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Multi-point search algorithm with rotation angle dependent on the best position

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Abstract—Canonical deterministic particle swarm optimization (abbr. CD-PSO) was proposed to analyze the dynamics theoretically. The CD-PSO is easy to implement because the system is deterministic. In this paper, we propose a multi-point search algorithm based on the behavior of the CD-PSO. The multi-point search algorithm improves more efficiency than the CD-PSO. We confirm the effectiveness of the multi-point search algorithm by using benchmark functions.

1. INTRODUCTION

Optimization problems to search the most suitable value of the given evaluation function are fundamental and important problems. However, it is generally difficult to search a feasible solution of large-scale problems within real time. In order to solve such large scale problems quickly, various heuristic optimization solvers are proposed. The particle swarm optimization(abbr. PSO) [1][2] is one of such optimization solvers. The PSO can search a feasible a solution quickly comparing with other heuristic optimization solvers. Each particle of the PSO has the best position information that gives the best evaluation value, and it shares in swarm.

Since the PSO contains stochastic factors, the rigorous analysis of the dynamics of the PSO is quite difficult[3]. To analyze the dynamics, we proposed a canonical deterministic PSO (abbr. CD-PSO)[4].

The coordinate system of CD-PSO is adapted a canonical coordinate system. Since CD-PSO is a deterministic system, the dynamics of the particle of CD-PSO can be clarified theoretically. The analysis results of CD-PSO indicate that the search range is shrunk with the update of the best location information. To overcome this situation, we proposed the method to keep the extent of the search region[5]. However, the method cannot search around the obtained best position. Also, the method cannot generate diversity of the solutions because CD-PSO is a deterministic system. The diversity is very important for heuristic optimization solvers. The diversity of the solution search performance of the deterministic system is poorer than the stochastic system. To overcome this situation, we consider a mechanism to generate the diversity for the deterministic system. The mechanism improves the solution search performance of the local search. In this article, the purpose of this study is to confirm the solution search performance. We will confirm the solution search performance by using the benchmark functions.

2. CD-PSO

The conventional PSO is described by the following equations.

$$\begin{cases} \boldsymbol{v}_i^{t+1} = w\boldsymbol{v}_i^t + c_1 r_1(\boldsymbol{pbest}_i^t - \boldsymbol{x}_i^t) + c_2 r_2(\boldsymbol{gbest}^t - \boldsymbol{x}_i^t) \\ \boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \boldsymbol{v}_i^{t+1} \end{cases} \tag{1}$$

where, $\boldsymbol{v}_i^t$ and $\boldsymbol{x}_i^t$ denote the velocity vector and the location vector of the i-th particle on the t-th iteration, respectively. $\boldsymbol{pbest}_i^t$ means the location that gives the personal best value of the evaluation function of the i-th particle until the t-th iteration. $\boldsymbol{gbest}^t$ means the location which gives the best value of the evaluation function on the t-th iteration in the swarm. $w \geq 0$ is an inertia weight coefficient, $c_1 \geq 0$, and $c_2 \geq 0$ are acceleration coefficients, and $r_1 \in [0, 1]$ and $r_2 \in [0, 1]$ are two separately generated uniformly distributed random numbers.

To analyze the dynamics of the conventional PSO, the random coefficients have been omitted from the conventional PSO. We rewrite the best location information as follows.

$$\begin{cases} \boldsymbol{p}_i^t = \dfrac{c_1\boldsymbol{pbest}_i^t + c_2\boldsymbol{gbest}^t}{c} \\ c = c_1 + c_2 \end{cases} \tag{2}$$

We normalize the location information by $\boldsymbol{p}_i^t$. Without loss of generality, we consider one-dimensional case. In this case, Eq. (1) is transformed into the following matrix form:

$$\begin{bmatrix} y_i^{t+1} \\ v_i^{t+1} \end{bmatrix} = \begin{bmatrix} w & -c \\ w & 1-c \end{bmatrix} \begin{bmatrix} y_i^t \\ v_i^t \end{bmatrix}, \tag{3}$$

where, $y_i^t = x_i^t - p_i^t$.

The behavior of the particle is governed by the eigenvalues of the matrix in Eq. (3). The eigenvalue λ is derived as follows.

$$\lambda = \frac{(1 + w - c) \pm \sqrt{(1 + w - c)^2 - 4w}}{2} \tag{4}$$

This system is a discrete-time system. Therefore, if the eigenvalues exist within the unit circle on the complex plane, the system is said to be stable. When the eigenvalues are complex conjugate numbers, the behavior of the trajectory of Eq. (3) in the phase space $v_i - y_i$ becomes a spiral motion. We have clarified that the system exhibits an excellent solution search performance when the particle exhibits the spiral motion in the phase space. In this case, the damping factor Δ and the rotation angle θ are given as follows.

$$\Delta = \sqrt{\mathrm{Im}(\lambda)^2 + \mathrm{Re}(\lambda)^2} = \sqrt{w}, \tag{5}$$

$$\theta = \arctan\frac{\mathrm{Im}(\lambda)}{\mathrm{Re}(\lambda)} = \arctan\frac{\sqrt{4w-(1+w-c)^2}}{(1+w-c)}. \tag{6}$$

To clarify the effect of the eigenvalues, we derive a canonical deterministic PSO (abbr. CD-PSO).

$$\begin{bmatrix} y_i^{t+1} \\ v_i^{t+1} \end{bmatrix} = \begin{bmatrix} \delta & -\omega \\ \omega & \delta \end{bmatrix}\begin{bmatrix} y_i^t \\ v_i^t \end{bmatrix} \tag{7}$$

The damping factor Δ and the rotation angle θ of CD-PSO are derived as

$$\Delta = \sqrt{\delta^2+\omega^2}, \tag{8}$$

$$\theta = \arctan\frac{\omega}{\delta}. \tag{9}$$

By using these parameters, Eq. (7) is rewritten as follows.

$$\begin{bmatrix} y_i^{t+1} \\ v_i^{t+1} \end{bmatrix} = \Delta\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} y_i^t \\ v_i^t \end{bmatrix} \tag{10}$$

If the rotational angle is set by the golden angle, the search position becomes not overlapped[6][7]. The golden angle ϕ is defined as

$$\phi = 180(3-\sqrt{5})[\mathrm{deg}]. \tag{11}$$

The particle of CD-PSO cannot escape from the local minimum if the particle traps the local minimum[6][7].

To improve the global search ability, we proposed a control method that set a lower limit of the rotation radius on the phase space[5]. When the rotation radius is smaller than a criterion value R, the proposed method resets the velocity of the particle as follows.

$$\begin{bmatrix} x_i^t \\ v_i^t \end{bmatrix} = \begin{bmatrix} p_i^t + R\cos\theta \\ R\sin\theta \end{bmatrix}, \text{if } \sqrt{y_i^{t2}+v_i^{t2}} < R. \tag{12}$$

We confirmed that the efficiency of the local search is reduced because the criterion of the rotation radius is set[5].

3. Proposed method

The time series of the search points of CD-PSO are shown in Fig. 1a. The search points are vibrated as that the center is the best location and the rotation radius is R. Therefore, the location information is described as the following equation.

$$x_i^t = R\cos(\theta t) + p_i^t \tag{13}$$

However, the searching motion is the local search performance is poor[5]. Since the rotation angle is a constant, the efficiency of the search in the vicinity of the best position is poor. Therefore, in order to improve efficiency of the local search ability, we propose a method which depends on the rotation angle at the best position. The proposed method is shown in the following

$$x_i^t = R\cos(\theta_i^t) + p_i^t \tag{14}$$

$$\theta_i^t = \theta_i^{t-1} + (\frac{|p_i^t - x_i^t|}{R}) \tag{15}$$

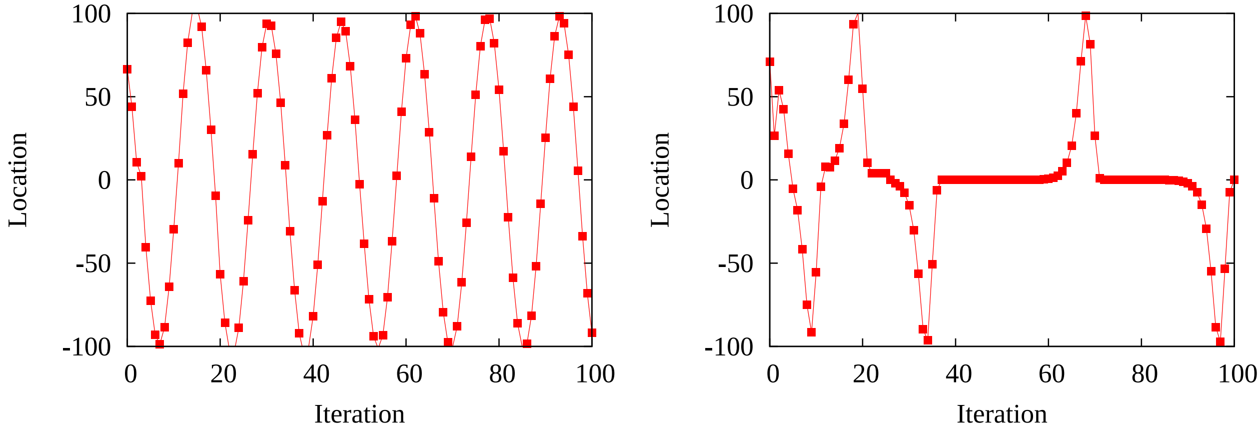
The step width of the particle of the proposed method varies with the rotation angle. If the amount of changing times the rotation angle is large, it performs a global search. On the other hand, when the rotation angle is small, the particle performs a local search. The rotation angle is determined by the best position and its location. Figure1 shows a changing time series of the search position of the particle. If the rotation angle is a constant, it performs generally search.

However, the proposed method can confirm that we search for a lot of position neighborhood best than rotation angle is a constant.

In this case, the rotation angle is synchronized. This situation is problem. When the rotation angle is synchronized, the biased search performance is reduced. Since the best position of each particle of the proposed method is different, there is no bias without different rotation angle synchronization. Figure 2 shows the behavior of the particles on the two dimensional space. If the rotation angle is a constant, the particle does not search on the second quadrant and the fourth quadrant on the phase space. However, the proposed method can be confirmed that it has been all quadrant search.

4. Numerical experiment

We confirm the performance with the benchmark functions. The applying benchmark functions are represented in Table 1. We compare the numerical simulation results of CD-PSO, PSO and the proposed method. Table 2 shows the results of CD-PSO, PSO, and the proposed method. These results indicate that the solution search ability of the proposed method is excellent comparing with the CD-PSO. Sphere function and Rosenbrock function is a unimodal function, Rastrigin functions and Griewank function is a multimodal function In the case of multi-modal evaluation function, the search results of the proposed method correspond to the conventional PSO. However, in the case of unimodal evaluation functions, the conventional PSO performance is better than the proposed method. The reason is the proposed system is not converged.



(a) The rotation angle is certain

(b) Proposed method

Figure 1: Time variation of the search position of the particle

Table 1: Benchmark functions

Function	Definition	Domain	Optimal
Sphere	$f(\boldsymbol{x}) = \sum_{i=1}^{D} x_i^2,$	$[-100, +100]^D$	$f(\boldsymbol{0}) = 0$
Rastrigin	$f(\boldsymbol{x}) = 10D + \sum_{i=1}^{D}\left(x_i^2 - 10\cos(2\pi x_i)\right),$	$[-100, +100]^D$	$f(\boldsymbol{0}) = 0$
Rosenbrock	$f(\boldsymbol{x}) = \sum_{i=1}^{D-1}\left(100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2\right),$	$[-100, +100]^D$	$f(\boldsymbol{1}) = 0$
Griewank	$f(\boldsymbol{x}) = 1 + \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}}),$	$[-600, +600]^D$	$f(\boldsymbol{0}) = 0$
Shifted Rastrigin	$f(\boldsymbol{y}) = 10D + \sum_{i=1}^{D}\left(y_i^2 - 10\cos(2\pi y_i)\right), \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-100, +100]^D$	$f(\boldsymbol{o}) = 0$
Shifted Rosenbrock	$f(\boldsymbol{y}) = \sum_{i=1}^{D-1}\left(100(y_i^2 - y_{i+1})^2 + (1 - y_i)^2\right), \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-100, +100]^D$	$f(\boldsymbol{1} + \boldsymbol{o}) = 0$

$\boldsymbol{o}$ denotes a random unform vector in the domain

5. Conclution

We proposed the multi-point search algorithm which is based on the behavior of the CD-PSO. In addition, We confirmed the effectiveness of the proposed method by using benchmark problems. It is necessary to confirm which parameter is effected to the solution search performance. Furthermore, we would like to construct a novel model of the deterministic PSO whose solution search ability is better than the conventional stochastic PSO. These are our future problems.

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Table 2: Simulation result

Function	Manner	Mean	Best	Worst	Std. Dev.
Sphere 関数	CD-PSO	5.57e+03	1.89e+03	1.37e+04	2.76e+03
	PSO	1.91e-42	1.19e-51	5.44e-41	9.75e-42
	Propsed	5.95e-05	1.08e-05	2.39e-04	5.68e-05
Rastrigin 関数	CD-PSO	6.31e+03	2.76e+03	1.36e+04	2.49e+03
	PSO	1.28e+02	5.27e+01	2.04e+02	3.92e+01
	Propsed	1.54e+02	5.91e+01	3.16e+02	5.37e+01
Rosenbrock 関数	CD-PSO	2.72e+08	3.18e+07	1.01e+09	2.64e+08
	PSO	3.13e+01	1.34e+00	2.05e+02	4.25e+01
	Propsed	5.69e+02	1.03e+01	3.39e+03	8.79e+02
Griewank 関数	CD-PSO	2.39e+00	1.36e+00	4.42e+00	6.99e-01
	PSO	2.15e-02	0.00e+00	9.82e-02	2.27e-02
	Propsed	2.93e-02	2.10e-08	7.68e-01	1.07e-01
Shifted Rastrigin 関数	CD-PSO	4.30e+04	1.76e+02	6.17e+04	1.31e+04
	PSO	1.74e+02	3.01e+01	5.24e+02	1.03e+02
	Propsed	1.56e+02	5.54e+01	3.74e+02	5.91e+01
Shifted Rosenbrock 関数	CD-PSO	1.38e+10	1.13e+03	2.81e+10	7.04e+09
	PSO	1.85e+01	1.00e-02	7.60e+01	2.29e+01
	Propsed	3.34e+02	1.16e+02	9.14e+02	1.82e+02

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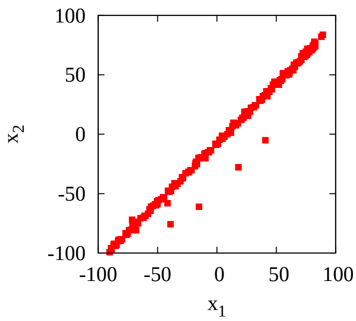
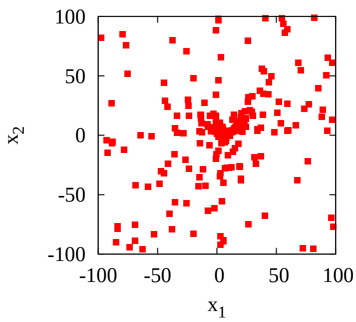


(a) The rotation angle is certain



(b) Proposed method

Figure 2: The behavior of the particles in on the 2-dimensional solution space