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# Revisiting delay embedding: dynamical reconstruction based on Sturm-Liouville theory

Naoto Nakano†‡

†JST PRESTO, 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan
‡Faculty of Science, Hokkaido Univerisity, Kita 10 Nishi 8, Kita-ku, Sapporo 060-0810, Japan
Email: n_nakano@math.sci.hokudai.ac.jp

**Abstract**—Delay embedding is well-known for nonlinear time-series analysis, and it is used in several research fields. Takens theorem ensures validity of the delay embedding analysis: embedded data preserves topological properties, which the original dynamics possesses, if one embeds into some phase space with sufficiently high dimension. This means that, for example, an attractor can be reconstructed by the delay coordinate system topologically. However, configuration of embedded data may easily vary with the delay width and the delay dimension, namely, "the way of embedding". In a practical sense, this sensitivity may cause degradation of reliability of the method, therefore it is natural to require robustness of the result obtained by the embedding method in certain sense.

In this study, we investigate the mathematical structure of the framework of delay-embedding analysis to provide Ansatz to choose the appropriate way of embedding, in order to utilise for time-series prediction. In short, mathematical theories of the Hilbert–Schmidt integral operator and the corresponding Sturm–Liouville eigenvalue problem underlie the framework. Using those mathematical theories, one can derive error estimates of mode decomposition obtained by the present method and a time evolution equation represented by the mode amplitude functions constructed exclusively by given time-series. Moreover, projecting datasets into a subspace spanned by the leading modes, we can detect the attractor and analyse the corresponding dynamics. In this talk, we will show some results for some numerical and experimental datasets to validate the present method.

In fact, this mathematical justification relies on the $L^2$ analysis and the modes of the decomposition corresponds to intrinsic modes of the autocorrelation function, namely, the intrinsic frequency modes. Hence, this methodology is expected to have some relevance to the Koopman mode decomposition, which is used to extract characteristic frequency of the signal.