1D Modified Unsplitted PML ABCs for truncating Anisotropic Medium

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***Abstract*-Based on the FDTD method in anisotropic medium, the implementation of the modified NPML absorbing boundary conditions for truncating anisotropic medium is presented. By using the partial derivatives of space variables stretched-scheme in the coordinate system, the programming complexity is reduced greatly. According to one dimensional numerical simulation analysis, the modified NPML absorbing boundary condition is validated.**

I INTRODUCTION

In order to simulate the open-domain electromagnetic scattering questions, the FD-TD methods introduce absorbing boundary conditions (ABCs) which is used to truncate an infinite problem space to a finite computation domain for us to simulate the electromagnetic scattering question. In 1981, Mur put forwarded the Mur ABCs with the FDTD discrete form in the computational domain truncation at the boundary [1]. This is an effective FDTD absorbing boundary condition and widely available. In 1994, 1996 Berenger proposed extended Maxwell's equations to splitted field form and constitutes a perfectly matched layer (PML), which is a highly effective absorbing boundary condition has been adapted in a variety of ways [2,3]. The Uniaxil Perfectly Matched Layer (UPML) were presented [4~7], which has been used successfully in the FD-TD computation for open-region electromagnetic problems.

Cummer has introduced a new kind of PML formulation named as Nearly Perfectly Matched Layer (NPML) [8], and was obtained by inserting the stretching factor of the PML within the derivatives on space of the curls. Hu and Cummer observed that the reflection from a NPML is as low as that from a normal PML [9]. The first implementation of the NPML for truncating nonlinear dispersive FDTD grids is presented by O. Ramadan [10]. Yang etc studied a novel non-splitted field perfectly matched layer ABC to truncate anisotropic magnetized plasma in FDTD computation [11]. The advantage of NPML is a simplicity ABC, and has the important application value in the truncated anisotropic medium.

Based on the FDTD method in anisotropic medium, a modified NPML ABC, this paper introduced a modified NPML ABC for truncating anisotropic medium. The programming complexity is reduced greatly by using the partial derivatives of special variables stretched-scheme in the coordinate system. The validity of this modified NPML ABC is proved through theoretical analysis and one-dimensional numerical simulation analysis.

II FDTD FORMULATION OF MODIFIED NPML ABSORBING BOUNDARY CONDITION FOR TRUNCATING ANISOTROPIC MEDIUM

For a homogeneous anisotropic medium, Maxwell's equations in time domain are given as follows:

$$\nabla \times \boldsymbol{H} = \boldsymbol{\varepsilon} \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{\sigma} \boldsymbol{E} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\boldsymbol{\mu} \frac{\partial \boldsymbol{H}}{\partial t} - \boldsymbol{\sigma_m} \boldsymbol{H} \tag{2}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ is the magnetic field, the permittivity tensor is given by $\boldsymbol{\varepsilon}=[\varepsilon_{ij}]$ and the permeability tensor is given by $\boldsymbol{\mu}=[\mu_{ij}]$, i,j=1,2,3. The conductivity tensor is given by $\boldsymbol{\sigma}=[\sigma_{ij}]$ and the magneto conductivity tensor is given by $\boldsymbol{\sigma_m}=[\sigma_{mij}]$, i,j=1,2,3.

Suppose the 1D TEM electromagnetic waves with fields vary in the -z direction of anisotropic medium, i.e. $\partial/\partial x=0$, $\partial/\partial y=0$. In Cartesian coordinates, Eq.(1) and Eq.(2) can be written as follows:

$$-\frac{\partial H_y}{\partial z} = \varepsilon_{11}\frac{\partial E_x}{\partial t} + \varepsilon_{12}\frac{\partial E_y}{\partial t} + \sigma_{11}E_x + \sigma_{12}E_y \tag{3}$$

$$\frac{\partial H_x}{\partial z} = \varepsilon_{21}\frac{\partial E_x}{\partial t} + \varepsilon_{22}\frac{\partial E_y}{\partial t} + \sigma_{21}E_x + \sigma_{22}E_y \tag{4}$$

$$-\frac{\partial E_y}{\partial z} = -\mu_{11}\frac{\partial H_x}{\partial t} - \mu_{12}\frac{\partial H_y}{\partial t} - \sigma_{m11}H_x - \sigma_{m12}H_y \tag{5}$$

$$\frac{\partial E_x}{\partial z} = -\mu_{21}\frac{\partial H_x}{\partial t} - \mu_{22}\frac{\partial H_y}{\partial t} - \sigma_{m21}H_x - \sigma_{m22}H_y \tag{6}$$

Based on the stretching coordinate transform, replacing ∂z by $\partial \tilde{z} = \left(1+\sigma_z / j\omega\right)\partial z$, then Eqs. (3)～(6) become

$$-\frac{\partial H_y}{\partial \tilde{z}} = \varepsilon_{11}\frac{\partial E_x}{\partial t} + \varepsilon_{12}\frac{\partial E_y}{\partial t} + \sigma_{11}E_x + \sigma_{12}E_y \tag{7}$$

$$\frac{\partial H_x}{\partial \tilde{z}} = \varepsilon_{21}\frac{\partial E_x}{\partial t} + \varepsilon_{22}\frac{\partial E_y}{\partial t} + \sigma_{21}E_x + \sigma_{22}E_y \tag{8}$$

$$-\frac{\partial E_y}{\partial \tilde{z}} = -\mu_{11}\frac{\partial H_x}{\partial t} - \mu_{12}\frac{\partial H_y}{\partial t} - \sigma_{m11}H_x - \sigma_{m12}H_y \tag{9}$$

$$\frac{\partial E_x}{\partial \tilde{z}} = -\mu_{21}\frac{\partial H_x}{\partial t} - \mu_{22}\frac{\partial H_y}{\partial t} - \sigma_{m21}H_x - \sigma_{m22}H_y \tag{10}$$

According to Ref. [9], we can obtain

$$\frac{\partial H_y}{\partial \tilde{z}} = \frac{\partial H_y}{(1+\sigma_z/j\omega)\partial z} = \frac{\partial\left(H_y/(1+\sigma_z/j\omega)\right)}{\partial z}$$

Let $\tilde{H}_{yz} = H_y/(1+\sigma_z/j\omega)$, Eq.(7) becomes

$$-\frac{\partial \tilde{H}_{yz}}{\partial z} = \varepsilon_{11}\frac{\partial E_x}{\partial t} + \varepsilon_{12}\frac{\partial E_y}{\partial t} + \sigma_{11}E_x + \sigma_{12}E_y \tag{11}$$

Similarly, applying this coordinate transformation and introducing new variables $\tilde{E}_{xz}$, $\tilde{H}_{yz}$ and $\tilde{H}_{xz}$ to Eqs.(8) ~(10), then we have

$$\frac{\partial \tilde{H}_{xz}}{\partial z} = \varepsilon_{21}\frac{\partial E_x}{\partial t} + \varepsilon_{22}\frac{\partial E_y}{\partial t} + \sigma_{21}E_x + \sigma_{22}E_y \tag{12}$$

$$-\frac{\partial \tilde{E}_{yz}}{\partial z} = -\mu_{11}\frac{\partial H_x}{\partial t} - \mu_{12}\frac{\partial H_y}{\partial t} - \sigma_{m11}H_x - \sigma_{m12}H_y \tag{13}$$

$$\frac{\partial \tilde{E}_{xz}}{\partial z} = -\mu_{21}\frac{\partial H_x}{\partial t} - \mu_{22}\frac{\partial H_y}{\partial t} - \sigma_{m21}H_x - \sigma_{m22}H_y \tag{14}$$

According to Eqs. (11)-(14), we obtain FDTD iterative formula of the electric and magnetic field yield as follows:

$$E_x^{n+1}(k) = \frac{1}{\delta}\left\{\begin{array}{l}\left(\frac{\varepsilon_{22}}{\Delta t}+\frac{\sigma_{22}}{2}\right)\left[\begin{array}{l}\frac{\tilde{H}_{yz}^{n+0.5}(k-\frac{1}{2})-\tilde{H}_{yz}^{n+0.5}(k+\frac{1}{2})}{\Delta z}\\ +\left(\frac{\varepsilon_{11}}{\Delta t}-\frac{\sigma_{11}}{2}\right)E_x^n(k)+\left(\frac{\varepsilon_{12}}{\Delta t}-\frac{\sigma_{12}}{2}\right)E_y^n(k)\end{array}\right]\\ +\left(\frac{\varepsilon_{12}}{\Delta t}+\frac{\sigma_{12}}{2}\right)\left[\begin{array}{l}\frac{\tilde{H}_{xz}^{n+0.5}(k-\frac{1}{2})-\tilde{H}_{xz}^{n+0.5}(k+\frac{1}{2})}{\Delta z}\\ +\left(\frac{\sigma_{21}}{2}-\frac{\varepsilon_{21}}{\Delta t}\right)E_x^n(k)+\left(\frac{\sigma_{22}}{2}-\frac{\varepsilon_{22}}{\Delta t}\right)E_y^n(k)\end{array}\right]\end{array}\right\} \tag{15}$$

where $\delta = \left(\frac{\varepsilon_{11}}{\Delta t}+\frac{\sigma_{11}}{2}\right)\left(\frac{\varepsilon_{22}}{\Delta t}+\frac{\sigma_{22}}{2}\right)-\left(\frac{\varepsilon_{12}}{\Delta t}+\frac{\sigma_{12}}{2}\right)\left(\frac{\varepsilon_{21}}{\Delta t}+\frac{\sigma_{21}}{2}\right)$.

Similarly, we consider the same derivation with $E_y^{n+1}(k)$, $H_x^{n+0.5}(k+0.5)$, and $H_y^{n+0.5}(k+0.5)$.

Now, we consider the new variable $\tilde{H}_{xz}$,

$$\tilde{H}_{xz} = \frac{H_x}{s_z} = \frac{H_x}{1+\frac{\sigma_z}{j\omega\varepsilon_0}} \tag{16}$$

Then Eq.(16) is rewritten as follows:

$$j\omega\varepsilon_0\tilde{H}_{xz} + \sigma_z\tilde{H}_{xz} = j\omega\varepsilon_0 H_x \tag{17}$$

By transformation, Eq.(17) becomes

$$\frac{\partial \tilde{H}_{xz}}{\partial t} + \sigma_z\tilde{H}_{xz} = \frac{\partial H_x}{\partial t} \tag{18}$$

Using the center differential scheme, we can discrete Eq.(18) as follows:

$$\tilde{H}_{xz}\Big|_{k+\frac{1}{2}}^{n+0.5} = \frac{1-\frac{\sigma_z\Delta t}{2}}{1+\frac{\sigma_z\Delta t}{2}}\tilde{H}_{xz}\Big|_{k+\frac{1}{2}}^{n-0.5} + \frac{1}{1+\frac{\sigma_z\Delta t}{2}}\left(H_x\Big|_{k+\frac{1}{2}}^{n+0.5} - H_x\Big|_{k+\frac{1}{2}}^{n-0.5}\right) \tag{19}$$

Similarly, we have

$$\tilde{E}_{xz}\Big|_k^{n+1} = \frac{1-\frac{\sigma_z\Delta t}{2}}{1+\frac{\sigma_z\Delta t}{2}}\tilde{E}_{xz}\Big|_k^n + \frac{1}{1+\frac{\sigma_z\Delta t}{2}}\left(E_x\Big|_k^{n+1} - E_x\Big|_k^n\right), \tag{20}$$

$$\tilde{E}_{yz}\Big|_k^{n+1} = \frac{1-\frac{\sigma_z\Delta t}{2}}{1+\frac{\sigma_z\Delta t}{2}}\tilde{E}_{yz}\Big|_k^n + \frac{1}{1+\frac{\sigma_z\Delta t}{2}}\left(E_y\Big|_k^{n+1} - E_y\Big|_k^n\right), \tag{21}$$

$$\tilde{H}_{yz}\Big|_{k+\frac{1}{2}}^{n+0.5} = \frac{1-\frac{\sigma_z\Delta t}{2}}{1+\frac{\sigma_z\Delta t}{2}}\tilde{H}_{yz}\Big|_{k+\frac{1}{2}}^{n-0.5} + \frac{1}{1+\frac{\sigma_z\Delta t}{2}}\left(H_y\Big|_{k+\frac{1}{2}}^{n+0.5} - H_y\Big|_{k+\frac{1}{2}}^{n-0.5}\right). \tag{22}$$

III VALIDATION AND NUMERICAL RESULTS

Suppose the TEM wave vertically incident on a half-space anisotropic medium, the reflection of propagating wave has been computed. In the simulations, the incident wave is a Gaussian pulse plane wave whose frequency spectrum from 0 GHz to 1 GHz. The space cells size Δz=75μm and the time step $\Delta t=\Delta z/2c$ is 0.125ps, where c is the velocity of electromagnetic wave. The modified NPML ABCs are applied to both ends of computation domain. The thickness of the NPML ABC is $6\Delta z$.

In the first simulation, we validate the reflection coefficients for uniaxial anisotropic media. In FDTD computation, ε_{11}=4, ε_{22}=4, ε_{33}=16, the reflection coefficients of uniaxially anisotropic media are computed by using the modified NPML ABCs. They are compared with those of the analytical solution [12], shown in Figure.1. Numerical results are presented to verify the effectiveness of the proposed method, and demonstrate that the modified NPML ABCs method is in good agreement with the analytical solution.

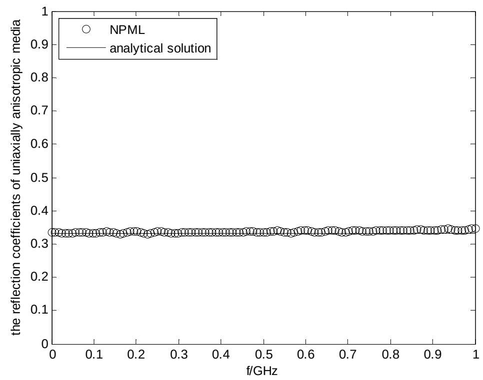

Figure 1 The reflection coefficients of uniaxially anisotropic media

In the second simulation, we let the parameters ε_{11}=9, ε_{22}=4, ε_{33}=16. By Numerical simulation, we obtain the Fig.2. The reflection coefficients of biaxial anisotropic media computed by using the modified NPML ABCs are compared with those of the analytical solution [13]. The results also show the modified NPML ABCs method is good agreement with the analytical solution, and the validity of the modified NPML ABC is proved, too. The results in Figure 2 also show that the modified NPML ABCs perform very well for the biaxial anisotropic media.

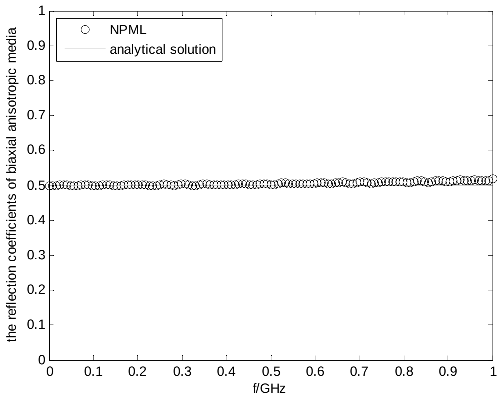

Figure2 The reflection coefficients of biaxial anisotropic media

IV CONCLUSION

In this paper, we have shown that based on the FDTD method for electromagnetic scattering by anisotropic medium and NPML, the modified NPML absorbing boundary conditions are presented, which can be used to truncate the anisotropic medium. Two numerical examples were given to illustrate that the proposed formulations can achieve a relatively simple programming complexity and the numerical examples also show that the modified NPML ABCs provide good absorbing performance. Therefore, the modified NPML ABCs have a large advantage in truncating anisotropic dispersion medium.

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