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Synchronization phenomena of piecewise-linear oscillators coupled by a hysterisis element

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Abstract—In this study, we consider two-dimensional piecewise-linear oscillators with a diode coupled by a hysteresis element. Coexistence phenomena of in-phase and an anti-phase synchronization are confirmed. We have examined parameter conditions where coexistence phenomena can be observed. Typical behavior are confirmed in laboratory.

1. Introduction

In nature and engineering fields, we observe often a synchronization phenomena which is one of the interesting non linear phenomena. For example, synchronization chirp of frog and synchronization of the neural network to perform the information processing of the human brain are well known[1]. However, because of complexity of large-scale network, it is often difficult to analyze the phenomena. Therefore, coupled systems of oscillators by using electrical circuits are good model for considering the synchronization phenomena. The van der Pol oscillator is one of the famous nonlinear oscillators, and it have been studied by many researches[2]. It is well-known that in-phase and anti-phase synchronization are observed in the van der Pol oscillators coupled by a resister and a native resister, respectively. On the other hand, in the case of the oscillators coupled by a inductor, coexistence phenomena of in-phase and anti-phase synchronization are observed. The coexistence phenomena are important to understand interesting nonlinear phenomena observed in large scale networks[3].

In our previous researches, we have focused on a coupled system of piecewise-constant oscillators. If the oscillators coupled by a voltage controlled current source (VCCS) with signum characteristic,either in-phase or anti-phase synchronization are observed, and if the oscillators coupled by a VCCS with hysteresis, they exhibit coexistence of in-phase and anti-phase synchronization[4]. Since difference between the signum function and the hysteresis is characterized by width of hysteresis, we can consider the bifurcation phenomena by choosing the hysteresis width as a bifurcation parameter. Therefore, we have been investigated the behavior of van der Pol oscillators coupled by a hysteresis element from both numerical and circuit experiments. As a result, van der Pol oscillators coupled by a hysteresis element exhibit the similar behavior as van der Pol oscillators coupled by inductors [6].Since hysteresis element is the memory element as well as inductor, the coexistence phenomenon may be confirmed. However, some discussion for the various oscillators are inadequacy.

In this paper, we consider two-dimensional piecewise-linear oscillators with a diode coupled by hysteresis element. The two-dimensional oscillator consists of a inductor, a capacitor, a negative conductor and a diode. If the diode is assumed to operate as ideal switch, the circuit equation is degenerate for one-dimensional when diode is on[5]. We confirm coexistence phenomena of in-phase and anti-phase synchronization in two-dimensional piecewise-linear oscillators with a diode coupled by hysteresis element in laboratory experiments. We have examined parameter conditions for coexistence phenomena of in-phase and an anti-phase synchronization by using the bifurcation diagram.

2. Piecewise-linear oscillators with a diode

Figure1 shows a circuit diagram. The oscillator is two-dimensional piecewise linear oscillators with a diode. The element $-g$ is a negative conductor.

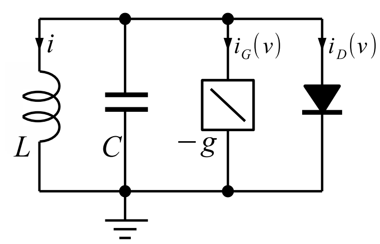
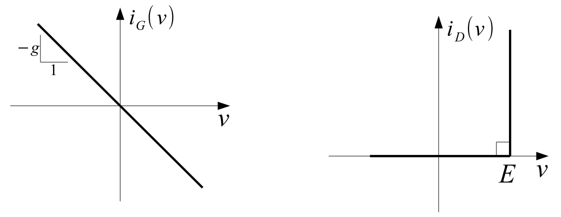


Figure 1: piecewise-linear oscillators including a diode.

We represent the $v-i$ characteristics of a negative conductor and an idealized diode (see Fig.2) . $i_G(v)$ presents the current of the negative conductor, and $i_D(v)$ presents the current through the diode. The diode is assumed an ideal switch with the threshold voltage of E. The circuit dynamics is represented by one-dimensional equation when the diode is on. In addition, the threshold voltages of the diode can be varied by connecting some diodes in series. The



(a) negative conductor.

(b) idealized diode.

Figure 2: $v-i$ characteristics of each elements.

circuit dynamics can be represented by the following two-dimensional piecewise-linear differential equation:
Diode OFF

$$\begin{cases} L\dfrac{di}{dt} = v \\ C\dfrac{dv}{dt} = -i - i_G(v), \end{cases} \tag{1}$$

Diode ON

$$\begin{cases} L\dfrac{di}{dt} = E \\ v = E(i_D = i - i_G(E)). \end{cases} \tag{2}$$

If Diode is OFF($v < E$), diode becomes ON when v reachese E.

If Diode is ON($i_D > 0$), diode becomes OFF when i_D reachese 0.

By using normalized variable and parameter as follows:
$\tau = \frac{1}{\sqrt{LC}}t,\ \delta = \frac{g}{2}\sqrt{\frac{L}{C}},\ x = \frac{1}{E}\sqrt{\frac{L}{C}}i,$
$y = \frac{v}{E},\ f_D = \frac{1}{E}\sqrt{\frac{L}{C}}i_D,$
the following normalized equation is obtained:
Diode OFF

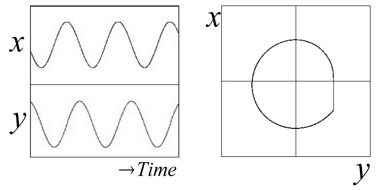
$$\begin{cases} \dfrac{dx}{d\tau} = y \\ \dfrac{dy}{d\tau} = -x + 2\delta y, \end{cases} \tag{3}$$

Diode ON

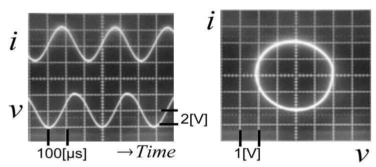
$$\begin{cases} \dfrac{dx}{d\tau} = 1 \\ y = 1.(f_D = x - 2\delta). \end{cases} \tag{4}$$

Each region is connected to another region by the following transition conditions:

Figure3 shows the numerical results and laboratory experiments. It was confirmed that these results match qualitatively.



(a)numerical result.



(b)Laboratory experiment.

Figure 3: output signal($\delta = 0.045$).

3. Piecewise-linear oscillators with a diode coupled by a hysteresis element

Figure4 shows a circuit diagram. This circuit is two-dimensional piecewise-linear oscillators with a diode coupled by a hysteresis element. VCCS with hysteresis characteristic as shown in Figure5. H is switched from 1 to -1 if $v_1 - v_2$ reaches to the threshold $-v_{th}$ and H is switched from -1 to 1 if $v_1 - v_2$ reaches to v_{th}.

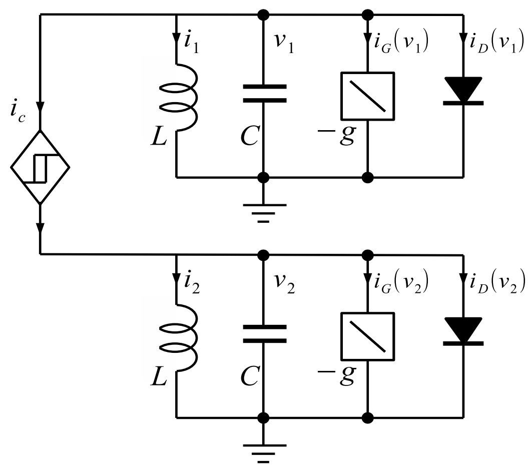


Figure 4: two-dimensional piecewise-linear oscillators with a diode coupled by hysteresis element.

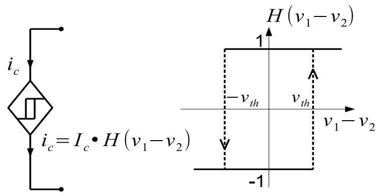


Figure 5: VCCS with hysteresis characteristic

The circuit dynamics can be represented as follows:
Diode OFF

$$\begin{cases} L\dfrac{di_1}{dt} = v_1 \\ C\dfrac{dv_1}{dt} = -i_1 - i_G(v_1) - i_c \\ L\dfrac{di_2}{dt} = v_2 \\ C\dfrac{dv_2}{dt} = -i_2 - i_G(v_2) + i_c, \end{cases} \quad (5)$$

Diode ON

$$\begin{cases} L\dfrac{di_1}{dt} = E \\ v_1 = E(i_D = i_1 - i_G(E) + i_c) \\ L\dfrac{di_2}{dt} = E \\ v_2 = E(i_D = i_2 - i_G(E) - i_c). \end{cases} \quad (6)$$

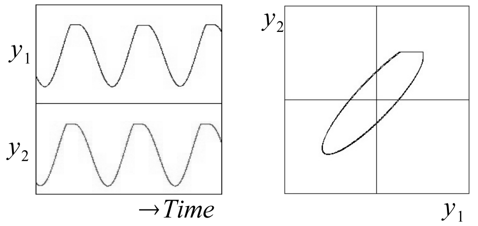
By using following variables and parameters:
$\tau = \frac{1}{\sqrt{LC}}t, \delta = \frac{\alpha}{2}\sqrt{\frac{L}{C}}, x_1 = \frac{1}{E}\sqrt{\frac{L}{C}}i_1, x_2 = \frac{1}{E}\sqrt{\frac{L}{C}}i_2,$
$y_1 = \frac{v_1}{E}, y_2 = \frac{v_2}{E}, \gamma = \frac{1}{E}\sqrt{\frac{L}{C}}i_c,$
$f_{D1} = \frac{1}{E}\sqrt{\frac{L}{C}}i_{D1}, f_{D2} = \frac{1}{E}\sqrt{\frac{L}{C}}i_{D2},$
the normalized equation is obtained:
Diode OFF

$$\begin{cases} \dfrac{dx_1}{d\tau} = y_1 \\ \dfrac{dy_1}{d\tau} = -x_1 + 2\delta y_1 - \gamma h(y_1 - y_2) \\ \dfrac{dx_2}{d\tau} = y_2 \\ \dfrac{dy_2}{d\tau} = -x_2 + 2\delta y_2 + \gamma h(y_1 - y_2) \end{cases} \quad (7)$$

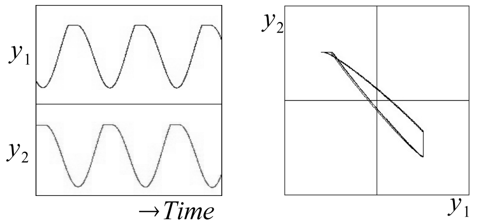
Diode ON

$$\begin{cases} \dfrac{dx_1}{d\tau} = 1 \\ y_1 = 1(f_{D1} = x_1 - 2\delta + \gamma h(y_1 - y_2)) \\ \dfrac{dx_2}{d\tau} = 1 \\ y_2 = 1(f_{D2} = x_2 - 2\delta - \gamma h(y_1 - y_2)). \end{cases} \quad (8)$$

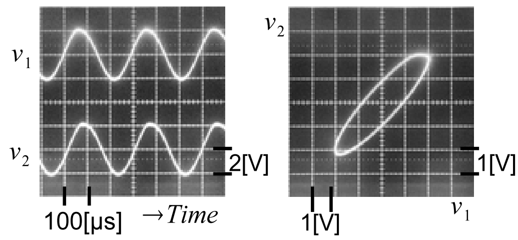
Figure6 shows laboratory experiments and numerical results under same parameter conditions. Figure6(a) shows in-phase synchronization, and Figure6(b) shows anti-phase synchronization. In these result, we confirm coexistence phenomenon of in-phase and anti-phase synchronization in this system. Further, from the results represented in Fig.6(c) and (d), it was identified the co-existence phenomenon of in-phase and anti-phase synchronization in the circuit experiment. These behaviors are qualitatively consistent with behavior of van der Pol oscillators coupled by a inductor.



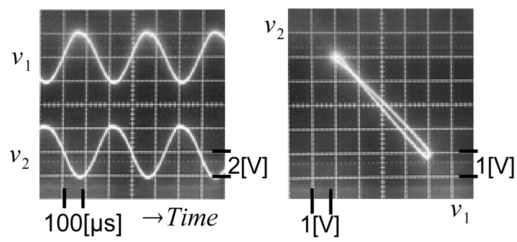
(a) in-phase (numerical result)



(b) anti-phase (numerical result)



(c) in-phase (Laboratory experiment)



(d) anti-phase (Laboratory experiment)

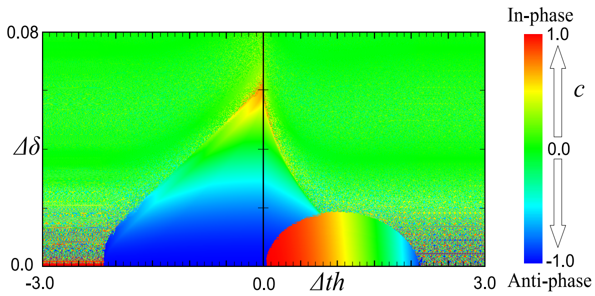
Figure 6: coexistence phenomenon of in-phase and anti-phase synchronization($\delta = 0.045, \gamma = -0.01$, initial value(a)$x_1 \approx x_2, y_1 \approx y_2$, (b)$x_1 \approx -x_2, y_1 \approx -y_2$,).

4. Parameter regions of in-phase and anti-phase synchronization

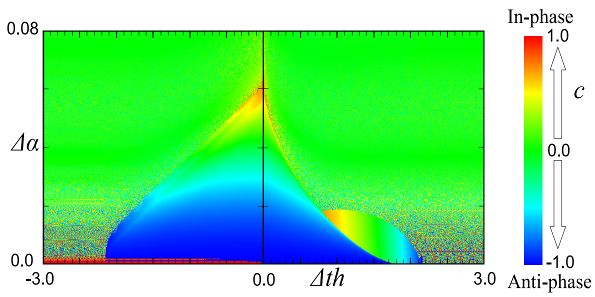
In order to derive the parameter regions of in-phase and anti-phase synchronization, we introduce correlation coefficients c with regard to y_1 and y_2.

$$c = \frac{\sum_{k=0}^{N}(x_{1k} - \overline{x_1})(x_{2k} - \overline{x_2})}{\sqrt{\sum_{k=0}^{N}(x_{1k} - \overline{x_1})^2}\sqrt{\sum_{k=0}^{N}(x_{2k} - \overline{x_2})^2}} \quad (9)$$

Figure7 shows the bifurcation diagram of in-phase and anti-phase synchronization. $\Delta\alpha$ is parameter mismatch of oscillator 1 and 2. Δth is hysteresis width of hysteresis element.



(a)Parameter bifurcation diagram.
(Easy initial value in-phase synchronization)



(b)Parameter bifurcation diagram.
(Easy initial value anti-phase synchronization)

Figure 7: bifurcation diagram of in-phase and anti-phase synchonization($\delta = 0.045, \gamma = -0.01$)

Nearly red region($c \to 1.0$) denotes in-phase synchronization, nearly blue region($c \to -1.0$) denotes anti-phase synchronization, and nearly yellow or green regions($c \to 0.0$) denotes asynchronous. In this way, we confirmed parameter conditions where coexistence phenomena of in-phase and an anti-phase synchronization.

5. Conclusion

In this paper, we consider two-dimensional piecewise-linear oscillators with a diode coupled by hysteresis element. Firstly, we confirmed the behavior of the two-dimensional oscillator including inductor, capacitor, negative conductor and a diode from the numerical results and laboratory experiments. These results match qualitatively. Next, we investigated the behavior of piecewise-linear oscillators coupled by a hysteresis element. As a result, we confirmed coexistence phenomena of in-phase and anti-phase synchronization. Finally, we illustrated parameter conditions where coexistence phenomena of in-phase and an anti-phase synchronization by using the bifurcation diagram.

We will investigate parameter conditions where coexistence phenomena of in-phase and an anti-phase synchronization in more detail.

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