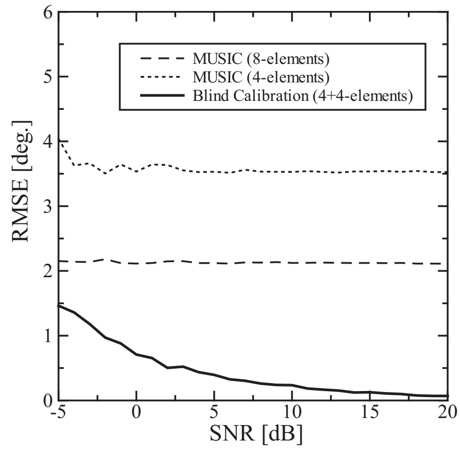
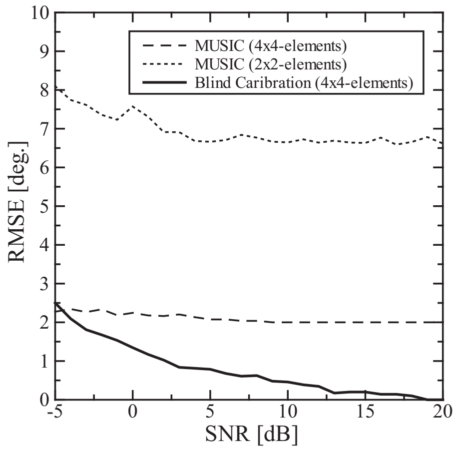
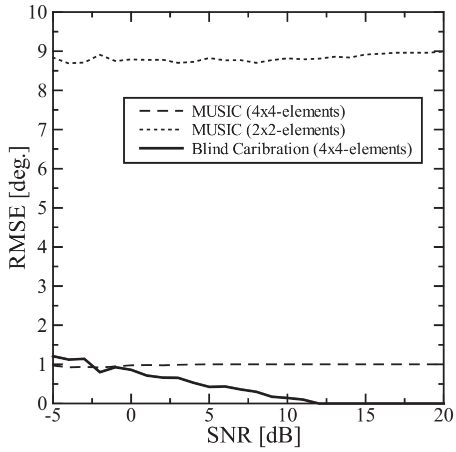


Figure 1: SNR dependency of RMSE for ULA



(a) RMSE for azimuth angle



(b) RMSE for elevation angle

Figure 2: SNR dependency of RMSE for URA

Table 2: Specifications of experiment

array configuration	ULA
# of original elements	4
# of total elements	8
array element interval	0.5λ
DOA in azimuth	0° to 60°
DOA in elevation	90°
carrier frequency	2.4GHz
# of snapshots	100

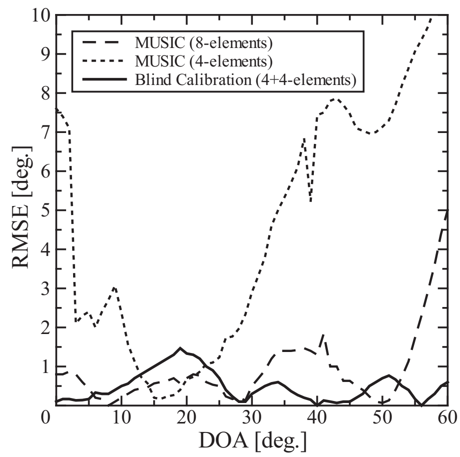


Figure 3: Angle dependency of RMSE