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Performance Evaluation and Analysis of Chaotic CDMA Considering Synchronization Acquisition

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Abstract– Chaotic sequences have been shown to be effective for Direct Sequence / Code Division Multiple Access (DS / CDMA). Because the spreading codes generated by the chaotic sequences have negative autocorrelation, the chaotic sequences can minimize interferences of the DS/CDMA system. However, the error of synchronization acquisition due to the negative autocorrelation characteristics in chaotic sequences is not considered. Therefore, we study the bit error rate (BER) of the Chaotic sequence in DS/CDMA system by considering the synchronization acquisition. As a result, optimal autocorrelation is changed compared to conventional results. More specifically, we derive both simulation analysis and theoretically analysis to find the optimal autocorrelation to minimize error caused by negative autocorrelation in chaotic sequence with taking synchronous acquisition.

1. Introduction

In conventional DS/CDMA, the spectrum is spread into wide band by using different random code, called spreading code. For example, M sequences, gold sequences have been used as the spreading code in conventional systems. In this paper, we consider other types of spreading code generated by chaotic dynamical systems. On contrary with the spreading code generated by the god sequence which has zero cross-correlation, the spreading code generated by the chaotic system has negative cross-correlation. It has been shown in previous studies that the chaotic codes with negative autocorrelation can help to reduce the multiple access interference of the DS/CDMA system in comparison with the gold sequences [1]-[3].

However, the chaotic code generates the negative autocorrelation between the spreading codes which affects to the synchronization acquisition. In these aforementioned works, the influence of the synchronization acquisition has not been considered.

In this paper, we evaluate performance of chaotic code by taking into account both the synchronization acquisition error caused by negative autocorrelation and the negative cross correlation due to the chaotic sequences. Then, we analyze the simulation result theoretically.

In the following, DS/CDMA by chaotic sequence and these conventional researches are explained in Section 2. Then, performance analysis considering degradation of autocorrelation characteristic in DS/CDMA is shown in Section 3. Then, theoretical analysis is shown in Section 4. Finally, summary is shown in Section 5.

2. Code Division Multiple Access by Chaotic Sequence

2.1. DS/CDMA

DS/CDMA is a communication scheme that spreads spectrum into wide band by using different random code, called spreading code, for each terminal. In other words, each transmitter divides the data sequence into certain time interval (Chip length), and multiplies each chip by spreading code. Due to that, each terminal is able to transmit data simultaneously in the same frequency band. In order to reduce multiple access interference, the low cross-correlation sequences for the spreading code in DS / CDMA are needed. For example, Gold sequences are used in conventional systems.

2.2. Chaotic CDMA

Chaotic sequences have been shown to be effective for DS/CDMA, when the synchronization of the transmission timing is not taken between each transmitter, that is the asynchronous DS/CDMA. Optimal spreading code, which minimizes interferences on the asynchronous DS/CDMA, can be generated by the chaotic maps.

In [1]-[3], the effectiveness of spreading code with negative autocorrelation has been shown in the asynchronous DS / CDMA. In [2], spreading code with a negative autocorrelation has been theoretically analyzed on asynchronous DS / CDMA by calculating the multiple access interference. The Fig.1 shows the models of two users using spreading code X and Y.

In Fig.1, l and ε are the time difference of the chips. When ε is zero, it corresponds to a chip-synchronous CDMA, and when ε is not zero, it is a chip-asynchronous CDMA.

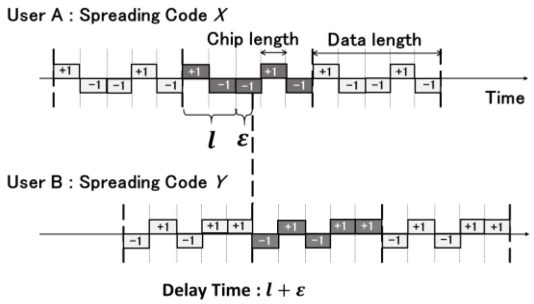


Fig.1: Interference model in asynchronous DS / CDMA.

The multiple access interference between code X and Y is expressed by following equation,

$$I=(1-\varepsilon)R_N^{E/O}(l;X,Y)+\varepsilon R_N^{E/O}(l+1;X,Y)\ , \quad (1)$$

where $R_N^{E/O}(l;X,Y)$ is even/odd cross-correlation, which is expressed by following equations,

$$R_N^{E}(l;X,Y)=\sum_{n=0}^{N-l-1}X_nY_{n+l}+\sum_{n=0}^{l-1}X_{n+N-l}Y_n\ , \quad (2)$$

$$R_N^{O}(l;X,Y)=\sum_{n=0}^{N-l-1}X_nY_{n+l}-\sum_{n=0}^{l-1}X_{n+N-l}Y_n\ . \quad (3)$$

The value of n-th chip of the spreading code X_n, is defined as follows,

$$X_n=\{-1,+1\} \quad (0\le n\le N-1)\ , \quad (4)$$

where N is the spreading factor, and, n is an n-th chip value of the spreading code ($0\le n\le N-1$).

Here, we assume that the spreading code of each user is generated by the binary Markov chain model as in Fig.2.

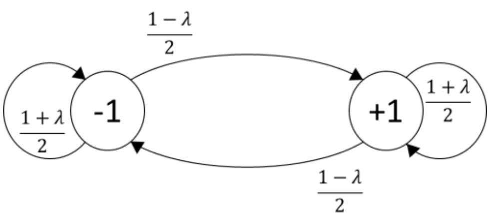


Fig.2: Markov chain model for spreading sequences.

According to the Markov chain in Fig.2, we obtain following equations,

$$E[X_nX_{n+l}]=\lambda^l\ , \quad (5)$$

$$E[Y_nY_{n+l}]=\lambda^l\ , \quad (6)$$

$$E[X_nY_{n+l}]=0\ , \quad (7)$$

$$E[X_n]E[Y_n]=0\ , \quad (8)$$

where $E[X]$ denotes the expectation of X. The equations (5) and (6) are corresponding to the autocorrelation of the spreading code shifted l chips. In other word, by changing λ, auto-correlation of the spreading code can be changed.

Using these equations, we can find the multiple access interference by using the equation (1). When N is large enough, the expectation of multiple access interference can be calculated as follows,

$$E\left[\left(\frac{I(\lambda)}{\sqrt{N}}\right)^2\right]=\frac{2(1+\lambda+\lambda^2)}{3(1-\lambda^2)}\ . \quad (9)$$

From (9), it can be clarified that the multiple access interference is minimized, when $\lambda=-2+\sqrt{3}$. It has shown that the spreading codes with parameters of $\lambda=-2+\sqrt{3}$ is able to reduce multiple access interference in the asynchronous DS/CDMA. In other word, cross correlation between users is reduced by using spreading code with negative autocorrelation.

To generate such spreading code, we use the chaotic map, called Kalman map. The Kalman map generates the spreading code according to the following equations,

$$x(t+1)=\frac{2x(t)}{1+\lambda} \quad (0\le x(t)\le\frac{1+\lambda}{2})\ , \quad (10)$$

$$x(t+1)=\frac{2x(t)}{1-\lambda}-\frac{2\lambda}{1-\lambda} \quad (\frac{1+\lambda}{2}\le x(t)\le 1)\ , \quad (11)$$

$$x(t+1)=\frac{2x(t)}{1-\lambda}-\frac{2}{1-\lambda} \quad (1\le x(t)\le\frac{3+\lambda}{2})\ , \quad (12)$$

$$x(t+1)=\frac{2x(t)}{1+\lambda}-\frac{2\lambda-2}{1+\lambda} \quad (\frac{3+\lambda}{2}\le x(t)\le 2)\ . \quad (13)$$

When $0\le x(n)\le 1$, spreading code $X_n=-1$. And, when $1\le x(n)\le 2$, spreading code $X_n=+1$. Fig.3 shows an example of the Kalman map.

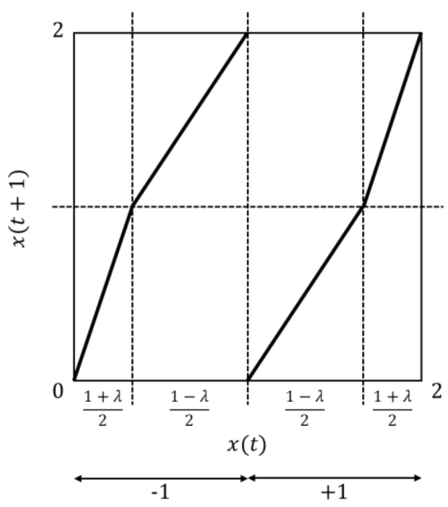


Fig.3: Kalman map for chaotic code.

In addition, we conduct the simulation to verify the our analysis in Fig.4 The horizontal axis is λ a parameter to determine the autocorrelation of spreading code. The vertical axis is the expectation of the multiple access interference. Here, the red line shows the expectation of the multiple access interference by calculation. Blue dot shows the multiple access interference by simulation using chaotic codes that is made by the Kalman map on spreading factor 511. It can be seen that the blue dot has a similar tendency to the red line. Furthermore, we also confirm that the interference is minimize when $\lambda = -2 + \sqrt{3}$.

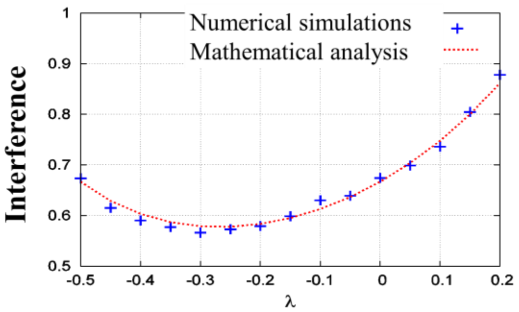


Fig.4: Expectation of the multiple access interference.

The effectiveness of the spreading code made by chaotic map in the asynchronous DS / CDMA is also shown in the simulation [3].

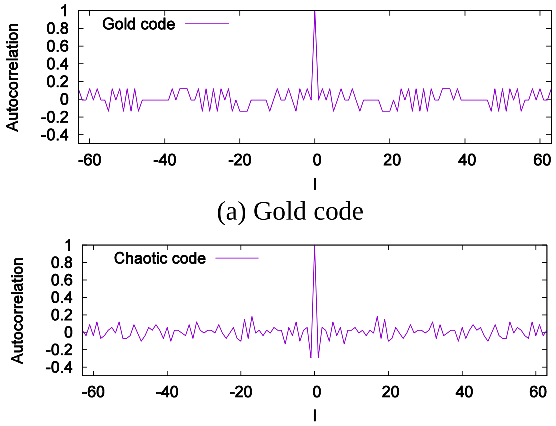
3. Performance Evaluation Considering Synchronization Acquisition

3.1. Influence of Autocorrelation in DS/CDMA

In previous section, optimal λ that minimizes the cross correlation between each spreading code has been found. Chaotic code with negative autocorrelation reduces multiple access interference. However, this analysis does not consider the effect of synchronization acquisition caused by negative autocorrelation. Generally, in DS/CDMA, the received signal is passed through to correlator, and then, determining the peak value to capture synchronization point. Then, if the spreading codes have negative autocorrelation, errors due to the occurrence of the second peak are increased.

Fig.5 shows an example of the autocorrelation properties of Gold code and chaotic code, when $\lambda = -2 + \sqrt{3}$, on spreading factor 127.

From Fig.5 (b), we observe that the second peaks are occurred in the point shifted by one chip from the peak. Synchronization acquisition errors increase because of these second peaks.



(a) Gold code

(b) Chaotic code.

Fig.5: Example of the autocorrelation properties.

In the following, we evaluate performance of chaotic code by considering the degradation of synchronization acquisition caused by negative autocorrelation, and also the cross correlation between each spreading code.

3.2. Simulation of Chaotic CDMA Considering Synchronization Acquisition

3.2.1. Simulation Model

In this paper, we evaluated the bit error rate (BER) of the DS/CDMA with synchronization acquisition by computer simulation. Fig.6 shows the simulation model.

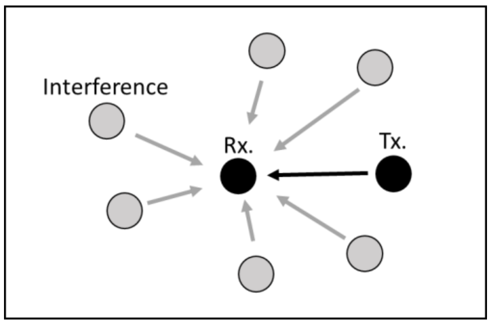


Fig.6: Simulation model.

There are one pair transmitter and receiver. The terminals are the interference sources which generate the interference to the receiver. All the terminals adjust their transmit power in order to get same receive power at the receiver. At the receiver side, the received signal is passed through to correlator, and determined the peak value that is positive or negative. Then, demodulated signal is compared with the transmission signal to compute the BER. Simulation parameters are shown in Table.1.

Table 1: Simulation parameters of Fig.6.

Method	Asynchronous DS/CDMA
Channel Noise	Nothing
Spreading Code	Chaotic code $(-4 \le \lambda \le 0.2)$
Code Length	127
Number of User	5,6,7,8,9,10

3.2.2. Simulation Results

The simulation result is shown in Fig.7. The vertical axis represents the BER, and the horizontal axis represents the value of λ.

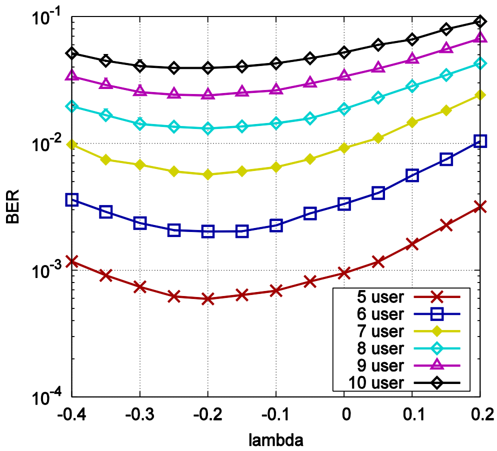


Fig.7: BER against parameter λ.

We see from Fig.7 that BER is minimize when nearby $\lambda = -0.2$ in each line. As compared with conventional research, optimal lambda approaches zero when considering synchronization acquisition.

4.Theoretical Analysis

This section describes why the optimal lambda changes from $\lambda = -2 + \sqrt{3}$ to nearby $\lambda = -0.2$.

The autocorrelation function of chaotic code is represented by using lambda in Fig.8.

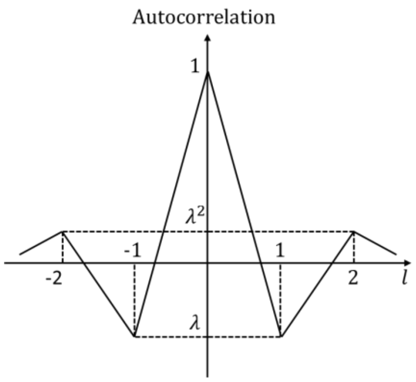


Fig.8: The autocorrelation function of chaotic code.

It can be seen from this figure that second peak is occurred when $l = -1$ or 1. This second peak is the cause of error, when synchronization acquisition is considered. Then, if negative autocorrelation is high, the BER of the system is increasing. However, from equation (9), it is shown that the effectiveness of the negative autocorrelation in order to reduce multi access interference by lowering cross correlation. Therefore, it is necessary to minimize the influence of both autocorrelation and cross correlation, as shown in the following objective function (14),

$$\min\left\{\alpha|\lambda| + \beta\frac{2(1+\lambda+\lambda^2)}{3(1-\lambda^2)}\right\}, \quad (14)$$

where, α and β is a parameter that determine the weight of each term. If the networks have less interference, the value of α becomes larger to reduce the BER caused by the synchronization acquisition. Otherwise, if the networks have high interference, value of β becomes lager to against the multiple access inference.

It can be seen from objective function (14) that the optimal lambda approaches zero when considering synchronization acquisition. This is consistent with the simulation in Fig.7.

5. Conclusion

In this paper, we study the DS/CDMA where the spreading codes has been generated by the chaotic sequence. We focus on the influence of negative autocorrelation of the spreading code which has not been considered in previous works. More specifically, we derive both simulation analysis and theoretically analysis to find the optimal autocorrelation to minimize error with taking the synchronous acquisition caused by negative autocorrelation in chaotic sequence.

As future prospects, it is necessary to clarify the cause-and-effect relationship of the number of interference and parameters α and β.

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