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Numerical Analysis on Synchronization of Four Metronomes

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Abstract—We numerically investigate behavior of four weakly coupled metronomes using equations of motion of the metronomes. In the numerical simulations, the parameter values of the equations of motion are experimentally decided from the experimental apparatus, where the four metronomes are put on a board hung by strings. As a result, we found that synchronization of metronomes depend on frequency and initial angles. In addition, we also found the importance of the individual difference between metronomes, which becomes clear by the numerical simulations.

1. Introduction

In our world, various rhythms exist, for example seasonal transitions, neuronal activities, and collective behavior of animals and insects[1][2]. In these real systems, even though each individual has its own rhythm, it is widely acknowledged that their rhythms synchronize by mutual couplings or external forces. We can also observe the synchronization by using weakly coupled metronomes[3][4][5]. For example, if we put a number of metronomes on a board hung by strings, the metronomes eventually synchronize.

In this paper, we numerically investigate the synchronization phenomenon of metronomes by using a mathematical model of the motion of four metronomes which are put on a board hung by strings. When we conduct numerical simulations, we used the parameter values in the mathematical model estimated from the handmade experimental apparatus. As a result, the time necessary for the in-phase synchronization becomes shorter when the frequency of the board becomes larger.

2. Mathematical model

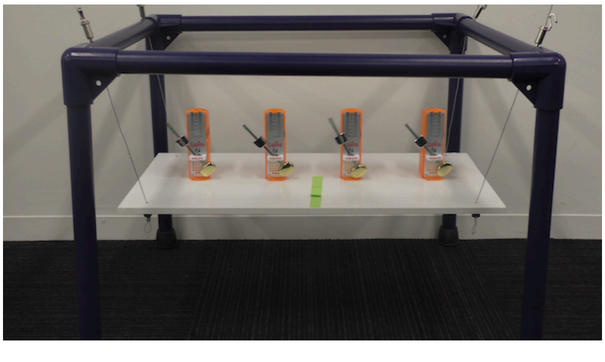


Figure 1: Experimental apparatus

Figure 1 shows a handmade experimental apparatus. The experimental apparatus is comprised of four metronomes put on a wooden board hung by four strings. The equation of motion of the experimental apparatus shown in Fig. 1 is approximately described as follows:

$$\ddot{\theta_i} = -2\zeta_i\omega_i\dot{\theta}_i - \omega_i^2 \sin\theta_i - (\omega_i/\omega_p)^2\ddot{\theta_p}\cos\theta_i, \quad (1)$$

$$\alpha\ddot{\theta}_p = -2\zeta_p\omega_p\dot{\theta}_p - \alpha\omega_p^2\theta_p - \gamma\eta\sum_{i=1}^{N}\ddot{\theta}_i(\theta_p\sin\theta_i + \cos\theta_i) - \omega_p\gamma\eta\sum_{i=1}^{N}\dot{\theta}_i^2(\theta_p\cos\theta_i - \sin\theta_i), \quad (2)$$

where θ [rad] is an angle, ω [rad/s] is a natural angular frequency, ζ is a damping ratio, $\alpha = (1 + N\gamma), \gamma$ is a ratio of mass of a metronome to that of the board, η is a ratio of the length of the pendulum attached the metronome to the length of the string. The subscript i of θ, ω, and ζ corresponds to the ith metronome, and p corresponds to the board. In the numerical simulations, we determined the parameter values in Eqs.(1) and (2) from the experimental apparatus shown in Fig. 1. The oscillation frequency f_i of the ith metronome is estimated from a metronome called 'Lupina' produced by Nikko Seiki Co., Ltd [4]. Even if we set $f_i (i = 1, 2, 3, 4)$ to 1.4 [Hz] (168[bpm]) by adjusting the sliding weight of the pendulum in Lupina, there exist individual differences between the metronomes. Then, oscillation frequencies are slightly different from each other. The estimated values are $f_1 = 1.385$ [Hz], $f_2 = 1.376$ [Hz], $f_3 = 1.389$ [Hz] and $f_4 = 1.382$ [Hz]. From the oscillation frequencies, the angular frequency is calculated by $\omega_i = 2\pi f_i$. In addition, the values of parameter of the pendulums are $\zeta_1 = 0.0226, \zeta_2 = 0.0228, \zeta_3 = 0.0231$, and $\zeta_4 = 0.0237$. The parameter values of the board are $\zeta_p = 0.00113, \gamma = 0.024, \eta = 0.01\omega_p^2/g$, and $\alpha = 1.072$. In the numerical simulations, the absolute value of the angular velocity ω_i of the pendulum is increased by 25.8 [deg/s] due to the impulsive force of the metronome when the angle θ_i becomes $\pm 10°$. Then, metronome can continue to oscillate.

3. Results

3.1. Time-series data

Figures 2 and 3 show the time-series of the angle of each metronome. The horizontal axis is time [s], and the vertical axis is the angle θ_i [deg].

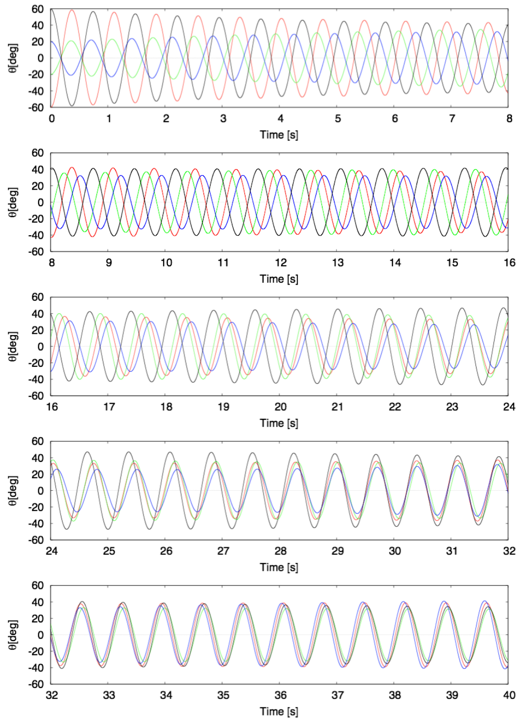


Figure 2: Time-series $\theta_i(t)$ of four metronomes. f_p = 1.200[Hz], $\theta_1(0)$ = 60° , $\theta_2(0)$ = 20°, $\theta_3(0)$= −20°, and $\theta_4(0)$ = −60°.θ_i's are plotted in red, green, blue and black curves.

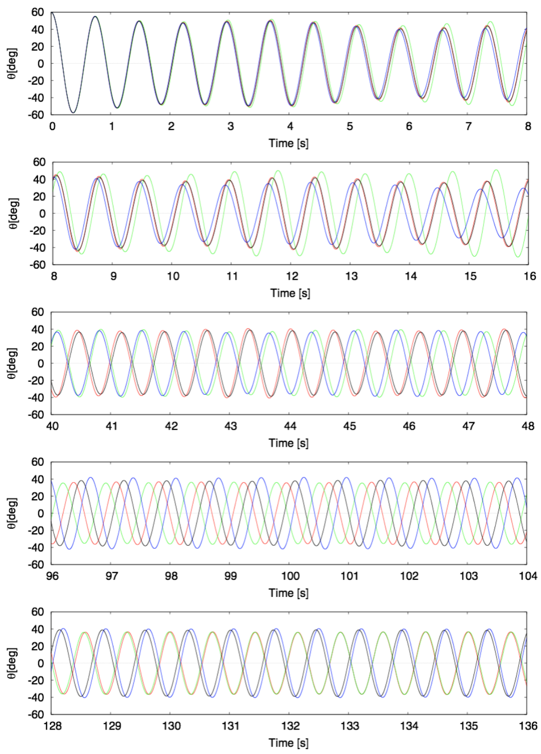


Figure 3: The same as Fig.1, but for f_p = 1.600[Hz], and $\theta_1(0) = \theta_2(0) = \theta_3(0) = \theta_4(0)$ = 60°.

In Fig. 2, the initial angles of the four metronomes are $\theta_1(0)$ = 60° , $\theta_2(0)$ = 20°, $\theta_3(0)$= −20°, and $\theta_4(0)$ = −60°. Although the amplitudes of θ_2 and θ_3 are small at the initial state, their amplitudes reach to their maximum level which is about 60° when the time is around 8 [s]. After 40 second later, the four metronomes starts to synchronize. In Fig. 3, $\theta_1(0) = \theta_2(0) = \theta_3(0) = \theta_4(0)$ = 60°. Although the initial angles of all metronomes are set to 60°, they start to separate from each other when the time is around 8 [s] because of the individual differences. We can then observe that a specific pair of metronomes synchronizes when $40 \leq t \leq 48$. However, their synchronized metronomes separate after 96 seconds, and then a different pair of metronomes synchronize.

3.2. Relationship between frequencies of the metronomes and the board

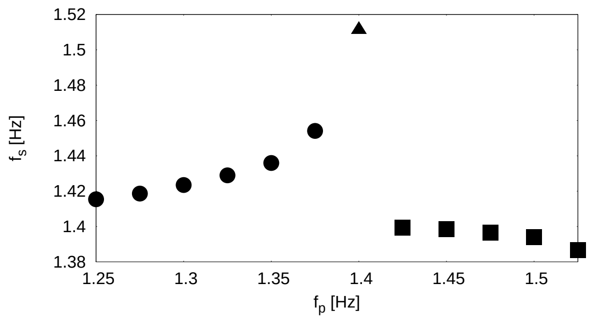


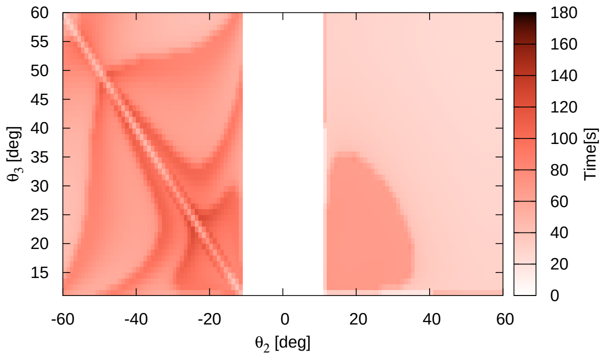
Figure 4: Relations between the mean frequency f_s of the metronomes in the synchronous state and the frequency f_p of the board.

In Fig. 4, the circles indicate the in-phase synchronization, the triangle indicates the state that the in-phase synchronization and the anti-phase synchronization coexist, and the squares indicates the anti-phase synchronization. The horizontal axis shows the frequency of the board, and the vertical axis shows the average frequency of the four metronomes.

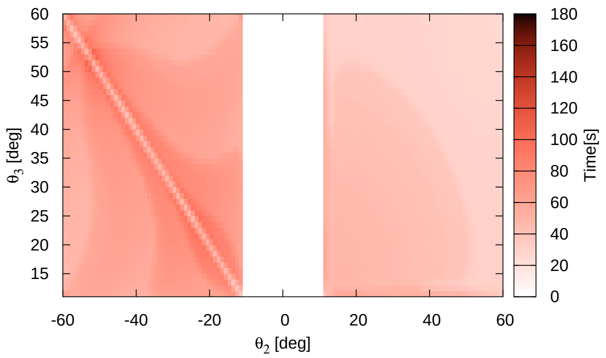
As shown in Fig. 4, when the frequencies of the metronomes are equal to that of board, f_s reaches the largest value. From these results, when f_s is large, the metronomes acquire energy from the board. When the frequencies of the metronomes approaches to that of the board, the metronomes are likely to acquire energy from the board.

3.3. Time necessary for the in-phase synchronization

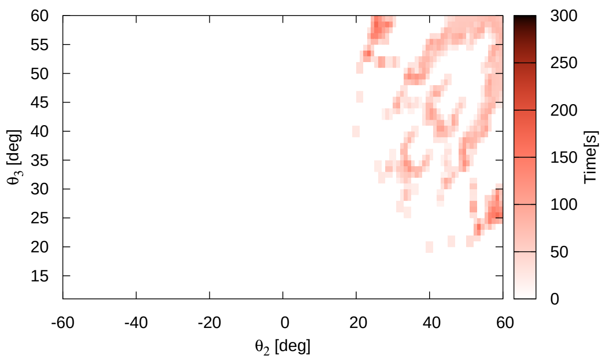
We next investigate the time required for the in-phase synchronization when we change the frequency of the board. In these numerical experiments, we set the initial states of the metronomes to $\theta_1(0)$ = 60°, $\theta_4(0)$ = −60°, $\theta_2(0) \in [-60°, 60°]$, and $\theta_3(0) \in [10°, 60°]$.



(a) f_p = 1.2 Hz

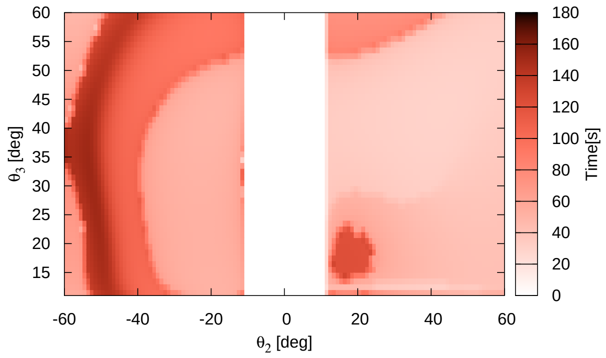


(b) f_p = 1.3 Hz

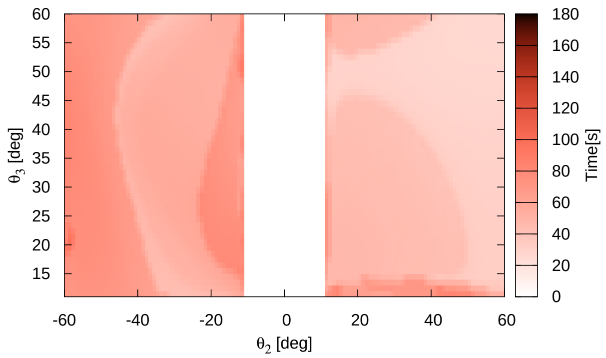


(c) f_p = 1.4 Hz

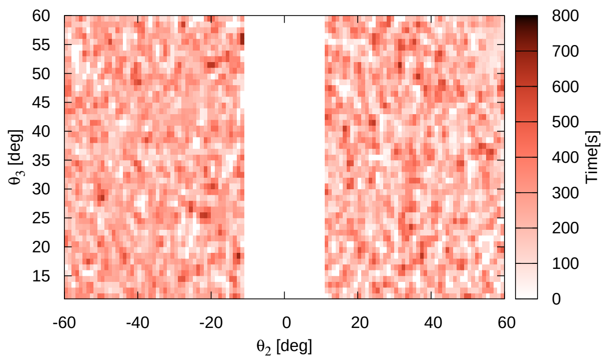
Figure 5: Time necessary for the in-phase synchronization when metronomes are identical.



(a) f_p = 1.2 Hz



(b) f_p = 1.3 Hz



(c) f_p = 1.4 Hz

Figure 6: Time necessary for the in-phase synchronization when metronomes are different.

The white areas in Figs. 5 and 6 indicate the damping oscillation. The colors show time necessary for the in-phase synchronization. From Fig. 5 (a) and (b), the time necessary for the in-phase synchronization becomes shorter when the frequency of the board becomes larger.

4. Conclusion

We fixed the frequencies of four metronomes to 1.4[Hz] in this study and performed numerical simulation of synchronization of four metronomes. When the frequency of the metronomes approaches to that of the board, the metronomes are likely to acquire energy from the board. When the frequency of the metronome is larger than that of the board, we showed that metronomes synchronize within approximately 40 seconds. In addition, we showed examples of the phase set to two specific initial states when the frequency of the metronome is smaller than the frequency of the board. We clarified that the frequency of the metronomes reaches the highest value when the natural frequency of the metronome is close to that of the board. In case of four metronomes, we can set initial angles into two pairs of symmetric angles in comparison with an experiment in the synchronization of three metronomes[5]. We confirmed that time necessary for the in-phase synchronization becomes shorter if there was no individual difference. Furthermore, we confirmed that the initial angles which accomplish the synchronization randomly exist when the frequency of the metronomes and the frequency of the board are identical. When the frequencies of the metronome and the board are identical and the metronomes synchronize, the frequency of the metronomes increases. However, the range of synchronization with no individual difference becomes small. Therefore, existence of the individual difference plays an important role for synchronization.

In this paper, we conducted only numerical simulations of the four metronomes. However, it is inevitable to compare numerical results and experimental results. As for the case of three metronomes, we have already reported several interesting results[5]. It is an important future issue to analyze the motion of four metronomes experimentally.

Acknowledgments

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