2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

# Chaos controlling with clock pulse modulation for DC-DC converter circuit

Daisuke Ito†

†Department of Electronic Systems Engineering, The University of Shiga Prefecture
2500 Hassaka-cho, Hikone-shi, Shiga, 522-8355 JAPAN
Email: ito.d@e.usp.ac.jp

**Abstract**—The DC-DC converters are one kind of hybrid dynamical systems, and they present typical nonlinear phenomena; bifurcation phenomena and chaotic attractors. On engineering view points, they are undesigned behavior because they cause abrupt motions or noise. To suppress them, various controlling techniques have been proposed. On DC-DC converters, the period of pulse inputs is an important parameter for behavior of the circuit. In this paper, we investigate the chaos controlling method by adjusting the interrupt dynamical events caused by pulse inputs, and propose two-type pulse modulation schemes.

## 1. Introduction

The systems with interrupt events that change dynamical behavior of them such as switches are treated as hybrid dynamical systems. These features are observed in many engineering's fields [1]. Due to the nonlinearity of interrupt events, rich complex behavior appears, i.e. bifurcation phenomena and chaotic attractors. They are also observed on the one-dimensional piece-wise linear system, and it is confirmed that border-collision bifurcations play an important role [3, 2, 4].

In the electrical circuit, hybrid dynamical systems exist by featuring the electrical switches, i.e. mechanical switches and MOS-FETs. A DC-DC converter circuit is one of hybrid dynamical systems. Some reports issue the bifurcation and chaos, and the influence for its electrical characteristic is investigated [5, 6]. On the view point of performance as converters, chaotic attractors are similar to noise-like responses, and they can be considered as the behavior to avoid. In recent studies, various controlling schemes via chaos controlling are proposed for DC-DC converter models [7, 8, 9]. Kousaka, et.al proposed the controlling scheme with varying the source voltage. We also have proposed the method with varying reference voltages of comparators, but they mean output voltage values, and it should be fixed as objective output voltage values. On the other hand, the pulse width modulation (PWM) input is used to drive the converter circuit. Its duty ratio decides the output voltage, and the period of the pulse have influence the circuit's behavior mainly. Accordingly, the frequency modulation has possibility to control the circuit, and chaos controlling by perturbation for frequencies.

In this study, we try to suppress chaotic phenomena on the DC-DC converter model circuit by the clock pulse modulation. The parameter perturbation based on the feedback control is used. The controller gain is designed with a pole assignment method. At first, we explain the design procedure, and show the gain range to stabilize objective values. Next, we demonstrate performance of our controller. by numerical simulations, feasibility and implementability is confirmed.

## 2. Circuit model

Let us consider a simple interrupt chaotic system [7] shown in Fig. 1 as an example. The switch is flipped by a certain rule depending on the state and the period. Assume that $v$ is the state variable, and then the normalized equation is given as follows:

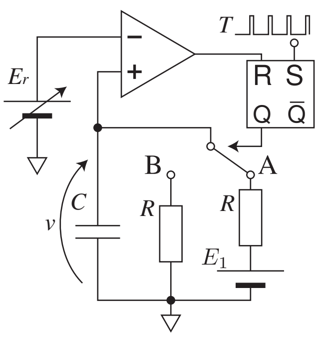



Figure 1: DC-DC converter model circuit.

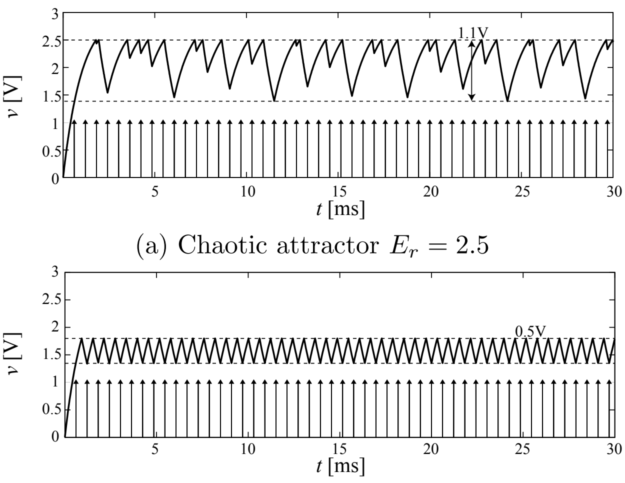
$$\frac{dv}{dt}(t) = v(t) - E \quad \begin{array}{l} \text{If } t = n\tau \quad \text{then } E = E_{\text{in}} \\ \text{If } v(t) > E_r \quad \text{then } E = 0 \end{array} \tag{1}$$

where $E_1$ and $E_r$ are a direct current bias and a switching threshold value, respectively. $\tau = T/RC$ is the period of the clock pulse input. If the Poincaré section is defined as $\Pi = v(t) \in \boldsymbol{R}; t = n\tau$, trajectories strike two types of solutions, and they can be solved exactly, see [3] Therefore the system can be discretized by the Poincaré section, and redefined as follows:

$$\begin{aligned} v_{k+1} &= g(v_k) \\ &= \begin{cases} (v_k - E_1)e^{-\tau} + E_1 & v_k < D \\ E_r \dfrac{v_k - E_1}{E_r - E_1} e^{-\tau} & v_k \geq D \end{cases}, \end{aligned} \tag{2}$$

$$D = (E_r - E_1)e^{\tau} + E_1. \tag{3}$$

A chaotic attractor and UPO with parameters $E_1 = 3$ and $\tau = 0.606$ are shown in Fig. 2. It is confirmed that the ripple voltages of the periodic orbit are about 0.6V smaller than the chaotic attractor. This model show various responses dependently on the output voltage $E_r$, and there is the case of large ripple voltages. Unstable periodic orbit similar to the Fig. 2 (b) is embedded into the chaotic attractor. If it can be stabilized by appropriate controlling input, the ripple voltage will be decreased.




(a) Chaotic attractor $E_r = 2.5$

(b) Periodic orbit: $E_r = 1.81$

Figure 2: Chaotic attractor and periodic orbit with $E_1 = 3$ and $\tau = 0.606$.

## 3. Controlling methods

In this paper, two types scheme for stabilization of UPOs are proposed based on a feedback control. The pole assignment method is used for design of a controller gain. Let us consider a 1-dimensional discrete time dynamical system as follows:

$$x_{k+1} = f(x_k,\ \lambda),\ x \in \boldsymbol{R},\ \lambda \in \boldsymbol{R}. \tag{4}$$

Assuming the controlling input $u_k = c(x^* - x_k)$ for the parameter, the system is described as $x_{k+1} = f(x_k,\ \lambda + c(x_k - x^*))$, where $x^*$ is a target value, and $c$ is the controler gain. Thus, the characteristic equation is shown as follows:

$$\chi(\mu) = A + Bc - \mu = 0, \tag{5}$$

where,

$$v^* = \frac{-E_r E_1 e^{-\tau}}{E_r(1 - e^{-\tau}) - E_1},\ A = \frac{\partial f}{\partial v^*} = \frac{E_r}{E_r - E_1} e^{-\tau}, \tag{6}$$

and $B = \partial f / \partial \lambda$. The conditions of stability for the target value $x^*$ is $|\mu| < 1$. Therefore the controller gain $c$ that $x^*$ becomes stable is derived as follows:

$$c = \frac{q - A}{B},\ -1 < q < 1 \tag{7}$$

In the target circuit model, various controlling scheme is proposed, e.g. Kousaka, et.al have used the source voltage $E_1$, and we also have proposed controlling scheme with the threshold value $E_r$. On the other hand, the time $\tau$ of the clock pulse input is also the adjustable parameter. The duty ratio of the clock influences the output voltage directory, but its period is of little relevance to them. However, the period is important parameters for behavior of the circuit, and could be efficient controlling parameter. Next, specific two-type controlling schemes with the perturbation for the period are explained.

### 3.1. PFM: the frequency of pulse width

The frequency $\tau$ of a clock signal is perturbed by controlling inputs $u_k$. This is one kind of the pulse frequency modulation (PFM). Figure 3 shows the sketch of the PFM type controlling. In this scheme, the controller can adjust the timing of the next pulse, and the voltage $v_{k+1}$ changing from the discharge to the charge is adjusted. Thus, this method influence only the map

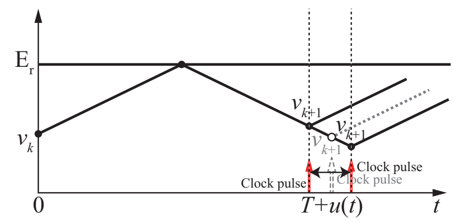



Figure 3: Sketch of PFM type controlling. Arrows mean additional interrupt events.

with $v_k < D$. The lower map of Eq. (2) is used for the objective one-periodic orbit obviously. the partial derivative of the map with (2) respect to the period $\tau$ is given as follows:

$$B = -E_r \frac{v^* - E_1}{E_r - E_1} e^{-\tau} \tag{8}$$

From Eq. (7) and (6), the ranges of stabilizable gains are determined without experimental results, and they are shown as follows:

$$\frac{-E_r + E_1 + E_r e^{-\tau}}{E_r(v^* - E_1)e^{-\tau}} < c < \frac{E_r - E_1 + E_r e^{-\tau}}{E_r(v^* - E_1)e^{-\tau}} \tag{9}$$

$$-1.7 < c < -0.79 \tag{10}$$

### 3.2. Occasional applied pulse input: forcing the interrupt event at appropriate time ahead original clock pulses

If the target circuit is already constructed, and controller can not adjust the frequency of the clock pulse, previous scheme does not apply to one. Next, let us consider the controlling scheme for the circuit with fixed clock pulses. The sketch of the controlling scheme is illustrated as Fig. 4. On the previous controller, it is designed based on changing the circuit phase forcibly to the charge at an appropriate time. On the other hand, this controller changes the mode to the discharge by appropriate pulse. The pulse is added to the RS-FF before the radical clock pulse. Thus, this controller does not influence the clock pulse, and it can apply to the circuit with the fixed clock pulse.

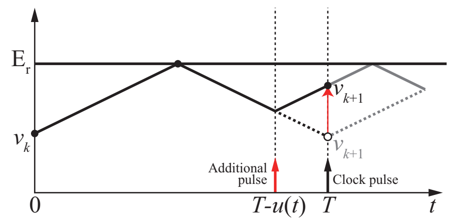



Figure 4: Sketch of Occasional applied pulse events. The red arrow means an additional interrupt pulse.

Figure 5 shows behaviour of the circuit with controlling inputs, it can confirm that four type trajectories exist dependently on the initial voltage $v_k$. (i) and (ii) are the same to the circuit's one without controller. (iii) and (iv) are new behaviour with influences of applied pulses. They can be derived as two operation mapping with $\tau - u_k$ and $u_k$. Therefore, it can be described as $v_{k+1} = g(g(v_k,\ \tau - u_k),\ u_k)$. As a result, the differential equation of the circuit with controller is described as follows:

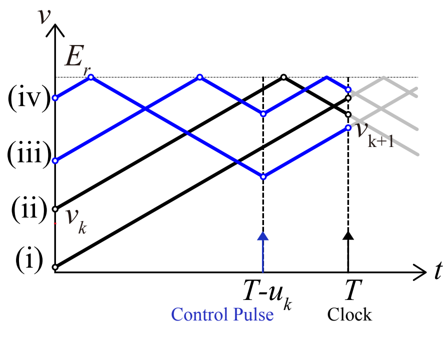



Figure 5: Four-type trajectories by the applied pulse.

$$v_{k+1} = g(g(v_k,\ \tau - u_k),\ u_k)$$

$$= \begin{cases} (v_k - E_1)e^{-\tau} + E_1 & \text{(i)} \\ E_r \dfrac{v_k - E_1}{E_r - E_1} e^{-\tau} & \text{(ii)} \\ E_r \dfrac{v_k - E_1}{E_r - E_1} e^{-\tau} + (1 - e^{-u_k})E_1 & \text{(iii)} \\ \left\{ \dfrac{E_r}{E_r - E_1} \right\}^2 (v_k - E_1)e^{-\tau} \\ \qquad - \dfrac{E_r E_1}{E_r - E_1} e^{-u_k} & \text{(iv)} \end{cases} \quad (11)$$

When $\xi << 1$, the input $u_k$ is also approximately zero. In this case, the map (iii) is applied, and the partial derivative of the map respect to the controlling input $u_k$ is given as follows:

$$B = e^{-u_k} E_1 \big|_{u_k = 0} = E_1 \quad (12)$$

From Eq. (7) and (6), the ranges of stabilizable gains are determined without experimental results, and they are shown as follows:

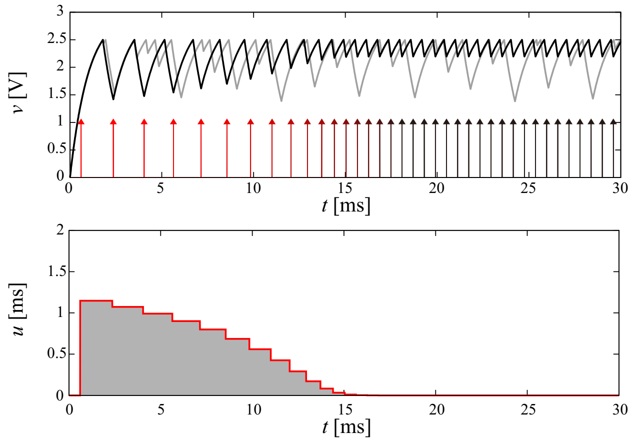
$$-\frac{1}{E_1} - \frac{E_r e^{-\tau}}{E_1(E_r - E_1)} < c < \frac{1}{E_1} - \frac{E_r e^{-\tau}}{E_1(E_r - E_1)} \quad (13)$$

$$0.58 < c < 1.24 \quad (14)$$

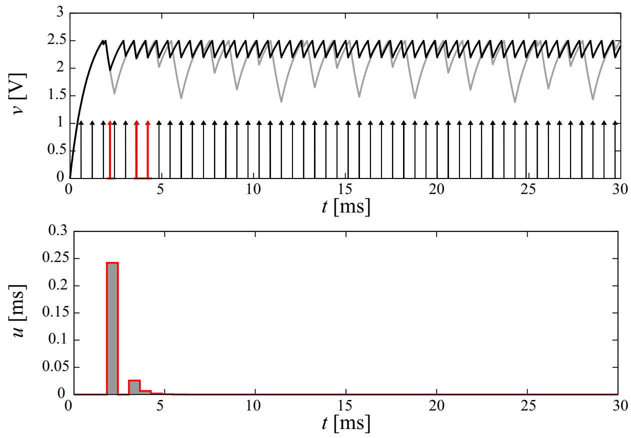
Note that, if $u_k < 0$, the applied pulse have no meaning because the clock pulses arrive before controlling pulse. Therefore, the limitation $0 < u_k$ for the controller is necessary, and pulse is applied occasionally to the objective circuit with only $u_k$ is positive values.

## 4. Controlling result

Figure 6 shows controlling results. The under arrows are clock pulses applied controlling input values, and the wave form means the capacitor voltage $v$ of the circuit. Under figures are amounts of controlling inputs. In Fig. 6 (a), it is confirmed that the frequency of clock pulse is decreased at the beginning of the simulation. However, it gets back the ideal value $\tau$ with decresing the controlling input value $u$. Finally, the voltage converged as one-periodic orbit with small ripples. In Fig. 6 (b), the frequency of the clock pulse is not changed, but additional pulse (red arrows) are added, and interrupt events are forced ahead original clock pulses. Under figures show amounts of the controlling inputs. When $v$ converges to the objective periodic orbit, the input $u(t)$ also converges to zero. Note that, there is the interval with $u = 0$ and no additional pulses, because this scheme has limitation for the amount of controlling input. If the input $u$ takes negative value, controller applies the additional pulse to the RS-FF after the clock pulse, and it has no meanings. Hence, when the controlling input takes negative value, it becomes zero. Thus, these behaviors are unstable periodic orbits and they are stabilized by controller.




(a) PFM type ($K = -1.38$)




(b) Occasional applied pulse type ($K = 1.01$)

Figure 6: Controlling results of two type controllers. Behavior of voltages $v$ converged as a one-periodic orbit with small ripples, and controlling inputs become $u(t) = 0$.

On comparison these result, it seems that the method 2) can control the circuit to the objective UPO from simulation results. Actually, setting times of two methods are $t_{s1} \approx 15$ and $t_{s2} \approx 4.5$, respectively. However, on the view point of robustness, the method 2) has limitation for the amount of controling input, and it causes the degration of robustness. The method 1) can applyes bipolar inputs, and can respond unexpected motions ( offcourse, this method has also limitation for the controlling input, and it is $u_k < \tau$, but it is slacker than the method 2)). Thus, the method 1) has high robustness than the method 2). Note that, these characteristics is changed by adjusting the parameter $q$ of the controller, and this paper does not disscuss appropriate vales of them.

## 5. Conclusion

In this study, we propose two types of chaos controlling for hybrid dynamical systems, and demonstrate the performance of them. As a result, our controller can stabilize unstable periodic orbits or DC-DC converter model circuits, and can reduce the ripples of them.

## Acknowledgment

This work was supported by JSPS KAKENHI Grant Number 16K18107.

## References

[1] M. Bernardo, C. J. Budd, A. R. Champneys, and P. Kowalczyk, *Piecewise-Smooth Dynamical Systems: Theory and Applications*, Springer-Verlag, London, UK, 2008.

[2] T. Inagaki and T. Saito, “Consistency in a chaotic spiking oscillator,” IEICE Trans. Fund., E91-A, 2240–2243, 2008.

[3] T. Kousaka, T. Kido, T. Ueta, H. Kawakami and M. Abe “Analysis of border-collision bifurcation in a simple circuit,” Proc. ISCAS 2000 Geneva, pp. 481–484.

[4] D. Ito, T. Ueta and K. Aihara, “Bifurcation analysis of two coupled Izhikevich Oscillators,” Proc. NOLTA 2010, pp. 627–630, 2010.

[5] G. Yuan, S. Banerjee, E. Ott, and J. A. Yorke, “Border-collision bifurcations in the buck converter,” Circuits Syst. I Fundam. Theory Appl. IEEE Trans., vol. 45, no. 7, pp. 707–716, 1998.

[6] A. El Aroudi and D. Fournier, “Bifurcation Behavior in a Two-Loop DC-DC Quadratic Boost Converter,” pp. 2489–2492, 2015.

[7] T. Kousaka, T. Ueta and H. Kawakami “Controlling chaos in a state-dependent nonlinear system,” Int. J. Bifurcation and Chaos 12, 1111–1119, 2002.

[8] D. Ito, T. Ueta, T. Kousaka, J. Imura and K. Aihara, “Controlling chaos of hybrid systems by variable threshold values,” International Journal of Bifurcation and Chaos, Vol. 24, No. 10, 1450125 (12 pages), Oct. 2014.

[9] T. Sasada, D. Ito, T. Ueta, H. Ohtagaki, T. Kousaka, and H. Asahara, “Controlling Unstable Orbits via Varying Switching Time in a Simple Hybrid Dynamical Systems,” Proc. NOLTA 2015, pp. 475–478, Hong Kong, China, Dec. 1–4, 2015.