

# Laser RCS Calculation of Conducting Targets in Random Media for E-Wave Polarization

Hosam El-Ocla

Department of Computer Science, Lakehead University  
955 Oliver Road, Thunder Bay, Ontario, Canada P7B 5E1

E-mail: hosam@lakeheadu.ca

**Abstract:** In the practical radar applications, plane wave incidence is not easy task to keep its entity as wide enough in the far field. Therefore, beam wave incidence should be used to calculate the laser radar cross-section (LRCS). This is vital when considering LRCS calculation of practical targets with inflection points in a random medium. E-wave polarization (E-wave incidence) is assumed with targets bigger than a one wavelength. The performance of the LRCS with changing the complexity of targets is studied in this work. Also, the enhancement in the LRCS (ELRCS) is numerically analyzed with different random media strength.

## 1 Introduction

Several methods were presented to calculate scattered waves from targets such as in [1]–[3]. Most of these techniques need heavy computation resources and a long processing time in accordance. Over the past years, a method has been presented to solve the scattering problem as a boundary value problem [4]. This method is characterized by the calculation of the current on the whole surface and not only on the illumination region as in the physical optics method. Therefore this method gives an accurate calculation of the waves intensity. Backscattering enhancement (BE) is a phenomenon that was investigated by researchers for decades [5]–[7]. Accordingly, the double passage effect on waves backscattering from point targets leads to that RCS in random medium is enhanced to be twice that in free space. In the recent years, the effects of target configuration, random media, and polarization on the RCS and BE for a beam wave incidence were analyzed in many of my publications (e.g., [8]–[10]). Spatial coherence length (SCL) of waves around the target is proved to have a key impact on the waves scattering. In this work, we calculate LRCS for targets of fairly complex cross sections in the free space and random media.

The mean size of the target could be smaller than the beam wave width but this is practically is not the usual case. As a result, it is considered here the case where the target size is greater than the beam width. Also, we assume that the beam width is fairly greater than the SCL to show the effect of the uncorrelated waves scattering from the part of the target located outside the SCL, but in the mean time inside the  $kW$ , on the waves intensity. The effect of the target complexity on the waves scattering would be analyzed. Using our method, LRCS is calculated for convex illumination region of concave-convex targets. We deal with the scattering problem two-dimensionally assuming E-wave incidence. The time factor  $\exp(-i\omega t)$  is assumed and suppressed in the following section.

## 2 Formulation

Geometry of the problem is shown in Figure 1. A random medium is assumed as a sphere of radius  $L$  around a target of the mean size  $a \ll L$ , and also to be described by the dielectric constant  $\epsilon(\mathbf{r})$ , the magnetic permeability  $\mu$ , and the electric conductivity  $\nu$ . For simplicity  $\epsilon(\mathbf{r})$  is expressed as

$$\epsilon(\mathbf{r}) = \epsilon_0 [1 + \delta\epsilon(\mathbf{r})] \quad (1)$$

where  $\epsilon_0$  is assumed to be constant and equal to free space permittivity and  $\delta\epsilon(\mathbf{r})$  is a random function with

$$\langle \delta\epsilon(\mathbf{r}) \rangle = 0, \quad \langle \delta\epsilon(\mathbf{r}) \delta\epsilon(\mathbf{r}') \rangle = B(\mathbf{r}, \mathbf{r}') \quad (2)$$

and

$$B(\mathbf{r}, \mathbf{r}) \ll 1, \quad kl(\mathbf{r}) \gg 1 \quad (3)$$

Here, the angular brackets denote the ensemble average and  $B(\mathbf{r}, \mathbf{r})$ ,  $l(\mathbf{r})$  are the local intensity and local scale-size of the random medium fluctuation, respectively, and  $k = \omega \sqrt{\epsilon_0 \mu_0}$  is the wavenumber in free space. Also  $\mu$  and  $\nu$  are assumed to be constants;  $\mu = \mu_0$ ,  $\nu = 0$ . For practical turbulent media the condition (3) may be satisfied. Therefore, we can assume the forward scattering approximation and the

scalar approximation [11]. Consider the case where a directly incident beam wave is produced by a line source  $f(\mathbf{r}')$  along the  $y$  axis. Here, let us designate the incident wave by  $u_{in}(\mathbf{r})$ , the scattered wave by  $u_s(\mathbf{r})$ , and the total wave by  $u(\mathbf{r}) = u_{in}(\mathbf{r}) + u_s(\mathbf{r})$ . The target is assumed to be a conducting cylinder of which cross-section is expressed by

$$r = a[1 - \delta \cos 3(\theta - \phi)] \quad (4)$$

where  $\phi$  is the rotation index and  $\delta$  is the concavity index. We can deal with this scattering problem two dimensionally under the condition (3); therefore, we represent  $\mathbf{r}$  as  $\mathbf{r} = (x, z)$ . Assuming a horizontal polarization of incident waves (E-wave incidence), we can impose the Dirichlet boundary condition for wave field  $u(\mathbf{r})$  on the cylinder surface  $S$ . That is,  $u(\mathbf{r}) = 0$ , where  $u(\mathbf{r})$  represents  $E_y$ .

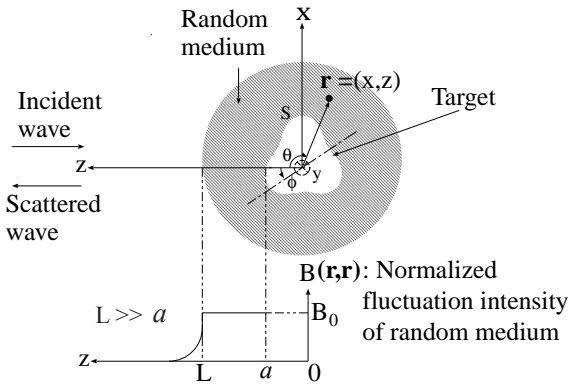


Figure 1: Geometry of the problem of wave scattering from a conducting cylinder in a random medium.

Using the current generator  $Y_E$  and Green's function in random medium  $G(\mathbf{r} | \mathbf{r}')$ , we can express the scattered wave as

$$u_s(\mathbf{r}) = \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 [G(\mathbf{r} | \mathbf{r}_2) Y_E(\mathbf{r}_2 | \mathbf{r}_1) u_{in}(\mathbf{r}_1 | \mathbf{r}_t)] \quad (5)$$

where  $\mathbf{r}_t$  represents the source point location and it is assumed as  $\mathbf{r}_t = (0, z)$  in section 3. We consider  $u_{in}(\mathbf{r}_1 | \mathbf{r}_t)$ , whose dimension coefficient is understood, to be represented as:

$$u_{in}(\mathbf{r}_1 | \mathbf{r}_t) = G(\mathbf{r}_1 | \mathbf{r}_t) \exp[-(\frac{kx_1}{kW})^2] \quad (6)$$

where  $W$  is the beam width. The beam expression is approximately useful only around the cylinder. Here,  $Y_E$  is the operator that transforms incident waves into surface currents on  $S$  and depends only on the scattering body. The current generator can be expressed in

terms of wave functions that satisfy Helmholtz equation and the radiation condition. More details about  $Y_E$  are available in my publications.

Therefore, the average intensity of backscattering wave for E-wave incidence is given by

$$\langle |u_{se}(\mathbf{r})|^2 \rangle = \int_S d\mathbf{r}_{01} \int_S d\mathbf{r}_{02} \int_S d\mathbf{r}'_1 \int_S d\mathbf{r}'_2 Y_E(\mathbf{r}_{01} | \mathbf{r}'_1) Y_E^*(\mathbf{r}_{02} | \mathbf{r}'_2) \exp[-(\frac{kx'_1}{kW})^2] \exp[-(\frac{kx'_2}{kW})^2] \times \langle G(\mathbf{r} | \mathbf{r}_{01}) G(\mathbf{r} | \mathbf{r}_{02}) G^*(\mathbf{r} | \mathbf{r}'_1) G^*(\mathbf{r} | \mathbf{r}'_2) \rangle \quad (7)$$

We can obtain the LRCS  $\sigma$  using equation (7)

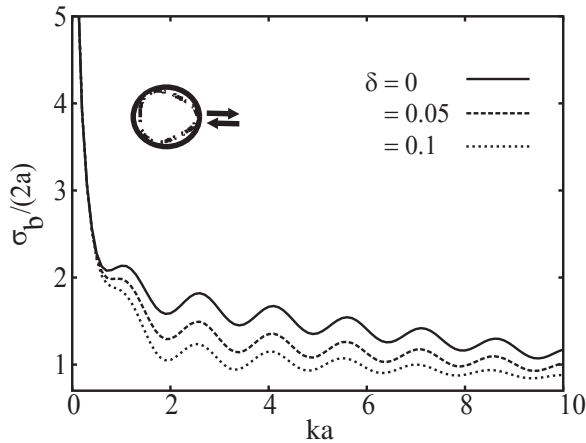
$$\sigma = \langle |u_s(\mathbf{r})|^2 \rangle \cdot k(4\pi z)^2 \quad (8)$$

### 3 Numerical Results

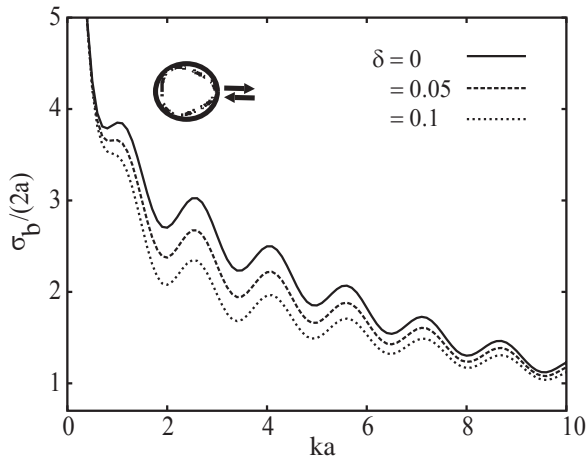
In the following, we conduct numerical results for the LRCS and normalized LRCS (NLRCS), defined as the ratio of LRCS in random media  $\sigma$  to LRCS in free space  $\sigma_0$ . In [9], some interesting cases were shown and that motivated us to continue our study to explore the parameters that affect the enhancement in the LRCS; in this regard we assume  $kW = 3$ .

#### 3.1 Radar cross-section RCS

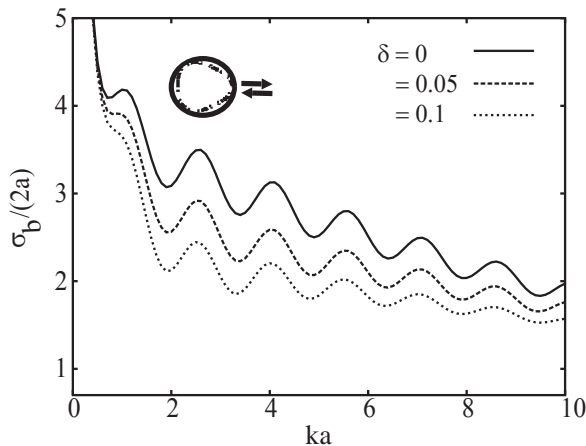
Numerical results for LRCS shown in figures 2 are discussed in this section. It is noted that LRCS decreases gradually with  $ka$  and this is attributed to the shortage in the surface current generated when  $ka$  gets greater than  $kW$ . On the contrary and with plane wave incidence, RCS is almost constant with  $ka$  because the generated surface current does not change since the target's front area confronting the incident wave is always illuminated and covered by the plane wave. LRCS undergoes oscillating behaviour due to the contributions from the stationary and inflection points. The scattered waves are sometimes in-phase so they add up and sometimes out-of-phase so they cancel out depending on the directions of scattering rays that in turn lead to such vast oscillating behavior [9]. As  $\delta$  increases, as the LRCS decreases as a result of the effective illumination region (EIR) shrink. For  $ka$  fairly greater than  $kW$ , LRCS slightly alter with  $\delta$  because the illumination region acts as being a plane surface. Also we notice that with small SCL, the rate of LRCS decrease with  $ka$  is greater than in the free space. This is because the waves are correlated around the target in free space and therefore they are in-phase mostly. More detailed discussion for the SCL effect on the LRCS will be presented in the next section.



(a)



(b)



(c)

Figure 2: LRCS vs. target size where the target is located in (a) free space, (b) random medium with  $SCL=3$ , (c) random medium with  $SCL=7.5$ .

### 3.2 Backscattering enhancement

In this section, we analyze the numerical results shown in figure 3 for the NLRCS. For  $ka \ll SCL$ , the NLRCS equals two, as a result of the double passage effect; this is realized, independent of the target complexity, i.e., independent of the concavity index  $\delta$ . In this range where  $ka < kW$ , the beam wave acts as if it is a plane wave incidence.

For  $ka \simeq SCL$ , the NLRCS dwindles with  $ka$  as a result of the lack in the surface current generated as pointed out earlier. In the mean time, NLRCS suffers from oscillating behaviour that is a function of the target complexity. The behaviour of the NLRCS changes with  $\delta$  and this is attributed to the difference in the effect of stationary and inflection points contributions that depend on the curvature of the surface in both cases of free space and random medium. Inflection points may locate inside  $kW$ , however, they locate outside the SCL depending on the  $ka$ . In this case, contributions from the pre-mentioned points in the free space are coherent, but on the contrary they are incoherent in the random medium and that makes such difference in the impact of these points and this is shown in particular in the figure 3-a where  $SCL = 3$  while  $2kW = 6$ . As  $\delta$  increases, as the number of the inflection points, which are near the shadow region and outside the SCL, increases as previously mentioned and this in turn magnifies the incoherent contributions. Therefore the difference in the LRCS between the free space and the random medium becomes bigger leading to larger fluctuations in the NLRCS. When SCL has a wider length as in the figure 3-b, the waves are more correlated and accordingly do not have such irregular oscillations. On the other hand and when  $SCL > 2kW$ , the contributions from the inflection points are more correlated in the random medium and, hence, the deviation of NLRCS from two is not that much and changes slightly with  $\delta$ .

It should be noted that when  $SCL < 2kW$ , not all the EIR is used which makes NLRCS is quite far from that when  $SCL > 2kW$  (in figure 3, compare the cases of NLRCS when  $SCL = 3$  and that at  $SCL = 7.5$ ). Therefore, to reduce the strength of the fluctuations in the NLRCS, the condition  $SCL \geq 2kW$  should be realized. In addition, SCL and accordingly  $kW$  should have wider sizes to have NLRCS closer to two. These observations can easily be noticed when having a comparison between our findings in this paper and the cases presented in [9] that handled smaller  $kW$ .

## 4 Conclusion

The parameters that affect the performance of wave scattering from targets were studied in this paper. In this regard, laser RCS (LRCS) was calculated and analyzed numerically. These parameters include the spatial coherence length (SCL) around the target and the target configuration including size and curvature complexity. Beam wave incidence with limited width is incapable of radar detection for targets with large size for the lack of the surfac current generated. LRCS suffers from oscillating behaviour and in particular with fairly complex cross sections owing to the inflection points contributions in addition to the specular reflections. Double passage is the effect that can be accepted where the backscattering enhancement is two. However, the enhancement in the LRCS deviates from two with the increase of the target size. To minimize such deviation, the following rules:  $SCL \geq kW$  and  $SCL \gg ka$ , should be maintained with having wider SCL. Actually in the real applications, there is no control over the SCL at the radar side, therefore, high frequency waves would be transmitted to hold the above rules.

## References

- [1] Joseph B. Keller and William Streifer, "Complex rays with an application to Gaussian beams", *J. Opt. Soc. Am.*, Vol. 61, No. 1, pp. 40-43, 1971.
- [2] Hiroyoshi Ikuno, "Calculation of far-scattered fields by the method of stationary phase", *IEEE Transactions on Antennas and Propagation*, Vol. AP-27, No. 2, pp. 199-202, 1979.
- [3] Bennett, C.L., Mieras, "Time domain scattering from open thin conducting surfaces", *Radio Science*, Vol. 16, No. 6, pp. 1231-1239, 1981.
- [4] Z. Q. Meng and M. Tateiba, "Radar cross sections of conducting elliptic cylinders embedded in strong continuous random media", *Waves in Random Media*, Vol. 6, pp. 335-345, 1996.
- [5] Yu. A. Kravtsov and A. I. Saishev, "Effects of double passage of waves in randomly inhomogeneous media", *Sov. Phys. Usp.*, Vol. 25, pp. 494-508, 1982.
- [6] E. Jakeman, "Enhanced backscattering through a deep random phase screen", *J. Opt. Soc. Am.*, Vol. 5, No. 10, pp. 1638-1648, 1988.
- [7] Akira Ishimaru, "Backscattering enhancement: from radar cross sections to electron and light localizations to rough surface scattering", *IEEE Antennas and Propagation Magazine*, Vol. 33, No. 5, pp. 7-11, 1991.

- [8] H. El-Ocla and M. Tateiba, "An indirect estimate of RCS of conducting cylinder in random medium", *IEEE Antennas and Wireless Propagation Letters*, Vol. 2, pp. 173-176, 2003.
- [9] H. El-Ocla, "Laser backscattered from conducting targets of large sizes in continuous random media for E-Wave polarization", *Journal of the Optical Society of America A*, Vol. 23, No. 8, pp. 1908-1913, 2006.
- [10] H. El-Ocla, "On laser radar cross section of targets with large sizes for E-polarization", *Progress in Electromagnetics Research PIER* 56, pp. 323-333, Cambridge, MA: EMW, 2006.
- [11] Akira Ishimaru, *Wave propagation and scattering in random media*, IEEE press, 1997.

## Acknowledgment

This work was supported in part by National Science and Engineering Research Council of Canada (NSERC) under Grant 250299-02.

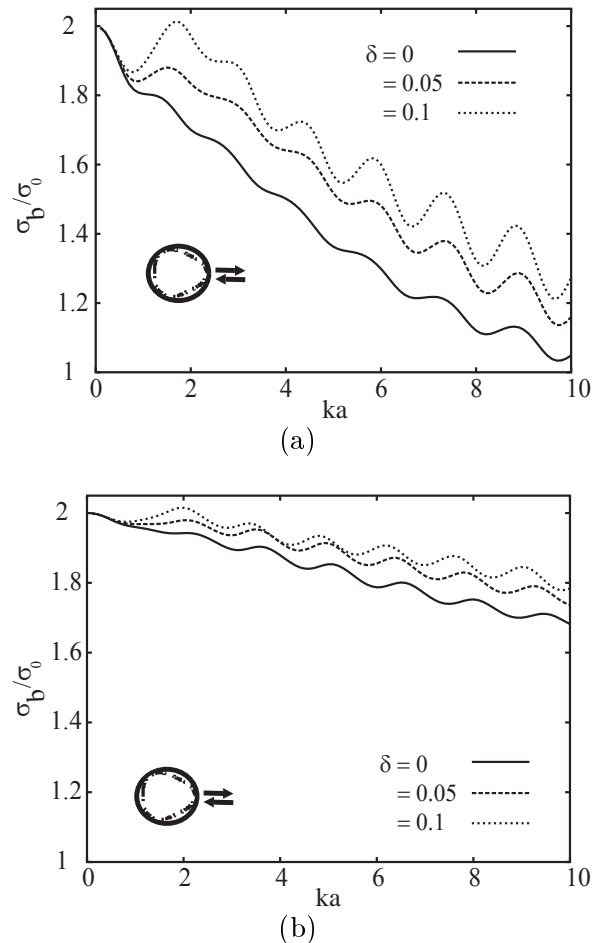


Figure 3: Normalized LRCS vs. target size at different  $\delta$  for  $kW = 3$  where (a)  $SCL=3$ , (b)  $SCL=7.5$  and  $\sigma_b, \sigma_0$  are LRCS in random media and in free space, respectively.