

# Electromagnetic Scattering from Rough Sea Surface Covered with Oil Films

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**Abstract-** A composite random rough surface model is presented for describing rough sea surface covered with oil films, the electromagnetic scattering from this sea surface is studied based on the Stratton-Chu integral equations. A general expression for the radar cross section is derived taking into account a modulation of the rough surface by long surface waves, and the formulae of bistatic scattering coefficient is obtained further. The curves of the bistatic scattering coefficient of HH polarization with varying of the scattering angle are obtained by numerical implementation, the influence of the root mean square and correlation function of small scale roughness, the ratio of the root mean square and correlation function of large scale roughness, wave number of space, the amplitude of spatial fluctuation, the electromagnetic wave irradiation area, the root mean square and correlation function of large scale roughness and the frequency of the incident wave on the bistatic scattering coefficient is discussed. The numerical results show that the influence of these on the bistatic scattering coefficient is very complex.

## I. INTRODUCTION

To detect and monitor oil films at sea is becoming increasingly important, because of the threats posed by such pollution to marine and wildlife [1]. In recent years, remote-sensing techniques and corresponding processing techniques have been developed for this purpose [2]. Gabriel Soriano et al. proposed a cutoff invariant Two-Scale Model in electromagnetic scattering from sea surfaces [3], Joel T. Johnson et al. proposed a numerical study of the retrieval of sea surface height profiles from low grazing angle radar data [4]. But few studies have been reported on electromagnetic wave scattering from rough sea surface covered with oil films.

In this letter, an approach for a description of the composite random rough surface is developed in order to have some progress in the solution of electromagnetic scattering from rough sea surface covered with oil films. The basis for the analysis is an approximate solution of the integral equation.

## II. FORMULATION

Consider electromagnetic scattering from a perfectly conducting random rough surface. Assume that a large surface wave with some random parameters modulates this surface. Here we consider only the case when both incident and scattered waves show horizontal polarization. The electric and magnetic fields inside a closed surface may be determined by the Stratton-Chu integral equations [5].

According to the reference [6], the first term of the solution of the integral equation for the electric field shows the form

$$E = \frac{ik_z e^{i\vec{k}\vec{R}}}{4\pi|\vec{R}|} \int dx e^{-i\gamma_x x - i\gamma_z f(x)} \quad (1)$$

Here  $|\vec{R}|$  means distance between a point at the surface and the observation point,  $\gamma_x = |\vec{k}|(\sin\theta_i - \sin\theta_s)$ ,  $\gamma_z = |\vec{k}|(\cos\theta_i + \cos\theta_s)$ ,  $k_z = |\vec{k}|\cos\theta_i$ , where  $\theta_i$  and  $\theta_s$  are the incidence and scattering angles,  $\vec{k}$  is the wave vector of the incident wave, and  $f(x)$  means surface height at the horizontal position  $x$ .

The intensity of the scattered field can readily write

$$I \sim \iint dx dx' e^{-i\gamma_x(x-x') - i\gamma_z[\bar{M}(x) - \bar{M}(x')] - i\gamma_z[\tilde{M}(x) - \tilde{M}(x')]} \quad (2)$$

Here  $\bar{M}(x)$  is the mean part of the large-scale roughness, and  $\tilde{M}(x) = f_1(x) + f_2(x)$  where  $f_1(x)$  and  $f_2(x)$  mean height of the small- and large-scale roughness.

The amplitudes of the statistically homogeneous small-scale components of the sea surface may be modulated by the large-scale components of the surface waves. As a result, a statistically inhomogeneous surface appears.

Let  $f_1(x)$  be a statistically homogeneous random field with correlation length  $l_1$ , and  $f_2(x)$  a modulating function with a scale  $l_2 \gg l_1$ . Hence we can choose its mean part as  $\bar{M}(x) = A \cos Kx$ .

For averaging we note that, one can write for normally distributed random variables  $f_k$

$$\left\langle \exp \left\{ i \sum_{k=1}^n q_k f_k \right\} \right\rangle = \exp \left\{ -\frac{1}{2} \sum_{r,s=1}^n W(f_r, f_s) q_r q_s \right\} \quad (3)$$

Where  $W(f_r, f_s) = \langle f_r f_s \rangle$

We introduce new variables of integration  $u = x - x'$  and  $v = x + x'$  substitute them into (2), and average analytically by using (3). As result we obtain

$$\bar{I} \sim \int_{-L}^L dv \int_{-\infty}^{\infty} du e^{-i\gamma_x u + 2i\gamma_z A \sin(kv/2) \sin(ku/2)} \cdot e^{-\gamma_z^2 [\langle h_1^2 \rangle (1-R_1) + \langle h_2^2 \rangle (1-R_2)]} \quad (4)$$

Here  $2L$  is the linear size of the illuminated area,  $R_1, R_2$  are the correlation functions, and  $h_1^2 = \langle f_1^2 \rangle$ ,  $h_2^2 = \langle f_2^2 \rangle$  mean rms height of the small- and large-scale roughness.

Consider the case of a Gaussian distribution of both the small- and large-scale components of the surface waves. It is worth noting that one can use another probability density of the spectrum, that is, a Pierson-Moscowitz spectrum.

In order to simplify Eqn. (4), we expand the term taking into account the large-scale roughness into a Taylor series up to the second term. We obtain instead of (4)

$$\bar{T} \sim e^{-\gamma_z^2 \langle h_1^2 \rangle} \int_{-L}^L dv \int_{-\infty}^{\infty} du e^{-i\gamma_x u + 2i\gamma_z A \sin(kv/2) \sin(ku/2)} \gamma_z^2 [\langle h_1^2 \rangle e^{-u^2/L^2} + \langle h_2^2 \rangle (u/L_2)^2] \quad (5)$$

Expanding the integrand of (5) into an infinite Taylor series, the radar scattering cross section is obtained

$$\sigma^0 = \sqrt{\pi} L e^{-\gamma_z^2 h_1^2} \sum_{m=0}^{\infty} \frac{1}{m!} (\gamma_z h_1)^{2m} S_0^{-1/2} \sum_{r=0}^{\infty} (\gamma_z A / 2)^{2r} \phi(r) T(r) \quad (6)$$

Where

$$\phi(r) = \frac{1}{(r!)^2} + 2(-1)^r \sum_{n=0}^{r-1} \frac{(-1)^k}{n!(2r-n)!} \sin c[KL(r-n)]$$

$$S_0 = \gamma_z^2 \frac{h_2^2}{l_2^2} + \frac{m}{2l_1^2}$$

$$T(r) = \sum_{s=0}^{2r} \frac{(-1)^s (2r)!}{s!(2r-s)!} e^{-[\gamma_x - K(r-s)]^2 / (4S_0)}$$

When the short-wave range of the electromagnetic waves is used for remote sensing of the sea surface, it is possible to simplify (6) by taking into account that the inequality

$|r-s|K \ll \gamma_x$  holds for a wide range of variations of

$r$  and  $s$ , we suppose that  $\gamma_x K / (\gamma_z \gamma)^2 \ll 1$ . Then one can write Eqn. (6) as

$$\sigma^0 = \sqrt{\pi} L e^{-\gamma_z^2 h_1^2} \sum_{m=0}^{\infty} \frac{1}{m!} (\gamma_z h_1)^{2m} S_0^{-1/2} e^{-\gamma_x^2 / (4S_0)} M(m) \quad (7)$$

Where

$$M(m) = \sum_{r=0}^{\infty} [\gamma_x \gamma_z A K / (4S_0)]^{2r} e^{-\gamma_x K r / (4S_0)} \phi(r)$$

If  $KL = m_1 \pi$ ,  $m_1 = 1, 2, \dots$ , the following identity can be used to simplify (7)

$$M(m) = J_0(y) \quad (8)$$

Where  $J_0(y)$  is the Bessel function of zeroth order, whose argument is given by

$$y(m) = i\gamma_x \gamma_z A K \exp[-\gamma_x K / (4S_0)] / (2S_0)$$

Note that for the cases of most practical importance, the value of  $KL$  is usually so large that the second term in the above expression for  $\phi(r)$  (sum over  $n$ ) is negligibly small as compared to the first one. Then one can use (7) with  $M(m)$  given by (8) even if  $KL \neq m_1 \pi$ .

In this way, we can obtain the scattering coefficient of rough sea surface covered with oil films as

$$\sigma = 10 \log_{10} \sigma^0 \quad (9)$$

### III. NUMERICAL RESULTS AND DISCUSSION

The sampling of the incident frequency is  $f = 900$  GHz, the sampling of the incident angle is  $\theta_i = 20^\circ$ . For length, HH polarization of bistatic scattering is numerically calculated only.

1. The influence of the root mean square of small-scale  $h_1$  on the scattering coefficient

Fig.1 depicts the distribution of  $\sigma$  with  $\theta_s$  for different  $h_1$  with  $l_1 = 0.42\lambda$ ,  $L = 100\lambda$ ,  $A = 0.1/K$ ,  $K = 2\pi$ , and  $h_2/l_2 = 0.1$ .

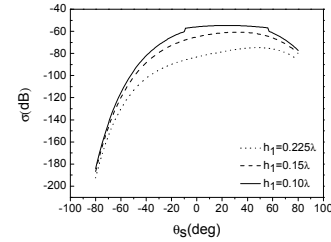


Fig.1 Distribution of  $\sigma$  with  $\theta_s$  for different  $h_1$

It is obvious that the influence of  $h_1$  on the scattering coefficient is big and obvious, the bigger of  $h_1$  is, the smaller of  $\sigma$  is, and scattering angle is closer the incident angle, the scattering coefficient with the more obvious changes in  $h_1$ , the more deviation incident angle, the scattering coefficient with the more obscure changes in  $h_1$ .

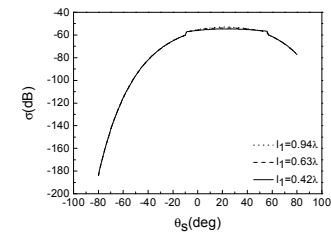


Fig.2 Distribution of  $\sigma$  with  $\theta_s$  for different  $l_1$

2. The influence of the correlation length of small-scale  $l_1$  on the scattering coefficient

The distribution of  $\sigma$  with  $\theta_s$  for different  $l_1$  with  $h_1 = 0.10\lambda$ ,  $L = 100\lambda$ ,  $A = 0.1/K$ ,  $K = 2\pi$ , and  $h_2/l_2 = 0.1$  is depicted in Fig.2.

All the curves of Fig.2 are almost coincide, and it is obvious that the influence of  $l_1$  on the scattering coefficient is small and obscure, and in the scattering angle is equal on both sides of the vicinity of the incident angle, the scattering coefficient with  $l_1$  changes slightly apparent than some.

### 3. The influence of $h_2/l_2$ on the scattering coefficient

Fig.3 depicts the distribution of  $\sigma$  with  $\theta_s$  for different  $h_2/l_2$  with  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $L = 100\lambda$ ,  $A = 0.1/K$ ,  $K = 2\pi$ .

There is stated regularity in distribution of  $\sigma$  with  $\theta_s$  for different  $h_2/l_2$  from Fig.3, that is, when  $\theta_s < \theta_i$ , the bigger of  $h_2/l_2$  is, the bigger of  $\sigma$  is, the scattering coefficient with  $h_2/l_2$  is very obvious changes in, in the vicinity of both sides of the scattering angle is equal to incident angle, the scattering coefficient almost no change with the change in  $h_2/l_2$ , when  $\theta_s > \theta_i$ , the bigger of  $h_2/l_2$  is, the bigger of  $\sigma$  is, the scattering coefficient with  $h_2/l_2$  is very obvious changes in, but it is not clear that such the case of  $\theta_s < \theta_i$ .

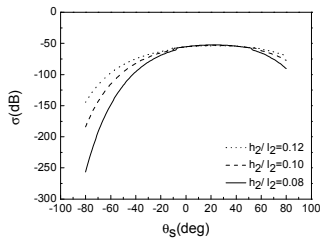


Fig.3 Distribution of  $\sigma$  with  $\theta_s$  for different  $h_2/l_2$

### 4. The influence of the wave number of space $K$ on the scattering coefficient

Distribution of  $\sigma$  with  $\theta_s$  for different  $K$  with  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $h_2/l_2 = 0.1$ ,  $L = 100\lambda$ ,  $A = 0.1/2\pi$  is depicted in Fig.4.

It is obvious that the influence of  $K$  on the scattering coefficient is big and obvious, that is, the bigger of  $K$  is, the bigger of the frequency of the curve oscillating is, and when  $\theta_s = \theta_i$ , the scattering coefficient has a sharp increase, but there is not stationary regularity for the influence of  $K$  on the magnitude of scattering coefficient.

### 5. The influence of the amplitude of spatial fluctuation $A$ on the scattering coefficient

Fig.5 depicts the distribution of  $\sigma$  with  $\theta_s$  for different  $A$  with  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $h_2/l_2 = 0.1$ ,  $L = 100\lambda$ ,  $K = 2.0\pi$ .

It is obvious that the influence of  $A$  on the scattering coefficient is big and obvious, that is, the bigger of  $A$  is, the bigger of the frequency of the curve oscillating is, it is obvious that the influence of  $A$  on the scattering coefficient is same

with the influence of  $K$ . Similarly, when  $\theta_s = \theta_i$ , the scattering coefficient has a sharp increase, but there is not stationary regularity for the influence of  $A$  on the magnitude of scattering coefficient.

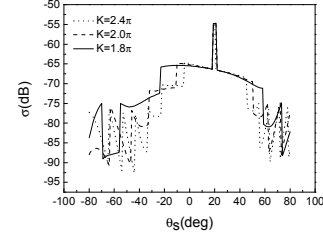


Fig.4 Distribution of  $\sigma$  with  $\theta_s$  for different  $K$

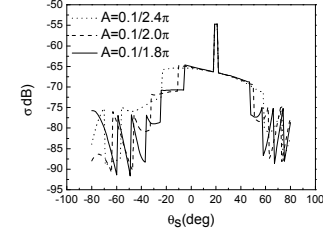


Fig.5 Distribution of  $\sigma$  with  $\theta_s$  for different  $A$

### 6. The influence of the electromagnetic wave irradiation area $L$ on the scattering coefficient

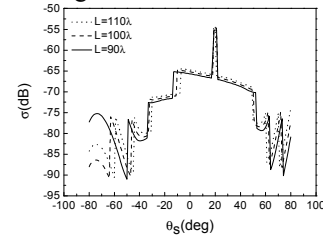


Fig.6 Distribution of  $\sigma$  with  $\theta_s$  for different  $L$

The distribution of  $\sigma$  with  $L$  in the condition that  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $h_2/l_2 = 0.1$ ,  $K = 2.0\pi$ ,  $A = 0.1/K$  is depicted in Fig.6.

The curve of the distribution of  $\sigma$  with  $L$  is oscillatory, and the bigger of  $L$  is, the bigger of the frequency of the curve oscillating is, but it is un conspicuous with  $L$  than with  $K$  and. Similarly, when  $\theta_s = \theta_i$ , the scattering coefficient has a sharp increase, but there is not stationary regularity for the influence of  $L$  on the magnitude of scattering coefficient.

### 7. The influence of rms of large scale roughness $h_2$ on the scattering coefficient

Fig.7 depicts the distribution of  $\sigma$  with  $\theta_s$  for different  $h_2$  with  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $l_2 = 127\lambda$ ,  $L = 100\lambda$ ,  $K = 2.0\pi$ ,  $A = 0.1/K$ .

It is obvious that the influence of  $h_2$  on the scattering coefficient is big and obvious, the bigger of  $h_2$  is, the bigger of  $\sigma$  is, and scattering angle the closer the incident angle, the scattering coefficient with the more obscure changes in the  $h_2$ , the more deviation incident angle, the scattering coefficient

with the more obvious changes in  $h_2$ , this is differ from the influence of rms of small scale roughness  $h_1$  on the scattering coefficient.

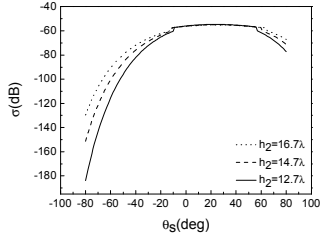


Fig.7 Distribution of  $\sigma$  with  $\theta_s$  for different  $h_2$

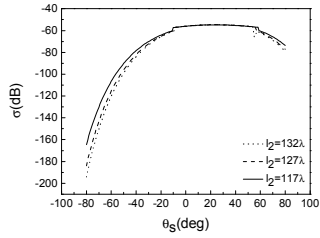


Fig.8 Distribution of  $\sigma$  with  $\theta_s$  for different  $l_2$   
8. The influence of correlation function of large scale roughness  $l_2$  on the scattering coefficient

The distribution of  $\sigma$  with  $l_2$  in the condition that  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $h_2 = 12.7\lambda$ ,  $L = 100\lambda$ ,  $K = 2.0\pi$ ,  $A = 0.1/K$  is depicted in Fig.8.

It is obvious that the influence of  $l_2$  on the scattering coefficient is big and obvious, the bigger of  $l_2$  is, the smaller of  $\sigma$  is, and scattering angle the closer the incident angle, the scattering coefficient with the more obscure changes in the  $l_2$ , the more deviation incident angle, the scattering coefficient with the more obvious changes in  $l_2$ , this is differ from the influence of correlation length of small scale roughness  $l_1$  on scattering coefficient.

9. The influence of the frequency of the incident wave on the scattering coefficient

The variation of  $\sigma$  with  $f$  under the condition that  $h_1 = 0.10\lambda$ ,  $l_1 = 0.42\lambda$ ,  $h_2/l_2 = 0.1$ ,  $L = 100\lambda$ ,  $K = 2.0\pi$ ,  $A = 0.1/K$  (i.e. a certain rough surface),  $\theta_i = 20^\circ$ ,  $\theta_s = 10^\circ, 40^\circ$  is depicted in Fig.9.

The curve of the variation of  $\sigma$  with  $f$  is oscillatory, generally speaking, when the incident frequency increases, the magnitude of the scattering coefficient is decreases, but this change is relatively slow, the oscillation frequency of curve decreases, the oscillation frequency of curve at  $\theta_s = 40^\circ$  is bigger than at  $\theta_s = 10^\circ$ , the curve is continuous, but is non-differentiable in certain frequency point. In the point of the same frequency, the corresponding scattering coefficient when

the scattering angle is equal to  $40^\circ$  is bigger than that when the scattering angle is equal to  $10^\circ$ .

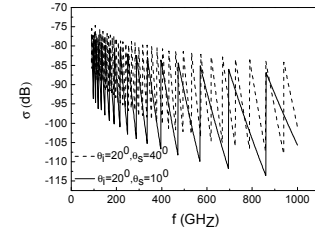


Fig.9 Distribution of  $\sigma$  with  $f$

It is obvious that the influence of  $h_1$  on the scattering coefficient is big and obvious, the bigger of  $h_1$  is, the smaller of  $\sigma$  is, and scattering angle the closer the incident angle, the scattering coefficient with the more obvious changes in  $h_1$ , the more deviation incident angle, the scattering coefficient with the more obscure changes in  $h_1$ .

#### IV. CONCLUSIONS

In this paper, the electromagnetic scattering from the rough sea surface covered with oil films is studied using a composite random rough surface model. A general expression for the radar cross section is obtained taking into account a modulation of the rough surface by long surface waves. The curves of the bistatic scattering coefficient of HH polarization with varying of the scattering angle are obtained by numerical implementation, the influence of the root mean square and correlation function of small scale roughness, the ratio of the root mean square and correlation function of large scale roughness, wave number of space, the amplitude of spatial fluctuation, the electromagnetic wave irradiation area, the root mean square and correlation function of large scale roughness and the frequency of the incident wave on the bistatic scattering coefficient is discussed. These results will be applicable for solving many engineering, technical, and scientific problems.

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