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Experimental verification of amplitude death in a pair of double-scroll circuits coupled by a one-way partial time-varying delay connection

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Abstract—This report experimentally investigates amplitude death in a pair of oscillators coupled by a one-way partial time-varying delay connection: the connection delay in one direction is varied, but the connection delay in the other direction is constant. Our circuit experiments show that the one-way partial time-varying delay connection can induce amplitude death for long connection delay. The analytical results agree with our experimental results.

1. Introduction

The dynamics of various physical, chemical, and engineering systems can be mathematically modeled by coupled oscillators [1]. In coupled oscillators, we observe various nonlinear phenomena such as synchronization, spacial-temporal chaos, chimera state, and so on. One of such phenomena is amplitude death [2], which is a stabilization of homogeneous steady state in diffusively-coupled oscillators. It is analytically shown that this phenomenon never occurs in coupled identical oscillators [3]. However, if there exists connection delay between oscillators, then amplitude death can occur even in coupled identical oscillators [4]. Amplitude death induced by the connection delay has been actively investigated in nonlinear science [2].

Amplitude death has been expected to suppress undesired oscillations in engineering systems such as coupled laser systems [5], dc micro grid [6], and coupled thermoacoustic oscillators [7]. This is because the usage of amplitude death does not need feedback controllers for stabilization. However, for implementation in real systems, amplitude death has one critical problem: if the connection delay is relatively long due to a practical constraint, then amplitude death cannot be induced [4]. In order to overcome this problem, the following three connections have been proposed: a distributed delay connection [8], a multiple delay connection [9], and a time-varying delay connection [10].

The time-varying delay connection would be easier to be implemented and would not cost compared with the other two connections [10]. The time-varying delay connection has been implemented in electronic circuits [11], and the topology and delay independent design procedure of connection parameters has been proposed [12]. However, all the connection delays have to be varied with high frequency. Thus, this connection is difficult to be implemented for large networks with a huge number of oscillators.

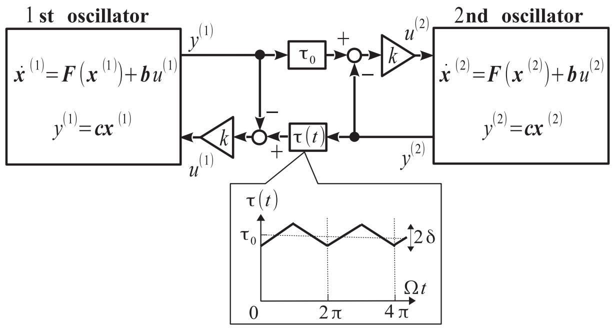


Figure 1: A pair of oscillators coupled by a one-way partial time-varying delay connection.

In order to defeat this difficulty, we first proposed a one-way partial time-varying delay connection for a pair of oscillators, in which the connection delay in a direction is varied, but that in the other direction is constant [13]. In addition, we proposed a two-way partial time-varying delay connection for networks, in which connection delays between some oscillators are varied, but the others are constant [12]. These partial time-varying delay connections are obviously easier to be implemented in large networks; however, to the best of our knowledge, there have been few reports about experimental investigations on amplitude death induced by the partial time-varying delay connections.

The present report experimentally investigates amplitude death in a pair of oscillators coupled by the one-way partial time-varying delay connection. The well-known double scroll circuit is employed as the oscillator, and the connection delay is mainly implemented by peripheral interface controllers (PICs) and DA converters. Our experiments show that the one-way partial time-varying delay connection can induce amplitude death even for long connection delay. The analytical results agree with our experimental results.

2. One-way partial time-varying delay connection

Let us consider a pair of m-dimensional oscillators (see Fig. 1),

$$\begin{cases} \dot{\boldsymbol{x}}^{(1,2)} = \boldsymbol{F}(\boldsymbol{x}^{(1,2)}) + \boldsymbol{b}u^{(1,2)} \\ y^{(1,2)} = \boldsymbol{c}\boldsymbol{x}^{(1,2)} \end{cases}, \tag{1}$$

where $\boldsymbol{x}^{(1,2)} \in \mathbb{R}^m$, $y^{(1,2)} \in \mathbb{R}$, and $u^{(1,2)} \in \mathbb{R}$ are respectively the state variables, the output signals, and the input signals of 1-st and 2-nd oscillators. $\boldsymbol{b} \in \mathbb{R}^m$ and $\boldsymbol{c} \in \mathbb{R}^{1\times m}$ denote the input and output vectors, respectively. We assume that each oscillator has at least one unstable fixed point $\boldsymbol{x}^* : \boldsymbol{F}(\boldsymbol{x}^*) = 0$. The input signals $u^{(1,2)}$ are given by

$$u^{(1)} = k\left\{y^{(2)}_{\tau(t)} - y^{(1)}\right\}, \;\; u^{(2)} = k\left\{y^{(1)}_{\tau_0} - y^{(2)}\right\}, \tag{2}$$

where $y^{(2)}_{\tau(t)} := y^{(2)}(t-\tau(t))$ and $y^{(1)}_{\tau_0} := y^{(1)}(t-\tau_0)$ are delayed output signals. $k > 0$ denotes the coupling strength. $\tau_0 > 0$ is the constant delay, and $\tau(t) > 0$ denotes the periodically time-varying delay (see Fig. 1) around the nominal delay τ_0 with the amplitude $\delta \in [0, \tau_0)$,

$$\tau(t) := \tau_0 + \delta f(\Omega t), \tag{3}$$

where $\Omega > 0$ is the frequency of a periodic sawtooth function $f(x)$,

$$f(x) := \begin{cases} +\dfrac{2x}{\pi} - 1 - 4n & \text{if } x \in [2n\pi, (2n+1)\pi) \\ -\dfrac{2x}{\pi} + 3 + 4n & \text{if } x \in [(2n+1)\pi, 2(n+1)\pi) \end{cases}, \quad n = 0, 1, 2, \ldots .$$

We will consider the local stability of a homogeneous steady state in a pair of oscillators (1), (2),

$$\left[\boldsymbol{x}^{(1)T}, \; \boldsymbol{x}^{(2)T}\right]^T = \left[\boldsymbol{x}^{*T}, \boldsymbol{x}^{*T}\right]^T. \tag{4}$$

Substituting the perturbation $\Delta\boldsymbol{x}^{(1,2)} := \boldsymbol{x}^{(1,2)} - \boldsymbol{x}^*$ into Eqs. (1), (2), it yields the dynamics around steady state (4),

$$\dot{\boldsymbol{X}} = \{\boldsymbol{I}_2 \otimes (\boldsymbol{A} - k\boldsymbol{bc})\}\,\boldsymbol{X} + \boldsymbol{B}_1\boldsymbol{X}_{\tau_0} + \boldsymbol{B}_2\boldsymbol{X}_{\tau(t)}, \tag{5}$$

where $\boldsymbol{A} := \{\partial \boldsymbol{F}(\boldsymbol{x})/\partial \boldsymbol{x}\}_{\boldsymbol{x}=\boldsymbol{x}^*}$ is the Jacobian matrix, and

$$\boldsymbol{X} := \begin{bmatrix} \Delta\boldsymbol{x}^{(1)} \\ \Delta\boldsymbol{x}^{(2)} \end{bmatrix}, \;\; \boldsymbol{B}_1 := \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ k\boldsymbol{bc} & \boldsymbol{0} \end{bmatrix}, \;\; \boldsymbol{B}_2 := \begin{bmatrix} \boldsymbol{0} & k\boldsymbol{bc} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

For sufficiently large Ω, the stability of linear time-varying system (5) is guaranteed if liner time-invariant system

$$\dot{\boldsymbol{X}} = \{\boldsymbol{I}_2 \otimes (\boldsymbol{A} - k\boldsymbol{bc})\}\,\boldsymbol{X} + \boldsymbol{B}_1\boldsymbol{X}_{\tau_0} + \frac{\boldsymbol{B}_2}{2\delta}\int_{t-\tau_0-\delta}^{t-\tau_0+\delta} \boldsymbol{X}(s)\mathrm{d}s, \tag{6}$$

is stable [14]. Thus, we will focus on the stability of system (6) instead of system (5). The characteristic equation of system (6) is given by,

$$G(s) := \det\left[s\boldsymbol{I}_{2m} - \{\boldsymbol{I}_2 \otimes (\boldsymbol{A} - k\boldsymbol{bc})\} - \boldsymbol{B}_1 e^{-s\tau_0} - \boldsymbol{B}_2 e^{-s\tau_0} H(s\delta)\right] = 0, \tag{7}$$

where

$$H(x) := \begin{cases} \frac{\sinh x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}.$$

Therefore, the local stability of steady state (4) is governed by the roots of Eq. (7).

For checking the stability of characteristic Eq. (7), we derive the marginal stability curves on the connection parameter (k, τ_0) space. Substituting $s = i\lambda$ ($\lambda \in \mathbb{R}$) into Eq. (7), we obtain,

$$G(i\lambda) = G_+(\lambda)G_-(\lambda) = 0, \tag{8}$$

where

$$G_\pm(\lambda) := \det\left[i\lambda\boldsymbol{I}_m - \boldsymbol{A} + k\boldsymbol{bc} \pm ke^{-i\lambda\tau_0}\sqrt{\Phi(\lambda\delta)}\boldsymbol{bc}\right],$$

$$\Phi(x) := \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}.$$

By solving Eq. (8) in terms of k and τ_0, we can derive the marginal stability curves on the connection parameter space [10]. Our stability analysis focuses on the local stability of the homogeneous steady state; hence, we cannot deal with the global stability.

3. Experimental circuits

In our experiments, we deal with coupled double-scroll circuits [15] (see Fig. 2),

$$\begin{cases} C_1\dfrac{\mathrm{d}v_1^{(1,2)}}{\mathrm{d}t} = \dfrac{1}{R}\left(v_2^{(1,2)} - v_1^{(1,2)}\right) - h\left(v_1^{(1,2)}\right) \\ C_2\dfrac{\mathrm{d}v_2^{(1,2)}}{\mathrm{d}t} = \dfrac{1}{R}\left(v_1^{(1,2)} - v_2^{(1,2)}\right) + i_{\mathrm{L}}^{(1,2)} + i_u^{(1,2)} \\ L\dfrac{\mathrm{d}i_{\mathrm{L}}^{(1,2)}}{\mathrm{d}t} = -v_2^{(1,2)} \end{cases}, \tag{9}$$

where $v_1^{(1,2)}$ [V] and $v_2^{(1,2)}$ [V] denote the voltages of capacitors C_1 [F] and C_2 [F], respectively. $i_{\mathrm{L}}^{(1,2)}$ [A] is the current through inductor L [H]. The current $h\left(v_1^{(1,2)}\right)$ [A] through the nonlinear resistor is given by

$$h(v) := m_0 v + \frac{1}{2}(m_1 - m_0)\left|v + B_{\mathrm{p}}\right| + \frac{1}{2}(m_0 - m_1)\left|v - B_{\mathrm{p}}\right|.$$

The two circuits are coupled through the coupling resistor r. Thus, the coupling signals are given by

$$i_u^{(1)} = \frac{1}{r}\left(v^{(2)}_{2,\tau(t)} - v_2^{(1)}\right), \;\; i_u^{(2)} = \frac{1}{r}\left(v^{(1)}_{2,\tau_0} - v_2^{(2)}\right), \tag{10}$$

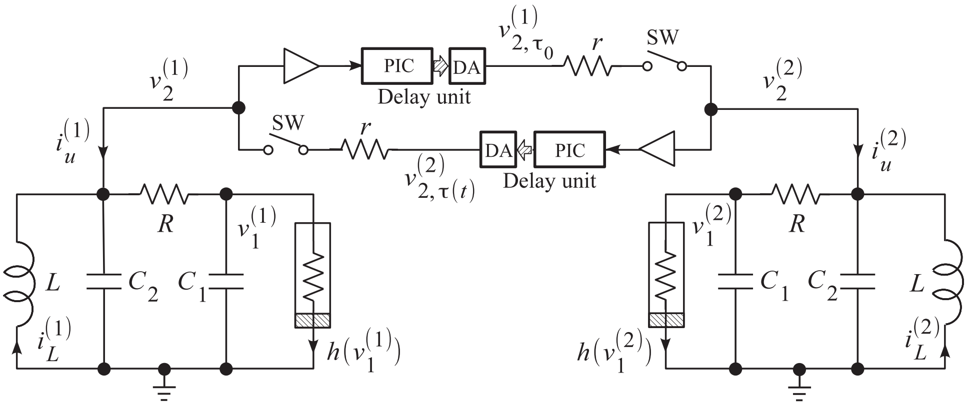


Figure 2: Experimental circuit diagram.

where $v^{(2)}_{2,\tau(t)} := v^{(2)}_2(t-\tau(t))$ and $v^{(1)}_{2,\tau_0} := v^{(1)}_2(t-\tau_0)$ are the delayed voltages. These delayed voltages are generated by the delay units in Fig. 2, which are implemented by PICs (PIC18F2550) and DA converters [11].

The non-dimensional form (1), (2) of coupled double scroll circuits (9), (10) is given with

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{bmatrix} \eta\{x_2 - x_1 - g(x_1)\} \\ x_1 - x_2 + x_3 \\ -\gamma x_2 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ \boldsymbol{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \quad (11)$$

where

$$x_1 := \frac{v_1}{B_\mathrm{p}},\ x_2 := \frac{v_2}{B_\mathrm{p}},\ x_3 := \frac{i_L R}{B_\mathrm{p}},\ k = \frac{R}{r},$$
$$\eta := \frac{C_2}{C_1},\ \gamma := \frac{R^2 C_2}{L},\ a := m_1 R,\ b := m_0 R,$$
$$g(x) := bx + \frac{1}{2}(b-a)\{|x-1| - |x+1|\}.$$

Note that in Eq. (11), the non-dimensional time $t/(RC_2)$ is used instead of the real time t. The double scroll circuit (11) has the three equilibrium points, $\boldsymbol{x}^*_\pm := \begin{bmatrix} \pm p & 0 & \mp p \end{bmatrix}^T$ and $\boldsymbol{x}^*_0 := \boldsymbol{0}$, where $p := (b-a)/(b+1)$. Here, we focus on the stability of $\boldsymbol{x}^*_+$. The Jacobian matrix around $\boldsymbol{x}^*_+$ is given by

$$\boldsymbol{A} = \begin{bmatrix} -\eta(b+1) & \eta & 0 \\ 1 & -1 & 1 \\ 0 & -\gamma & 0 \end{bmatrix}.$$

4. Experimental results

The parameters of circuit (9) are fixed at

$$C_1 = 0.1 \times 10^{-6}\ \mathrm{F},\ C_2 = 1.0 \times 10^{-6}\ \mathrm{F},$$
$$L = 180 \times 10^{-3}\ \mathrm{H},\ R = 1,800\ \Omega,$$
$$B_p = 1.0\ \mathrm{V},\ m_0 = -0.4 \times 10^{-3},\ m_1 = -0.8 \times 10^{-3}. \quad (12)$$

The circuit with parameter (12) shows the well-known double scroll attractor [15]: a resonant frequency of each oscillator at the equilibrium point is approximately 3.45. The frequency of time-varying delay (3) is fixed at a large value $\Omega = 23$.

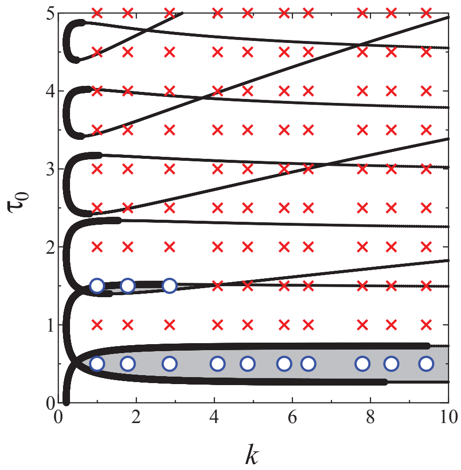
Figure 3 shows the stability regions (i.e., shaded areas) for (a) $\delta = 0$ (i.e., time-invariant delay connection) and (b) $\delta = 0.35$ (i.e., one-way partial time-varying delay connection) on (k, τ_0) space. These regions are derived from the marginal stability curves which are solutions of Eq. (8). Comparing the region in Fig. 3(a) with that in Fig. 3(b), we see that the one-way partial time-varying delay connection expands the region substantially. Especially, for $k > 7.1$ in Fig. 3(b), there are no curves; that is, we can use long connection delay τ_0 to induce amplitude death.

The symbol ○ (×) in Fig. 3 denotes the occurrence (non-occurrence) of amplitude death experimentally. It can be confirmed that most of the experimental results (i.e., symbols ○ and ×) agree with our analytical results (i.e., the shaded areas). A few parameter sets of experimental results do not agree with the analytical results because of parameters mismatch between the two oscillators. Figure 4 shows the time-series data of the voltages $v^{(1)}_1$ and $v^{(1)}_2$ at point A: $(k, \tau_0) = (7.80, 1)$ and point B: $(k, \tau_0) = (2.85, 3)$ in Fig. 3(b). The two double-scroll circuits are coupled at $t = 60$ ms; that is, the switch SW in Fig. 2 is turned on. For point A, the voltages converge onto the equilibrium point after coupling. On the other hand, for point B, the voltages continue to oscillate even after coupling [1].

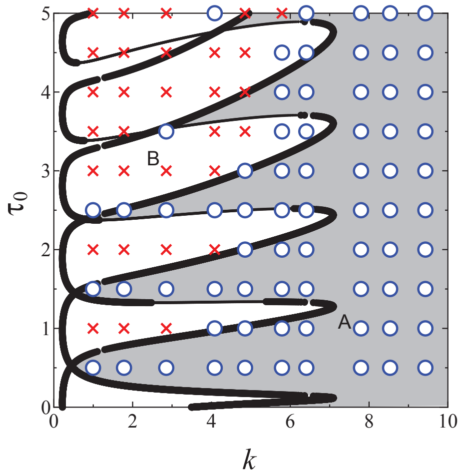
5. Conclusion

This report has experimentally investigated amplitude death induced by a one-way partial time-varying delay connection in a pair of double scroll circuits. It has been experimentally verified that the one-way partial time-varying delay connection can induce amplitude death even for long connection delay. Our experimental results agreed well with analytical results.

[1] The time-series data in Fig.4(b) seems to be a periodic solution. Since our results are based on the local stability of steady state (4), we cannot deal with global behavior, such as the periodic solution.

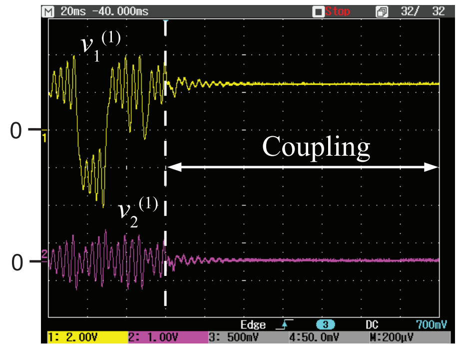


(a) δ = 0 (time-invariant delay connection)

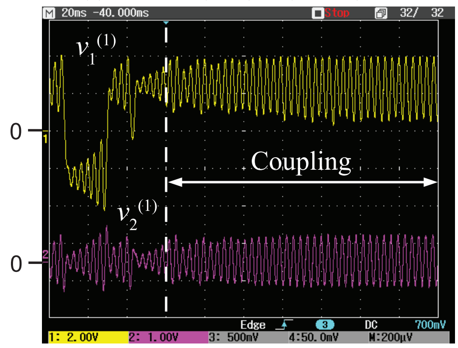


(b) δ = 0.35 (one-way partial time-varying delay connection)

Figure 3: Stability regions (i.e., shaded areas) for a pair of double scroll circuits. The symbol ○ (×) denotes the occurrence (non-occurrence) of amplitude death in our experiments ($\Omega = 23$).



(a) Point A: $(k, \tau_0) = (7.80, 1)$



(b) Point B: $(k, \tau_0) = (2.85, 3)$

Figure 4: Time-series data at points A and B in Fig. 3(b). Horizontal axis: 20 ms/div; Vertical axis: 2 V/div for $v_1^{(1)}$ and 1 V/div for $v_2^{(1)}$.

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References

[1] A. Pikovsky *et al.*, Cambridge University Press, 2001.

[2] G. Saxena *et al.*, Phys. Rep., vol.521, pp.205 – 228, 2012.

[3] D. Aronson *et al.*, Physica D, vol.41, pp.403–449, 1990.

[4] D. Reddy *et al.*, Phys. Rev. Lett., vol.80, pp.5109–5112, 1998.

[5] A. Prasad *et al.*, Phys. Lett. A, vol.318, pp.71–77, 2003.

[6] S. Huddy and J. Skufca, IEEE Trans. Power Electron., vol.28, pp.247–253, 2013.

[7] T. Biwa *et al.*, Phys. Rev. Applied, vol.3, p.034006, 2015.

[8] F.M. Atay, Phys. Rev. Lett., vol.91, p.094101, 2003.

[9] K. Konishi *et al.*, Phys. Rev. E, vol.81, p.016201, 2010.

[10] K. Konishi *et al.*, Phys. Lett. A, vol.374, pp.733–738, 2010.

[11] Y. Sugitani *et al.*, Nonlinear Dyn., vol.70, pp.2227–2235, 2012.

[12] Y. Sugitani *et al.*, Phys. Rev. E, vol.92, p.042928, 2015.

[13] Y. Sugitani *et al.*, Proc. of Nonlinear Dynamics of Electronic Systems, pp.261–264, 2012.

[14] W. Michiels *et al.*, IEEE Trans. Autom. Control, vol.50, pp.493–504, 2005.

[15] T. Matsumoto *et al.*, IEEE Trans. Circuit and Sys., vol.32, pp.797–818, 1985.