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A Study on Performance of Hopfield-Tank Neural Networks Running on Coherent Ising Machine

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Abstract—Coherent Ising Machine (CIM) which are composed with mutually interacted LASER network, and that have been studied as ultra-fast calculator for various optimization problem. It have been shown that CIM can obtain exact solution of MAX-CUT problem rapidly with high probability. We have proposed to use Hopfield-Tank Neural Network (HTNN) running on CIM to solve various optimization problems, such as traveling salesman problem. HTNN is one of combinatorial optimization method, and it have been shown that HTNN have ability to obtain exact solution on many kinds of combinatorial optimization problem and its real problem. In this paper, we have applied HTNN to CIM, and solved traveling salesman problem by HTNN running on CIM. We also investigate performance of HTNN running on CIM with using some kinds of noises.

1. Introduction

Coherent Ising Machine (CIM)[1] have been studied as one of combinatorial optimization method. CIM can obtain exact solution of several combinatorial optimization problem with high probability. For applying CIM to many kinds of combinatorial optimization problem, feasibility and implementation is studied. Especially, it have been shown that CIM can obtain exact solution of the max-cut problem[2] that is one of combinatorial optimization problem.

Combinatorial optimization method by CIM is based on the phenomenon that the energy of spin network minimize. Hopfield-Tank neural network (HTNN) is also mutually connected network, and have ability to solve combinatorial optimization problem such as traveling-salesman problem (TSP), quadratic assignment problem (QAP), and so on. Optimization method by HTNN is based on energy reducing in a monotone manner by network update, and which have disadvantage that the updating have tend to trap on local minimum. Therefore, it is necessary to improve the performance of HTNN by introducing stochastic fluctuation by Boltzman machine[4] or by using chaotic fluctuation with chaos neural network[5], but it is very hard to obtain exact solution even by these improving method.

In this paper, we propose fast optimization method using HTNN running on CIM which can obtain exact solution. First, we evaluate the feasibility of proposed method by applying the method to TSP. In this case, we introduce noise sequences to solve TSP by the HTNN model. Therefore, we investigate relationship between noise amplitude and performance of the model with changing the Gaussian sequences to other chaotic sequences.

2. Coherent Ising Maachine

CIM is the system which can obtain ground state of Ising Hamiltonian. The state of spin glass is defined as following equations (1) and (2).

$$\frac{dc_i}{dt} = (-1 + p - c_i^2 - s_i^2)c_i + \sum_{j=1}^{N} \xi_{ij} c_j, \tag{1}$$

$$\frac{ds_i}{dt} = (-1 - p - c_i^2 - s_i^2)s_i + \sum_{j=1}^{N} \xi_{ij} s_j. \tag{2}$$

Where, c_i is normalized in-phase state, s_i is normalized quadrature phase state, p is pump rate, ξ_{ij} is mutual injection.

Each spin have binary states such as up and down. Total energy of mutually coupled spin network is expressed by following equation (3).

$$H = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \sigma_i \sigma_j + \sum_{i=1}^{N} \lambda_i \sigma_i. \tag{3}$$

Where, J_{ij} corresponds coupling strength, $\sigma_i = \{\pm 1\}$ corresponds state of each spin.

Network becomes stable at ground state on appropriate pumping rate with updating the state of spins by equation (1) and (2). When CIM is used as optimization method, it is necessary to determine Hamiltonian to correspond with maximizing or minimizing objective function, and determine the connection weights between each spin based on the Hamiltonian. Hamiltonian becomes minimum state when CIM becomes ground state by updating, and that makes it possible to obtain solutions of minimum or maximum value of the objective function.

3. Hopfield-Tank Neural Network

Hopfield-Tank neural network is mutually connected neural network. Updating of Hopfield-Tank neural network is expressed by following equation.

$$\frac{dx_{ij}(t)}{dt} = -\frac{x_{ij}(t)}{\tau} + \sum_{k=1}^{N}\sum_{l=1}^{N} W_{ijkl}X_{kl}(t) - \theta_{ij}(t). \tag{4}$$

Where, W_{ijkl} is coupling weight between neuron x_{ij} and x_{kl}. W_{ijkl}= W_{klij} and W_{ijij}=0. θ_{ij} means threshold of neuron x_{ij} and τ means attenuation. $X_{ij}(t)$ means output of neuron x_{ij}, which have continuous range between 0 and 1. This output $X_{ij}(t)$ is decided by following sigmoid function as output function of $x_{ij}(t)$.

$$X_{ij}(t) = \frac{1}{1+\exp\left(\frac{-x_{ij}(t)}{\epsilon}\right)}. \tag{5}$$

Where, ϵ is parameter of sigmoid function. The energy of mutually coupled neural network by neuron of equation (4) is expressed by following equation (6).

$$E(t) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} W_{ijkl}X_{ij}(t)X_{kl}(t) + \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_{ij}X_{ij}(t). \tag{6}$$

The energy of network is decreased according to neuron updating expressed by equation 4.

4. Optimizing Method of TSP by Hopfield-Tank Neural Network

Traveling salesman problem (TSP) is one of combinatorial optimization problem to find the shortest path to visit all destination. On TSP, i is index of city and j is order of visiting, and $x_{ij} = 1$ means city i is visited at order j. Total path length can be expressed $\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=1}^{N} d_{ik}(x_{k,j+1} + x_{k,j-1})$. x_{ij} can be 1 for each i only once, because of that each city can be visited only once. Therefore, it is necessary to be $\sum_{i=1}^{N}(\sum_{j=1}^{N} x_{ij} - 1)^2 = 0$.

Similarly, $\sum_{j=1}^{N}(\sum_{i=1}^{N} x_{ij} - 1)^2$ should be 0. These are constraint function of TSP. By these path length and constraint functions, objective function of optimization on TSP can be expressed by following equation (7).

$$E = A\sum_{i=1}^{N}\left(\sum_{j=1}^{N} x_{ij} - 1\right)^2 + B\sum_{j=1}^{N}\left(\sum_{i=1}^{N} x_{ij} - 1\right)^2 + C\sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=1}^{N} d_{ik}\left(x_{k,j+1} + x_{k,j-1}\right). \tag{7}$$

A and B correspond coefficient of constraint term and C corresponds coefficient of path length.

Here, by comparison energy of HTNN and objective function of TSP, we can obtain injection weight W_{ijkl} and threshold θ_{ij} as following:

$$\begin{aligned} W_{ijkl} &= - A\delta_{ik}(1-\delta_{jl}) - B\delta_{jl}(1-\delta_{ik}) \\ &\quad - Cd_{ik}(x_{j,l+1} + x_{j,l-1}), \end{aligned} \tag{8}$$

$$\theta_{ij} = -\frac{(A+B)}{2}. \tag{9}$$

Energy function, objective function of TSP, converge to minimum solution by updating neuron state based on equation (4) with parameters W_{ijkl} and θ_{ij}.

5. Hopfield-Tank Neural Network Running on Coherent Ising Machine

In this paper, we investigate the performance of HTNN running on CIM. The output of HTNN expressed by the equation (5) have binary output 0 or 1. On the other hand, CIM spin that expressed by the equations (1) and (2) have binary output of -1 or 1. To homologize the output of HTNN and CIM, we have developed the equation of HTNN that have ± 1 output. The output function of ± 1 HTNN can be expressed by following equation (10) by assigning $\hat{X}_i = 2X_i - 1$ to equation (4).

$$\frac{dx_{ij}(t)}{dt} = -\frac{x_{ij}(t)}{\tau} + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{W_{ijkl}}{2}\hat{X}_{kl}(t) - \left(\theta_{ij}(t) - \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{W_{ijkl}}{2}\right). \tag{10}$$

Connection weight and threshold of HTNN can be decided by equation (10), and expressed by following equation (11).

$$\hat{W}_{ijkl} = \frac{W_{ijkl}}{2}, \quad \hat{\theta}_{ij} = \theta_{ij} - \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{W_{ijkl}}{2}. \tag{11}$$

Deeming $c_{ij} > 0$ as firing of neuron, we obtain following equation (12).

$$\frac{dc_{ij}}{dt} = (-1 + p - c_{ij}^2 - s_{ij}^2)c_{ij} + W_{scale}\sum_{k=1}^{N}\sum_{l=1}^{N}\hat{W}_{ijkl}c_{kl} + T_{scale}\hat{\theta}_{ij}. \tag{12}$$

By this equation (12), it is made possible to run HTNN on CIM.

To investigate the performance of this new model, we try to obtain optimal solution of TSP 10 city problem by proposed method. Figure 1 shows rate of obtaining optimal solution with changing pump rate and noise amplitude. From figure 1, we can find that there are parameters that HTNN can obtain optimal solution. Therefore, we have found that HTNN running on CIM is feasible to solve combinatorial optimization problem such as TSP.

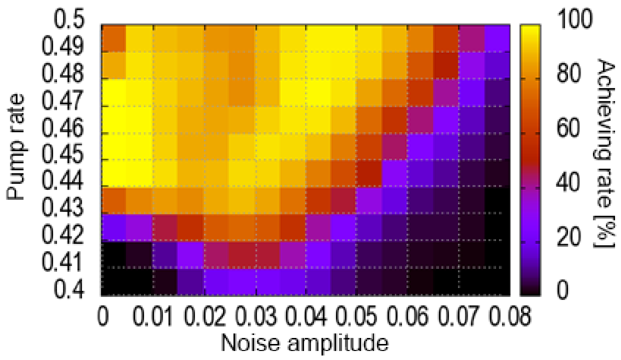

Figure 1: Rate of obtaining optimal solution with changing pump rate and noise amplitude using Hopfield-Tank neural network running on CIM.

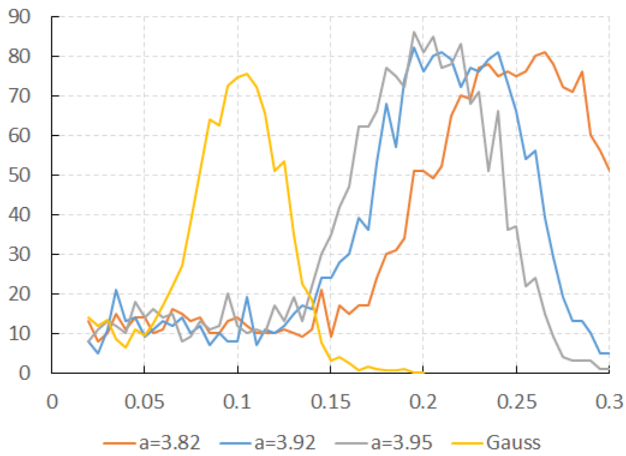

Figure 2: Relationship between achieving rate of obtaining optimal solution and noise amplitude.

We have also confirmed whether performance of HTNN is improved by noise amplitude and noise sequence. Figure 2 shows relationship between achieving rate of obtaining optimal solution and noise amplitude. From figure 2, it can be found that there is appropriate noise amplitude to improve performance of HTNN. We also introduced logistic equation that have negative autocorrelation as noise sequence to HTNN. It have been found that solution searching performance of HTNN is improved slightly by logistic sequence. To improve the performance, it is also necessary to consider noise sequence that can be introduced to real machine of CIM.

6. Conclusion

In this paper, we have compared output function of CIM and HTNN and homologized the outputs of two models. By the proposed method, HTNN have been able to run on CIM. We have solved 10 city problem of TSP with proposed update function, and found that HTNN running on CIM have the ability to search the optimal solution.

In this paper, we have shown one example of methods to solve various combinatorial optimization problems. However, because real implementation of CIM is under considering, it is necessary to continue improving applicability of our proposed method. It may be necessary to modify equations to run HTNN on real machine of CIM. We also study other methods to extend applicable area of CIM on combinatorial optimization problems.

Acknowledgments

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