An Adaptive Frequency Sweeping Algorithm of MoM Impedance Matrices in Full-Wave Analysis of Microstrip Patch Antennas

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***Abstract*-In this paper, a study on frequency interpolation algorithms of method of moments (MoM) impedance matrices is discussed in detail, which is successfully applied into the full-wave analysis of a microstrip patch antenna in a relatively wide band. By using Lagrange interpolation scheme in an adaptive system, two interpolating rules are realized and their accuracies are defined by Frobenius norms of the impedance matrices in entire frequency band. A microstrip fed patch antenna is considered to verify the algorithm from 1 to 5 GHz. The numerical results have shown that by selecting the Chebyshev zeros in the frequency band for polynomial interpolation is of high accuracy and the simulation efficiency can be highly elevated simultaneously. Besides, a statistical conclusion on the tradeoff between accuracy and efficiency issue has also been made quantitatively.**

I. INTRODUCTION

With the fast development in microwave and millimeter wave integrated circuit design and VLSI technology, more and more attention has been paid to the rigorous, accurate and fast modeling and simulation methods of layered circuits. By applying the MoM with layered medium dyadic Green's functions into the mixed potential integral equation (MPIE) has been one of the most popular methods for microstrip structures [1-5]. As is well-known, the spectral-domain Green's functions of layered medium structures can be expressed in closed form [6], and then inversed to spatial domain through the Sommerfeld integrals (SI). Based on our recent works [7, 8], the Green's functions in spatial domain have been fast and accurately obtained by means of the combination of the discrete complex image method (DCIM) and the all modes method in near and non-near region, respectively. With the closed-form spatial domain dyadic Green's functions, the MoM have been constructed for modeling and simulation for planar layered circuits, which is based on the RWG basis functions [9] and Delta-Gap voltage excitation model [10]. Although the computer codes could reach almost the same efficiency as some commercial software at single frequency point, it becomes inefficient for wide-band frequency sweeping situation. Considering the smooth property of MoM impedance matrix elements, it is possible to build interpolating schemes in a relative wide band, therefore, the efficiency of MoM can be highly elevated.

In this paper, a study on frequency interpolation algorithms of MoM impedance matrices is discussed in detail, which has been successfully applied into the full-wave analysis of a microstrip patch antenna in a relatively wide band. By using Lagrange interpolation scheme, two sampling rules are realized and their accuracy are defined by Frobenius norms of the impedance matrices in entire frequency band. A microstrip fed patch antenna is considered to verify the algorithms from 1 to 5 GHz. The numerical results have shown that by selecting the Chebyshev zeros in the frequency band for polynomial interpolation is of high accuracy and the simulation efficiency can be highly elevated simultaneously. Very good agreement on S-parameters between the proposed method and commercial software have been found.

II. MPIE FORMULATION AND PARAMETER EXTRACTION

By enforcing the boundary condition that revokes the vanishing of the total tangential electric field on the conductor surface, the EFIE governing the total current density can be established. However, to avoid the two-dimensional infinite integrals with highly oscillating, slowly decaying and hyper-singular kernel involved in the EFIE, the MPIE has been widely used in layered structures, which is composed of vector and scalar potentials with weakly singular kernels. The MPIE can be formulated as below [11]:

$$\hat{n}\times\vec{E}^{imp}=\hat{n}\times\begin{bmatrix} j\omega\mu_0\left\langle \underline{\underline{G}}^A(\bar{r}\,|\,\bar{r}');\vec{J}(\bar{r}')\right\rangle \\ -\dfrac{1}{j\omega\varepsilon_0}\nabla\left(\left\langle G^{\Phi}(\bar{r}\,|\,\bar{r}'),\nabla'_S\cdot\vec{J}(\bar{r}')\right\rangle\right)\end{bmatrix} \tag{1}$$

where $\vec{E}^{imp}=V_p\delta(\bar{r}-\bar{r}_p)\cdot\hat{n}_p$ stands for the impressed electric field, $\bar{r}_p$ is the location of the port and $\hat{n}_p$ is the outward normal parallel to the feed line. $\underline{\underline{G}}^A(\bullet)$ and $G^{\Phi}(\bullet)$ refer to the dyadic and scalar Green's functions of the vector and scalar potential, respectively. With the spatial Green's functions, the MoM can be applied to converting the MPIE into an matrix equation. For the sake of modeling the arbitrarily shaped geometries, the RWG triangular patches are adopted in this paper. With the Galerkin's procedure, (1) becomes [11]

$$\begin{aligned}&-j\omega\varepsilon_0\int_{T_m}\vec{E}_t^{imp}(\bar{r})\cdot\vec{f}_m(\bar{r})ds\\&=k_0^2\sum_{n=1}^{N}I_n\int_{T_m}\int_{T_n}\vec{f}_n(\bar{r}')\cdot\underline{\underline{G}}^A(\bar{r}\,|\,\bar{r}')\cdot\vec{f}_m(\bar{r})ds'ds\\&+\sum_{n=1}^{N}I_n\cdot\int_{T_m}\nabla\left(\int_{T_n}G^{\Phi}(\bar{r}\,|\,\bar{r}')\left(\nabla'_S\cdot\vec{f}_n(\bar{r}')\right)ds'\right)\cdot\vec{f}_m(\bar{r})ds\end{aligned}\tag{2}$$

where $\vec{f}_n$ and $\vec{f}_m$ stand for the RWG basis and weighting functions, respectively. T_n and T_m are the triangular pairs containing the source ($\bar{r}'$) and field ($\bar{r}$) point, respectively. By using the Green's identity and the numerical Gaussian integral over triangular meshes, the integral equation (2) can be deduced as an algebraic linear system. As is proposed in [10], the delta-gap voltage model is adopted to excite the physical port, and the matrix element involved can be calculated as [11]:

$$\begin{aligned} Z_{mn} &= k_0^2 \int_{T_m}\int_{T_n} \vec{f}_n(\bar{r}')\cdot \underline{\underline{G}}^A(\bar{r}\,|\,\bar{r}')\cdot \vec{f}_m(\bar{r})ds'ds \\ &-\int_{T_m}\int_{T_n}\left(\nabla'_S\cdot \vec{f}_n(\bar{r}')\right)G^{\Phi}(\bar{r}\,|\,\bar{r}')\left(\nabla_S\cdot \vec{f}_m(\bar{r})\right)ds'ds \end{aligned} \quad (3)$$

For the microstrip planar circuits, the S-parameters are usually extracted, which depend on the incident and reflected wave of the dominant mode. To observe the recognizable standing-wave feature on the feed line, the reference planes should be selected away from not only the discontinuities but also the exciting ports. The generalized pencil-of-function (GPOF) is adopted in this paper. After the curve-fitting operation, the current distribution can be written as [11]:

$$\begin{aligned} I(z) &\approx \sum_{i=1}^{N} p_i \exp(\gamma_i z) \\ &= \sum_{i=1}^{N} p_i \exp\left[(\alpha_i + j\beta_i)z\right] \;, \qquad z>0 \end{aligned} \quad (4)$$

where p_i is the amplitude of the i-th mode. α_i and β_i stand for the propagation constant of the i-th mode. From the physical point of view, the first two terms, namely, (p_1 α_1 β_1) and (p_2 α_2 β_2) are just the incident and reflected wave of the dominant mode. Therefore, the S_{11} can be easily obtained.

III. IMPEDANCE MATRIX INTERPOLATION SCHEME

It is a well-known fact that although most parameters of microstrip structures, such as S-parameters and the induced current distributions, varies rapidly with frequencies, however, the impedance matrix elements appear much smoother behaviors. Therefore, it enlighten us to introduce Lagrange polynomial interpolations to fit the impedance matrix elements.

In this paper, two sampling rules are realized, namely, equally spaced sampling and sampling with Chebyshev zeros in the frequency band of interest, as are shown in (5) and (6), respectively.

$$f_i = f_l + (f_h - f_l)\frac{i}{N} \quad (5)$$

$$f_i = \frac{(f_h - f_l)}{2}\cdot\cos\left(\frac{2i+1}{2N+2}\right) + \frac{(f_h + f_l)}{2} \quad (6)$$

where $[f_l, f_h]$ is the frequency band of interest and the number of sampling points is N+1. With accurate calculate the impedance matrix at these sampling frequencies, the rest can be approximated by Lagrange polynomial interpolation:

$$Z_{mn}^{\text{Inter}}(f) = \sum_{i=0}^{N} Z_{mn}^{\text{sample}}(f_i)\Psi_i(f) \quad (7)$$

$$\Psi_i(f) = \prod_{j=0,\,j\neq i}^{j=N}\left(\frac{f - f_j}{f_i - f_j}\right) \quad (8)$$

To estimate the relative error of the interpolation scheme, as well as to establish an adaptive algorithm, the Frobenius norms of impedance matrices are adopted [13] and the relative error can be expressed as:

$$\delta_{P+1} = \frac{\left\|[Z]_{P+1} - [Z]_P\right\|_{\text{Frobenius}}}{\left\|[Z]_P\right\|_{\text{Frobenius}}} \quad (9)$$

where the subscripts P stand for the number of sampling points. Considering the imagine parts of impedance matrices are much larger than the real parts, it is more reasonable to calculate δ for real and imagine parts separately.

IV. NUMERICAL EXAMPLES

In this paper, a microstrip-fed patch antenna is modeled [12] and analyzed by the proposed method. By using the "PDETOOL" in Matlab, totally 353 triangular elements are generated, corresponding to 486 RWG pairs, as is shown in Fig. 1. According to the definition of relative error in (9), the threshold should be appointed before the adaptive scheme. The behavior of the relative errors is shown in Fig. 2, in which both equally spaced sampling and sampling with Chebyshev zeros in the whole frequency band of interest are calculated. Fig. 3 shows the relative errors between the interpolating impedance matrix and the standard matrix from direct MoM.

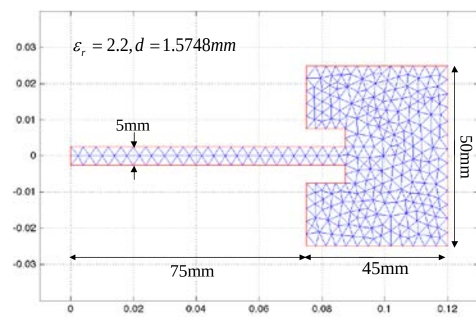

Fig.1.Planform of a patch antenna meshed by triangles

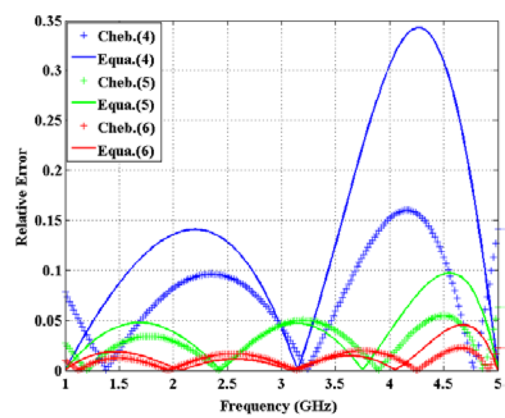

Fig 2. Relative errors of impedance matrix by different interpolation schemes

From these two figures, as the number of sampling nodes N increases, the relative error decreases rapidly. Also, the relative error of Chebyshev zeros interpolation scheme, comparing to that of equally spaced sampling interpolation scheme, tends to be more stable within the whole frequency band. Fig.4 shows the scattering parameter, S_{11}, calculated by the two interpolation schemes and the direct MoM, where 6 sampling points are selected in each interpolation scheme. Very good agreements can be found in Fig.4. However, from the stable point of view, interpolation by sampling with Chebyshev zeros is preferred because of its steady behavior of the relative error in the whole frequency band.

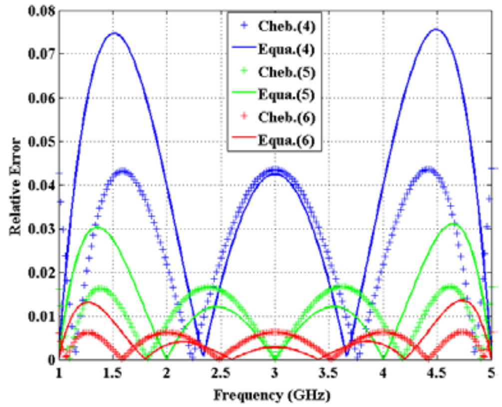

Fig. 3 Relative errors between impedance matrix interpolated by proposed schemes and that by direct MoM

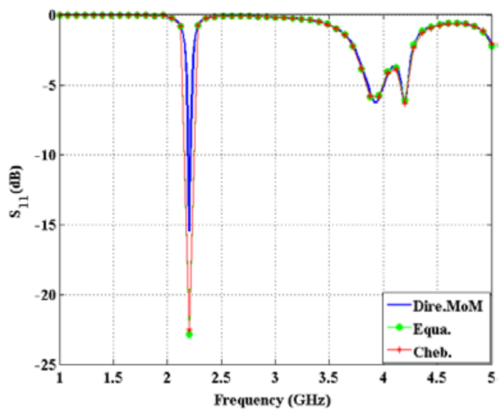

Fig. 4 S_{11} parameter for different methods

Table 1. Accuracy and Efficiency of the proposed method

RWG: 468, [1:0.02:5] GHz || Macbook Pro @ 2.5GHz，16G RAM		
Scheme	Time Cost (s)	Max Relative Error (Compare to Dire. MoM)
HFSS(discrete)	3,686	—
Dire. MoM	25,037	—
3-Cheb. Interp.	529	11.15% (-10dB)
4-Cheb. Interp.	690	4.37% (-14dB)
5-Cheb. Interp.	836	1.67% (-18dB)
6-Cheb. Interp.	949	0.63% (-22dB)
7-Cheb. Interp.	1071	0.26% (-26dB)
8-Cheb. Interp.	1189	0.11% (-30dB)

Table. 1 shows the time consumptions for different methods performed on a Macbook Pro with 2.5GHz and 16G RAM. From this table, we can find that interpolation by sampling with 6 Chebyshev zeros consumes only one third and one twenty-fifth of time by ANSYS HFSS (discrete model) and direct MoM, respectively, while the relative error is only 0.63%, which verifies the high efficiency and accuracy of the proposed scheme. Besides, a good statistical property can be found from the last two columns in this table, that is, as the sampling nodes increase by one, the time cost for calculation increases 132s in average, while the maximum relative error decreases 4dB.

V. CONCLUSION

An adaptive frequency sweeping algorithm based on Lagrange polynomial is investigated for interpolating impedance matrix of MoM. Both equally spaced sampling and sampling with Chebyshev zeros are realized and discussed. By introducing the Frobenius norms, the relative error of the interpolation matrices are effectively evaluated. From the numerical example, sampling with Chebyshev zeros has the advantage of error control within the whole frequency band. This algorithm yields accurate result in scattering parameters as those from ANSYS HSFF and direct MoM, while enhancing the efficiency more than 3 and 25 times, respectively.

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REFERENCES

[1] T. Itoh, Ed., *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, New York: Wiley, 1989, ch,3.

[2] R. E. Collin, *Field Theory of Guided Waves*, New York: McGraw-Hill, 1960.

[3] W. C. Chew, *Waves and Fields in Inhomogeneous Media*, ser. Electromagn. Waves Piscataway, NJ: IEEE Press, 1995.

[4] R. F. Harrington, *Field computation by Moment Methods*. Melbourne, FL: Krieger, 1983.

[5] D. G. Fang, *Antenna Theory and Microstrip Antennas*. Beijing: Science Press, 2006.

[6] D. G. Fang, J. J. Yang, and G. Y. Delisle. “Discrete image theory for horizontal electric dipoles in a multilayered medium above a conducting ground plane,” *IEE Proc. H*, pp.135 (5): 297-303, 1988.

[7] Z. Song, H. -X. Zhou, J. Hu, W. –D. Li and W. Hong, “Accurate Location of All Surface Wave Modes for Green’s Functions of a Layered Medium by Consecutive Perturbations,” *SCIENCE CHINA Information Sciences*. 2010;53(11):2363-2376.

[8] Song Z, Zhou H-X, Zheng K-L et. al, “Accurate Evaluation of Green’s Functions for Lossy Layered Medium by Fast Locating Surface and Leaky Wave Modes”. *IEEE Antennas and Propagation Magazine*. Vol. 55, pp.92-102.

[9] S. M. Rao, D. R. Wilton, A. W. Glisson., "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat*., vol. 30, pp. 409-418, 1982.

[10] G. V. Eleftheriades and J. R. Mosig, "On the network characterization of planar passive circuits using the method of moments," *IEEE. Trans. Microwave Theory Tech.*, vol. 44, pp. 438-445, 1996.

[11] Z. Song, Fast Algorithms for Spatial Domain Dyadic Green's Functions of Stratified Medium and its Applications in Electromagnetic Simulation of Layered Passive Circuits. Ph. D. Thesis in Southeast Univ. 2011.

[12] Yeo J, Mittra R. An algorithm for interpolating the frequency variations of method-of-moments matrices arising in the analysis of planar microstrip structures [J]. Microwave Theory and Techniques, IEEE Transactions on. 2003;51(3):1018-1025.

[13] W. D. Li, H. X. Zhou, J. Hu, Z. Song and W. Hong, “Accuracy improvement of cubic polynomial inter/extrapolation of MoM matrices by optimizing frequency samples”. Antenna and Wireless Propagation Letter, IEEE. 2011;10:888-891.