2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

Tabu Search Method for Solving Covering Salesman Problem with Nodes and Segments

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Abstract—As a new covering problem, we have already mathematically formulated Covering Salesman Problem with Nodes and Segments (CSPNS). In the CSPNS, a city distribution is given. The goal of the CSPNS is to identify the shortest tour of a subset of all given cities, such that each city which is not on the tour is within radius r of any city or segment on the tour. In addition, we have already proposed a local search method for the CSPNS. In this paper, to find better solutions of the CSPNS, we propose a heuristic method by using a tabu search.

1. Introduction

The Covering Salesman Problem with Nodes and Segments (CSPNS) is a new covering problem [1]. In the CSPNS, a city distribution is given. The CSPNS is to identify the shortest tour of a subset of all given cities, such that a city which is not on the tour is within radius r of any city (node) or path (segment) on the tour. To solve the CSPNS, we have already proposed a local search method [1]. In the method, first, an initial tour passing through all cities is constructed, that is, the Traveling Salesman Problem (TSP) is solved. Second, many visited cities are eliminated from the tour. Finally, a visited city and an unvisited city are exchanged to make a shorter tour. Although, the method quickly finds solutions of the CSPNS, the obtained solutions are local optimal solutions.

To find better solutions of combinatorial optimization problems, many metaheuristics were proposed such as the simulated annealing (SA) [2], the genetic algorithm (GA) [3], and the tabu search (TS) [4–6]. Among them, the tabu search shows good performances for many combinatorial optimization problems for example traveling salesman problem [6, 7], quadratic assignment problem [8, 9], and vehicle routing problem [10]. In this paper, to find better solutions of the CSPNS, we propose a new method by using the tabu search. As a result, the proposed method shows good performances for the CSPNS.

2. Covering Salesman Problem with Nodes and Segments

In the CSPNS, a set of cities $V = \{1, 2, ..., n\}$, distances d_{ij} between city i and city j, perpendicular distances c_{ijk} between city i and path jk, and a covering distance $r > 0$ are given. If distance d_{ij} is less than or equal to r, a city i can cover a city j. If perpendicular distance c_{ijk} is less than or equal to r, a path jk can cover a city i. The constraint conditions of the CSPNS are as follows: (1) the salesman starts from the city 1 and goes back to the city 1, (2) the salesman can visit each city at most once, and (3) all cities are within radius r of any visited city or path on the tour. The goal of the CSPNS is to identify the shortest tour which satisfies the constant conditions. Figure 1 shows a graphical example of the CSPNS.

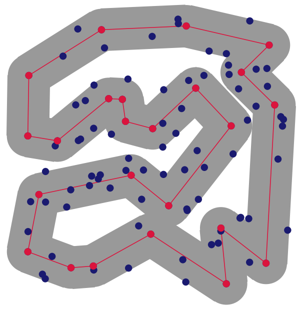


Figure 1: Graphical interpretation of the Covering Salesman Problem with Nodes and Segments. In this example, red circles are visited city and blue circles are unvisited city. All unvisited cities are covered by any visited city or path on the tour.

3. Proposed Methods

3.1. Local Search Methods

We have already proposed local search methods [1] for solving the CSPNS. In the method, first, an initial tour passing through all given cities is constructed, that is, the TSP is solved. Second, the length of the initial tour is improved by a local search method. Next, many visited cities are eliminated from the tour. Finally, a visited city and an unvisited city are exchanged to make a shorter tour.

The procedure of the local search method is shown as follows [1]:

Step 1: Constructing an initial tour

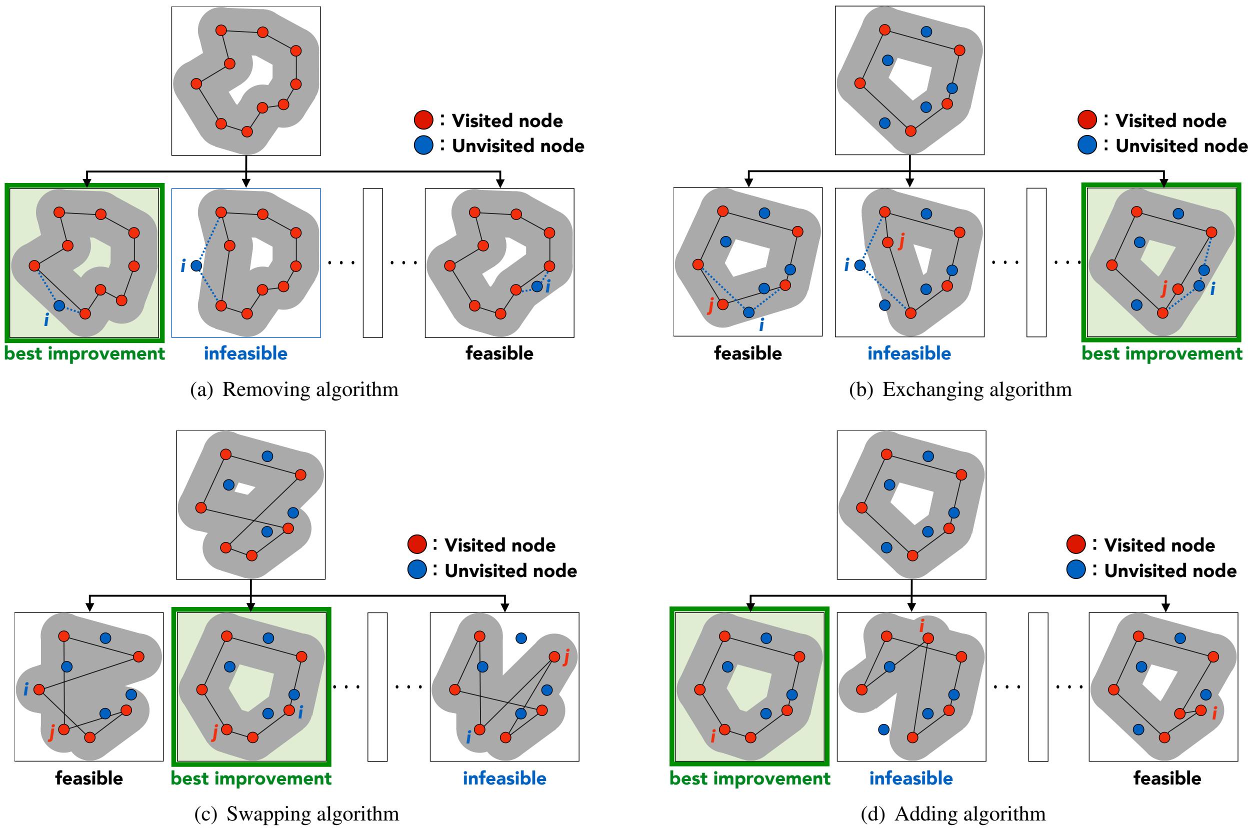


Figure 2: Local search methods used in the proposed method [1]

A random tour passing through all given cities is constructed.

Step 2: Improving the length the initial tour
To make a shorter tour, the initial tour is improved by the Lin-Kernighan heuristic [11]. The Lin-Kernighan heuristic is applied until no further improvement cannot be obtained.

Step 3: Removing algorithm
A visited city i is removed from a current tour, if a new tour is feasible solutions (Fig.2(a)). If many good improvements can be found, the best improvement is carried out.

Step 4: Exchanging algorithm
To make a shorter tour, a visited city i and an unvisited city j are swapped (Fig.2(b)). If many good combinations are found, the best exchange is carried out.

Step 5: Swapping algorithm
To make a shorter tour, a visited city i and other visited city j are swapped (Fig.2(c)).

Step 6: Repeating each local search
Until a local optimum solution is obtained, the removing algorithm, the exchanging algorithm, and the swapping algorithm are repeated.

3.2. A Method by using Tabu Search

To find good near-optimal solutions of the CSPNS, we propose a metaheuristics method by using the tabu search [4–6]. The local search method moves from one solution S to another improving solution S' in neighborhood solutions of the solution S, until a local optimum solution is found. To escape from the local optimum solution, the tabu search moves from one solution S to the best solution S' in neighborhood solutions of the solutions S, even though the solution S' is worse than the solution S. Then, to avoid period exploration process, the solution S is added to a tabu list. The information of the tabu list is used to guide the move from a current solution to a next solution. The solutions in the tabu list cannot be selected as a next solution for a certain temporal duration. Its duration is called tabu tenure.

The proposed method has four movements: (i) removing step, in which a visited city is removed from the tour (Fig.2(a)), (ii) exchanging step, in which a visited city in the tour is exchanged by a unvisited city (Fig.2(b)), (iii) swapping step, in which one visited city and another visited city are swapped (Fig.2(c)), and (iv) adding step, in which a unvisited city is added into the tour (Fig.2(d)). In the CSPNS, if a unvisited city is added into a current tour, the length of the new tour always becomes longer. However, when the number of visited cities is low, it is difficult to construct feasible neighborhood solutions in the removing step, the swapping step, and the exchanging step. Therefore, we add the unvisited city into the tour at the adding step.

Then, three tabu lists are constructed in the method. One tabu list memories a deleted city from the tour (*Deleted list*). The cities listed in *Deleted list* cannot be added to the tour for τ_d tenure. Another list memories an added city into the tour (*Added list*). The cities listed in *Added list* cannot be deleted from the tour for τ_a tenure. The other list memories swapped cities in the swapped step (*Swapped list*). The cities listed in *Swapped list* cannot be exchanged for τ_s tenure in Exchanged step. In the removing step, when a visited city i is removed from the tour, the city i is listed in the *Deleted list*. In the exchanging step, when a visited city i is exchanged for an unvisited city j, the city i is listed in the *Deleted listed* and the city j is listed in the *Added list*. In the swapping step, when visited cities i and j are swapped, the city i is listed in the *Swapped list*. In the proposed method, the removing step, the exchanging step, the swapping step and the adding step are carried out just one time in one iteration. In the proposed method, if the best solution is updated, the corresponding tour is improved to obtain a local optimal solution by the proposed local search methods.

4. Simulations and Results

To investigate performances of the proposed method, we used DIMACS [12] which is one of the benchmark problems for the Traveling Salesman Problem. The number of city n is set to 50 and 100. The cities are uniformly distributed in the $10^6 \times 10^6$ square and seed for making the instances is set to 1.

The covering distance r is set to $20,000, 40,000, 60,000,$ and $80,000$. The tabu tenure τ_d, τ_a, and τ_s are set to several values. In this simulation, the values of τ_a and τ_s are set to same values. The iteration of the tabu search method is 500. We conducted simulations using the Intel compiler on a Mac Pro (2.8 GHz Intel Core i7) with 16GB memory running Mac OS X 10.10.5.

By using the formulation of the CSPNS [1] and a mixed-integer programming solver, we can obtain an optimal solution. Table 1 shows a length of an optimal tour or a near-optimal tour obtained by the gurobi optimizer [13] in one hour. In this table, if the gurobi optimizer cannot find an optimal tour in one hour, the obtained best solution is shown by italic face. For $n = 50$, we can find an optimal solution of the CSPNS. These results are used to investigate performances of the proposed method.

Table 2 shows the attained results for the local search method and the tabu search method. In this table, the first column is covering distance r, the second column is the average percentage of gaps between the solution of the local search method and the best solution obtained by the gurobi optimizer (Table 1), the third column is the average percentage of gaps between the solution of the tabu search method and the best solution obtained by the gurobi optimizer, and the fourth column describes an improvement rate of the length of tour. From the results, when the covering distance is long ($r = 80,000$), the local search method cannot obtain good results. By using the tabu search, the better solutions can be found.

Figure 3 shows the results for several length of the tabu tenure. From Fig.3, when the length of tabu tenure is small, the proposed method cannot obtained good solutions because the method cannot escape local minima. When we appropriately set to the length of of the tabu tenure, the proposed method obtained good results for all instances and the covering distance.

Table 1: An obtained optimal solution or a *near-optimal solution (italic face)* by a gurobi optimizer [13] in one hour.

	Covering distance r			
n	20,000	40,000	60,000	80,000
50	5,973,311	5,807,486	5,700,104	4,991,860
100	*7,865,796*	*7,171,844*	*6,495,132*	*5,513,677*

Table 2: The results of the local search method and the tabu search method. Results are expressed by percentages of average gaps from obtained solution and the best solution obtained by the gurobi optimizer (Table 1). In third column, the numbers in a parenthesis show the tabu tenure τ_a, τ_d, and τ_s when the average percentage is obtained (τ_a, τ_d and τ_s)).

(a) $n = 50$

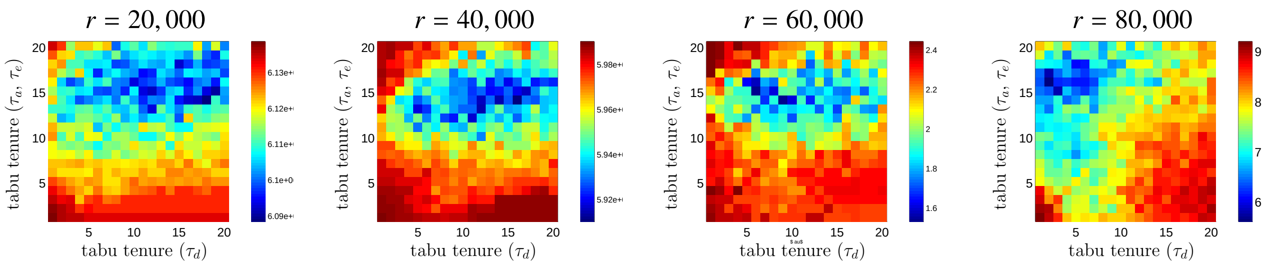
r	Local Search	Tabu Search	Improvement rate
20,000	2.83	2.01 (16,14)	0.95 %
40,000	3.41	2.01 (12,14)	1.35 %
60,000	2.76	1.53 (9,15)	1.19 %
80,000	9.43	5.57 (2,16)	3.52 %

(b) $n = 100$

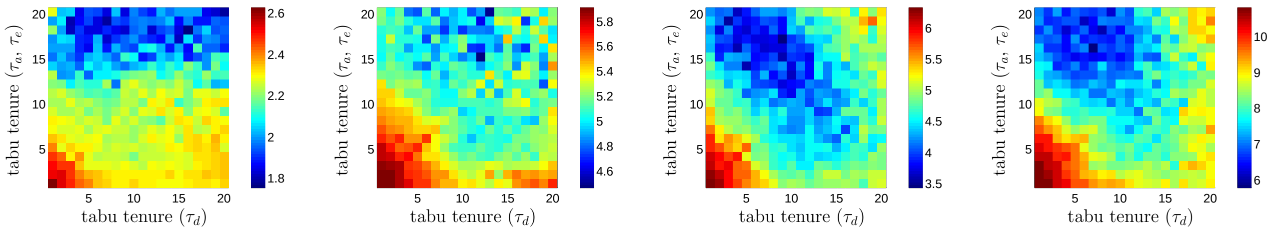
r	Local Search	Tabu Search	Improvement rate
20,000	2.63	1.75 (16,18)	0.86 %
40,000	5.93	4.45 (12,15)	1.40 %
60,000	6.26	3.40 (9,17)	2.69 %
80,000	10.86	5.73 (7,16)	4.63 %

5. Conclusions

In this paper, we proposed a metaheuristics method by using the tabu search for solving the CSPNS. From the computational results, although the proposed method uses simple local search methods and the tabu search, it obtains good solutions. In the future work, it is important to develop an effective adjustment method of tabu tenure. We



(a) n = 50



(b) n = 100

Figure 3: Performance of the proposed method. The average gaps (%) between an optimal solution (a near-optimal solution) obtained by the gurobi optimizer and solutions of the proposed method in 50 trials are indicated by color bar.

also develop variable neighborhood search to find good solutions. It is desirable to solve much larger problems, such as 10^5 order problems.

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