

# Physics in Resonant Transmission on the Small Aperture Coupling

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## 1. Introduction

Light (or Electromagnetic energy) transmission through a small aperture was first studied by Bethe in 1944. Bethe's theory tells us that the transmitted power through the small hole is very low and the transmission efficiency is very poor, in more detail, the transmittance normalized to the area scales as  $(a/\lambda)^4$  where  $a$  is the hole (small aperture) radius and  $\lambda$  is the wavelength.

Recently observations of extraordinary optical transmission (EOT) through a single small hole (or aperture) or through arrays of subwavelength holes in metal films have spurred on intense research activities into understanding and utilizing this phenomenon. The main findings of those recent works are that elliptical or rectangular holes can dramatically influence on the intensity of the transmission. Later, the origin of the transmission resonance was explained with a theory that incorporated the coupling between the modes inside the hole with the free-space regions on either side of the film. The physical nature of this effect was attributed to a Fabry-Pérot like resonance due to multiple reflections of the mode within the hole at the interfaces with the free-space regions.

Also, in the area of near-field optical applications such as optical data storage, nanolithography, and nano-microscopy, the high-transmission mechanism of the C-shaped aperture was investigated by testing the aperture transmission response for various values of the ridge geometries of the C-aperture [1].

Though the two transmission resonance structures – the Fabry-Pérot like resonance structure and the C-shaped aperture structure – seem to be quite different, they share common features from the viewpoint of the transmission resonance phenomena which has not been recognized fully. Hence, we are going to discuss the common features between the two structures with main interest centering on the transmission resonance phenomena.

## 2. Equivalent Circuit Descriptions for Two Structures

In this work, we are going to deal with the equivalent circuit representations for the above two structures. The one is a small circular aperture with a ridge in a thin conducting screen are shown in Fig. 1. In this structure, the radiation admittance at the port  $a-b$  can be made to be real by choosing appropriately the values of the ridge width  $W$  and the gap  $g$  between two ridges[2].

In the case of resonance, the structure can be represented by the equivalent circuit in Fig. 2. Here  $G_0$  denotes the conductance seen by the port  $a-b$  looking into the single half-space region ( $z < 0$  or  $z > 0$  region) and is given by  $G_0 = 4\pi/(3\eta_0\lambda^2)$  where  $\lambda$  and  $\eta_0$  are the wavelength of the incident wave and the intrinsic impedance of the free space, respectively. So the dissipated power  $|H_0|^2/(2G_0)$  in the right side conductance  $G_0$  means the transmitted power  $P_{\text{trans}}$  into the right half space ( $z > 0$ ), which is given as

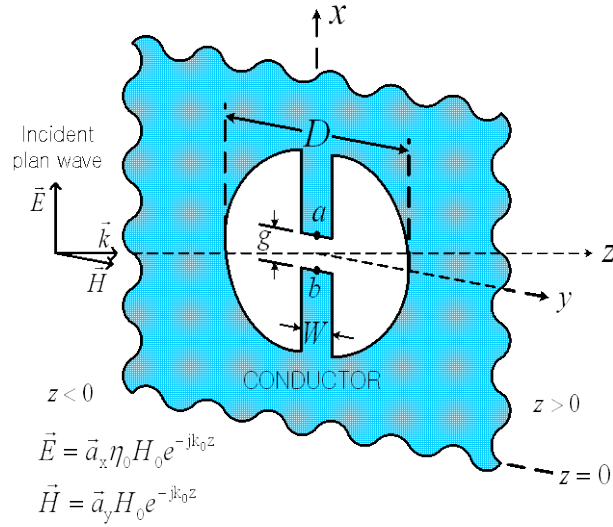


Figure 1: Small Aperture with a Ridge in a Thin Conducting Screen

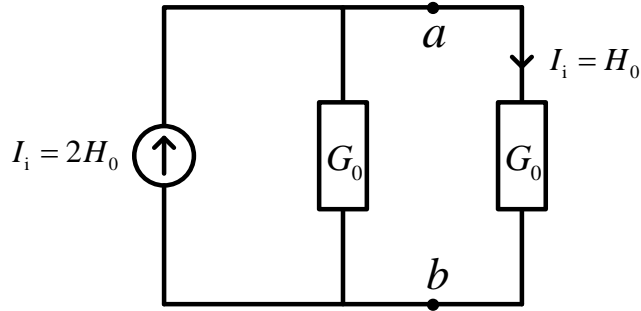


Figure 2: Equivalent Circuit Representation for the Small Circular Aperture with a Ridge for the Case of Resonance

$$P_{\text{trans}} = \frac{1}{2} |H_0|^2 \frac{3\eta_0 \lambda^2}{4\pi}. \quad (1)$$

If this transmitted power through the aperture into the right half space is divided by the incident power density  $P_{\text{inc}} = \eta_0 |H_0|^2 / 2$ , the transmission cross section  $\tau$  is defined to be

$$\tau \equiv \frac{P_{\text{trans}}}{P_{\text{inc}}} = \frac{3\lambda^2}{4\pi}. \quad (2)$$

It is interesting to note that this transmission cross section for the aperture at resonance is independent of the size of the aperture.

The other structure is the resonant transmission cavity structure embedded in a thick conducting screen as shown in Fig. 3. The section of waveguide with a resonant section is formed by two transverse walls with small coupling holes (apertures) in them.

The equivalent circuit representation for the case of Fig. 3 can be given as shown in Fig. 4, in which  $Y_C$  and  $\beta$  are the characteristic admittance and the wavenumber of the fundamental  $\text{TE}_{10}$  mode, respectively. Here the real part of  $Y_L$ , i.e.,  $G_1$  is given by  $4\pi / (3\eta_0 \lambda^2)$ , which is the same as that for  $G_0$  in Fig. 2.

In this structure, the transmission resonance, for which maximum power transmission occurs, is observed when  $\tan \beta l = -2Y_C / B_1$ . Under this condition, the total admittance becomes

purely real as  $Y_{in} = 2G_1$  and the dissipated power in  $G_1$  (in the right side  $Y_L$ ), meaning the transmitted power into the region for  $z > l$ , amounts to be  $P_{trans} = \frac{1}{2}|H_0|^2 \cdot \frac{3\eta\lambda^2}{4\pi}$ . Normalizing this  $P_{trans}$  with respect to the incident power density  $P_{inc} = \eta_0 |H_0|^2 / 2$ , the transmission cross section  $\tau (= P_{trans} / P_{inc})$  is obtained to be  $\tau = 3\lambda^2 / (4\pi)$  [m<sup>2</sup>], which is exactly the same as that in (2). This observation has been validated by numerical simulation [3]. It is to be noted that  $G_1 \ll B_1$  and  $Y_C \ll B_1$ . So the resonant cavity depth  $l_{res}$  is determined by the susceptance  $B_1$  of the small coupling hole as follows;  $l_{res} \cong \frac{\lambda_g}{2} \left( 1 - \frac{2Y_C}{\pi B_1} \right)$ , where  $\lambda_g$  means the wavelength of the TE<sub>10</sub> rectangular waveguide mode.

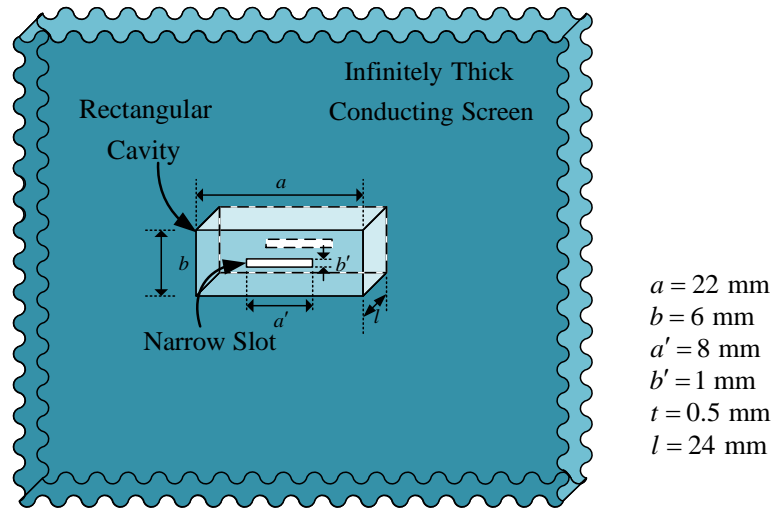


Figure 3: Transmission Cavity Embedded in a Thick Conducting Screen

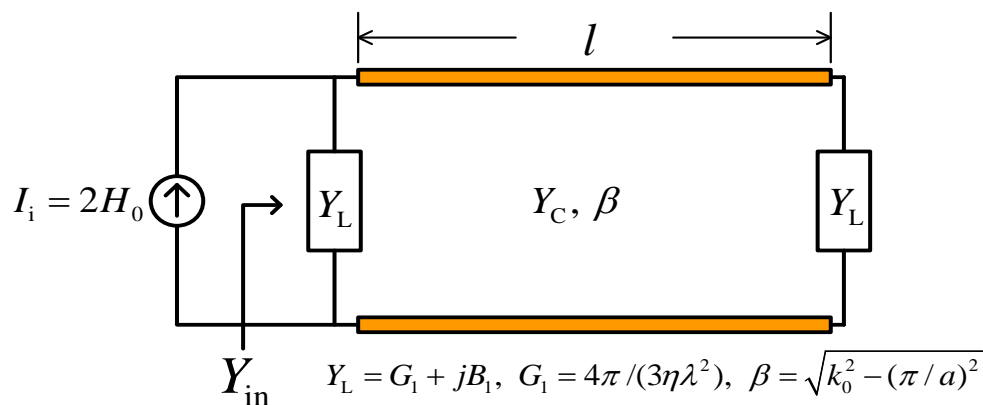
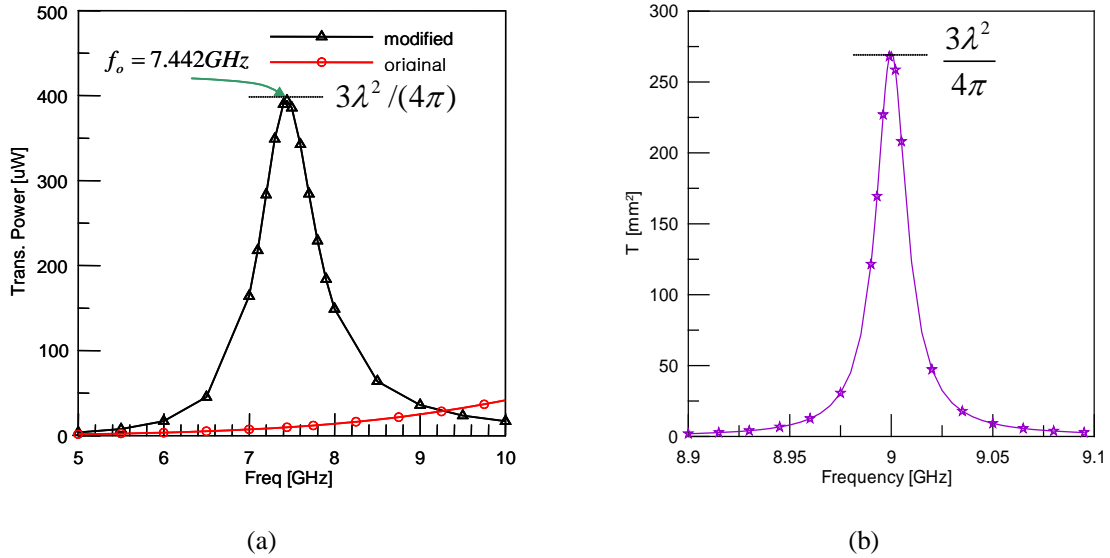


Figure 4: Equivalent Circuit Representation of the Resonant Transmission Cavity in Fig. 3

### 3. Resonant Transmission Physics

We have discussed transmission cross section between two resonant transmission structures. Based upon the discussion, it can be said that the incident power upon the transmission cross section

area(much larger than the physical coupling hole area) is funnelled into the small coupling hole and is transmitted through the above two resonant structures into the opposite region to the incident side. This funnelling mechanism is thought to be essential to the resonant transmission phenomena. Sampled data for the transmission cross section are given in Fig. 5(a) – (b), respectively, for the structures of Figs. 1 and 3. It is seen that the peaks in the figures correspond to the values of  $3\lambda^2/(4\pi)$ .



(a) (original:  $D=10$  mm,  $W=0$  mm)  
 (modified:  $D=10$  mm,  $W=3$  mm,  $g=0.478$  mm)

(b) ( $a=22$  mm,  $b=6$  mm,  $a'=8$  mm,  
 $b'=1$  mm,  $t=0.5$  mm,  $l=24$  mm)

Figure 5: Sampled Data for the Transmission Cross Section

It is well understood that the dissipated powers in the conductance of the right side in Figs. 2 and 4 correspond to the transmitted powers through the resonant structures into the opposite half space. From the equivalent circuits in Figs. 2 and 4, it is seen that the same powers as the transmitted power are back scattered for both circuits. This back scattered field is generated by the induced current over the area corresponding to the transmission cross section after short circuiting the aperture according to the equivalence principle. This back scattered field is cancelled by the field due to the magnetic current over the shorted aperture domain. This is thought to constitute the underlying physics on the transmission resonance.

## 4. Conclusion

In this paper, transmission resonance phenomena in two structures have been considered with main interest centering on the common feature. Underlying physics on the common feature have been discussed. This discussion may help in understanding the filter mechanism and in designing better NSOM probe.

## References

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