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Network analyses of chaotic systems

Miki U. Kobayashi† and Yoshitaka Saiki‡

†Faculty of Economics, Rissho University, 4-2-16 Osaki, Shinagawa-ku, Tokyo 141-8602, Japan
‡Graduate School of Commerce and Management, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan
Email: miki@ris.ac.jp

Abstract—It is found that chaotic dynamical systems possess infinitely many unstable periodic orbits embedded in a chaotic set. A chaotic motion can be considered as an irregular transition process among the unstable periodic orbits. This irregular transition process is better described as a network. Recently, we have proposed a concrete method for describing a chaotic motion as a network, in which the unstable periodic orbits are regarded as nodes and links are provided based on the transition process of a chaotic motion among the unstable periodic orbits.

General networks can be roughly divided into the following three kinds; regular, random and complex networks. Various real networks are complex networks, i.e. power and neural networks, networks in linguistics, and world wide web and so on. It is known that smaller average path length, larger clustering coefficient and scale free degree distribution are typical properties of complex networks. A problem for kinds of networks of chaos is open.

We consider the Lorenz system: $\dot{x} = -10x + 10y, \dot{y} = -xz + rx - y, \dot{z} = xy - (8/3)z$. Networks of the Lorenz system are constructed by our scheme. Average path length, clustering coefficient and degree distiribution are analysed in the network structures of the Lorenz system. We reveal that networks of Lorenz chaos is a kind of complex networks. Futhermore, we discuss network structures of the system with $r = 28$ whose the system is singular hyperbolic and with $r = 60$ whose the system is non-hyperbolic.

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