



## Effect of a Root-Raised-Cosine Filtered BPSK Signal on a Stochastic Resonance Receiver

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**Abstract**—Signal filtering is necessary for wireless communication however it fluctuates the signal amplitude and affects the performance of a stochastic resonance (SR) receiver. In this paper, we evaluate the bit error rate (BER) performance of filtered binary phase shift keying (BPSK) on an SR receiver. The result shows that filtering improves the BER performance of the SR receiver because the amplitude fluctuation contributes to improving the SR effect, indicating the applicability of the SR receiver to bandlimited BPSK signals.

### 1. Introduction

Stochastic Resonance (SR) is a nonlinear phenomenon that can enhance the response of a system by adding noise under certain conditions [1]. In contrast to many other systems that deal with noise negatively, SR positively utilizes noise. A particular advantage of SR is that it can detect weak signals buried in noise.

The characteristic of this interesting phenomenon has been discussed in the context of nonlinear physics [2,3], and some applications of SR to wireless communication have been proposed. SR is expected to be utilized for spectrum sensing in cognitive radio [4], signal detection [5,6], and improvement of receiver sensitivity [7].

We focus on applying SR to a receiver, for communication with weak signals. As application of SR to a receiver may have better sensitivity compared to a conventional receiver, and it can detect a significantly weak signal [8]. In previous research, an SR receiver in radio frequency (RF) has been proposed [8] and implemented [9]. The SR receiver in RF shows better performance than that in baseband [8].

However, in those studies, the transmitted signal was not bandlimited. In a real situation, transmitted signals must be bandlimited by a transmit-filter for removing spurious power and preventing interference with other channels. Filtering causes a fluctuation of the signal amplitude and fluctuating amplitude affects an SR receiver because the SR effect highly depends on the input signal amplitude.

Herein, we evaluate the effect of a filtered signal on the SR Receiver. We consider RF binary phase shift keying (BPSK) signals with and without filtering and evaluate per-

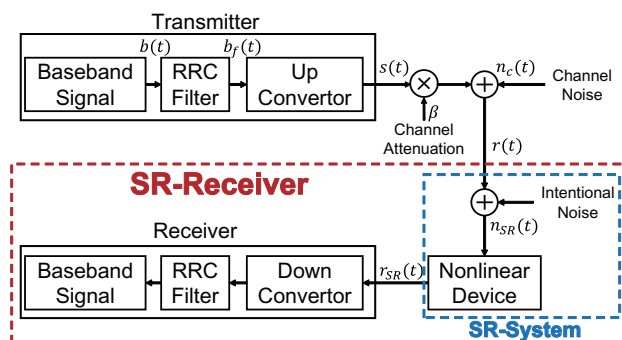


Figure 1: System model of the SR receiver

formance using bit-error rate (BER).

This paper is organized as follows. First, we show the system model of an SR receiver with filtering in Sec. 2. In Sec. 3, we evaluate experimentally the BER performance of the SR receiver with and without filtering, and compare those results. Conclusions are given in Sec. 4.

### 2. System model

Figure 1 shows the system model of the SR receiver. We assume that the transmitted signal is filtered and the received signal power is too weak to be received by a conventional receiver, i.e., the received signal level is lower than the minimum level that the receiver can detect. In such a situation, we use an SR receiver to receive such a weak (subthreshold) signal and communicate with it.

#### 2.1. Transmitter

We assume that a BPSK signal is transmitted. The BPSK signal in baseband  $b(t)$  is given by

$$b(t) = \sum_i d_i g(t - iT), \quad (1)$$

where  $d_i$  is the  $i$ th symbol of a binary data sequence, which takes  $\{\pm 1\}$  and  $g(t)$  is a rectangular pulse that has duration  $T$  with unity amplitude.

We denote  $b_f(t)$  as the BPSK signal bandlimited by the root-raised-cosine (RRC) filter  $x_{RRC}(t)$ , given by

$$b_f(t) = x_{RRC}(t) * b(t), \quad (2)$$

where  $*$  represents convolution. The frequency characteristic of  $x_{RRC}(t)$ ,  $X_{RRC}(f)$  is given as

$$X_{RRC}(f) = \begin{cases} \sqrt{T} & 0 \leq |f| \leq \frac{1-\alpha}{2T} \\ \sqrt{\frac{T}{2} \left\{ 1 + \cos \left[ \frac{\pi T}{\alpha} \left( |f| - \frac{1-\alpha}{2T} \right) \right] \right\}} & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases} \quad (3)$$

The RRC filter is commonly used in wireless communication systems, because it helps in minimizing intersymbol interference. The roll-off factor  $\alpha$  determines the excess bandwidth of a signal. When  $\alpha=0.5$ , the excess bandwidth is 50 %. A small roll-off factor results in strict bandlimiting, but causes time-domain ripples and distortion. Therefore, in wireless communication systems, the roll-off factor is generally set to 0.2-0.5.

Figure 2 shows examples of a power spectrum density of a BPSK signal with (a) non-filtering and (b) filtering with an RRC filter. The non-filtered signal illustrated in Fig. 2 (a) has a spurious power spectrum. The filtered signal is bandlimited and the removing spurious power spectrum is as shown in Fig. 2 (b).

After applying the RRC filter,  $b_f(t)$  is upconverted to carrier frequency  $f_c$  and the RF BPSK signal  $s(t)$  is transmitted, which is given by

$$s(t) = b_f(t) \sin f_c t. \quad (4)$$

The transmitted signal  $s(t)$  is propagated in a wireless communication channel. Through that channel, the transmitted signal is attenuated by factor  $\beta$ , and a channel noise  $n_c(t)$  is added to the attenuated signal. In general, channel noise  $n_c(t)$  is assumed to be a zero-mean Gaussian noise. The received signal  $r(t)$  is given by

$$r(t) = \beta s(t) + n_c(t). \quad (5)$$

## 2.2. SR receiver

We construct an SR receiver composed of an SR system and a conventional RF receiver as shown in Fig. 1. The SR system is connected to the front stage a conventional RF receiver.

We assume that the attenuated signal  $\beta s(t)$  is lower than the minimum level  $\xi_{RX}$  that the receiver can detect. Thus

$$|\beta s(t)| < \xi_{RX}. \quad (6)$$

In this system, the received signal  $r(t)$  is the input to the SR system. The SR system is composed of intentional

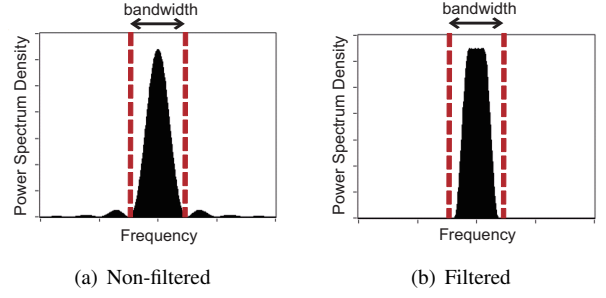


Figure 2: Examples of the power spectrum density of BPSK signal with (a) non-filtering and (b) filtering with the RRC filter ( $\alpha = 0.5$ )

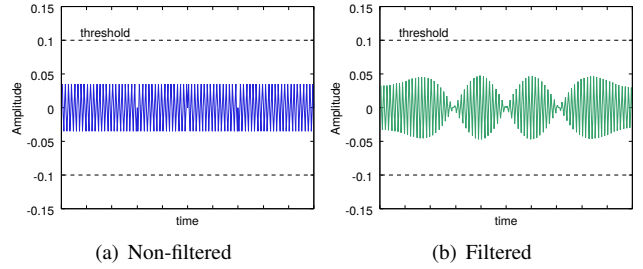


Figure 3: Examples of the BPSK signal with (a) non-filtering and (b) filtering with the RRC filter ( $\alpha = 0.5$ )

noise  $n_{SR}$  and a nonlinear-device exhibiting SR. We use the SR system to enhance the received signal, and then the conventional receiver can process the output signal of the SR system  $r_{SR}(t)$ . When channel noise power is not large enough for the SR system to exhibit its optimal performance, we require additional intentional noise  $n_{SR}$  and adjustment of the noise power to the optimal noise power of the SR [7,9]. Note that the output level of the SR system is enough larger than  $\xi_{RX}$ .

If the SR system has a threshold smaller than or equal to the sensitivity of the conventional receiver, the SR receiver can have better performance than that of the conventional one. This is because unlike the conventional receiver, the SR receiver can detect subthreshold signal.

The conventional receiver applies the RRC filter to the received signal in baseband. The receiver-filter is the same as the transmit-filter.

## 2.3. Filtering

In this section, we discuss how filtering affects the SR receiver.

Figure 3 shows examples of a BPSK signal in the time-domain, with (a) non-filtering and (b) filtering with an RRC filter. In Fig. 3 (a), the signal peak level is constant in its symbol duration. However, in Fig. 3 (b), the peak level fluctuates significantly. This fluctuation is caused by filtering and cutting the spurious power spectrum.

We focus on the fluctuation of the signal level caused

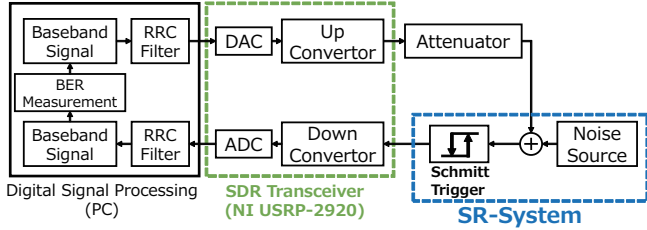


Figure 4: The measurement system of the BER of SR receiver

by filtering. In the SR system, subthreshold signals can be detected by adding intentional noise and then the signal peak level exceeds the threshold. The performance of the SR system depends on the difference between the threshold and amplitude. Smaller difference exhibit better SR performance. In this sense, the SR system is sensitive to the received signal level, and the fluctuation by filtering has some effects on its performance.

In this paper, we evaluate the effect of fluctuation by filtering. The SR performances between the filtered signal and non-filtered signals are not identical because the large-amplitude parts of the signal are easily detectable while the small-amplitude parts are hard to detect. The unevenness of the received signal amplitude may have some effects on the SR receiver's performance.

### 3. Experiment

In this section, we experimentally evaluate the effect of a filtered BPSK signal on the SR receiver. We use BPSK signals with and without filtering, and evaluate the effect of filtering by comparing the results.

#### 3.1. BER measurement

The BER measurement system is shown in Fig. 4, and the parameter settings are shown in Table 1 and 2. We use a Schmitt trigger as the nonlinear device, and a software-defined radio (SDR) transceiver (NI USRP 2920) as a conventional transceiver. The baseband signals are filtered by digital signal processing. The threshold level of the Schmitt trigger  $\xi_{SR}$  is higher than the that of the SDR receiver.

For SR at RF, the Schmitt trigger is designed with a high speed comparator (Analog Devices ADCMP607), which has a wide input bandwidth of 750MHz [10]. The input noise, which is the sum of the channel and intentional noise, is assumed to be a zero-mean Gaussian noise. We add the noise of bandwidth of 100MHz using a the vector signal generator (Agilent Technologies N5182A).

The transmitted signal is attenuated by an attenuator, and the signal with noise is fed into the Schmitt trigger. Table 2 shows the amplitude of the received signals, which are sub-thresholds of the Schmitt trigger. Both of the filtered and non-filtered signals are set to have the same mean square

Table 1: Parameter settings for BER measurements

Parameter	Value
Threshold of Schmitt Trigger $\xi_{SR}$	100[mV]
Modulation scheme	BPSK
Symbol rate $1/T$	250[kHz]
Carrier frequency $f_c$	70[MHz]
Transmitted data bits	10000
Number of trials	100
Noise distribution	Gaussian
Noise bandwidth	100[MHz]
I/Q sampling rate	4[MHz]
Filter type	RRC
Filter length	8
Roll-off factor $\alpha$	0.5

Table 2: Signal amplitude parameter

Filter type	none	RRC
Average amplitude[mV]	35	32.78
Maximum amplitude[mV]	35	51.1
Mean square amplitude[mV <sup>2</sup> ]	1.225	1.225

amplitude, meaning they have the same signal power. This parameter setting result in the same BER performance as that of a conventional receiver.

The filter length determines the order of the finite impulse response filter. We set this parameter large enough for the experiment. We set the roll-off factor  $\alpha=0.5$ , which is the practical value.

Figure 5 shows the BER performance of the SR receiver. As we see from the figure, the BER performance of the SR receiver improves in a specific noise power region. This is a typical phenomenon exhibiting SR. In this region, the input noise power is optimal for SR, so the SR receiver can detect a subthreshold signal and the BER performance can be improved.

#### 3.2. Effect of filtering on BER performance

Figure 6 shows the result in the region where noise power is lower than optimal noise power of the SR. In this region, filtered BPSK shows better BER performance than non-filtered BPSK.

The reason exhibiting such a result is the maximum amplitude. In SR system, signal can be detected by exceeding the threshold. In the region where noise power is lower than optimal noise power of the SR, BER performance are diminished because a weak signal having low power noise cannot exceed the threshold and hence cannot be detected. In the SR system, the suprathreshold signal is critical for detecting a weak signal. As Table 2 shows, the maximum amplitude of the filtered (RRC) BPSK signal is larger than

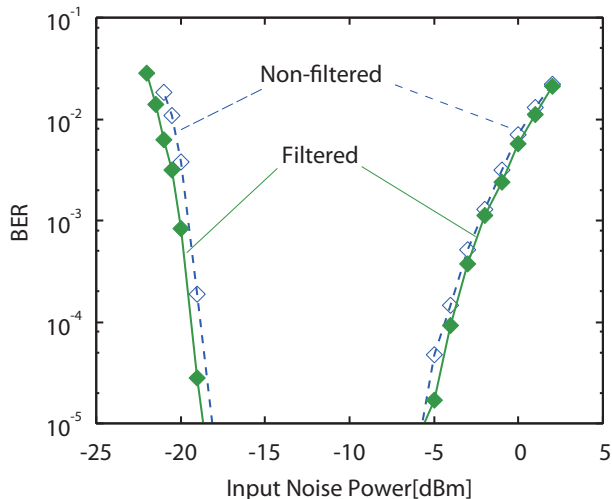


Figure 5: BER performance of the SR receiver

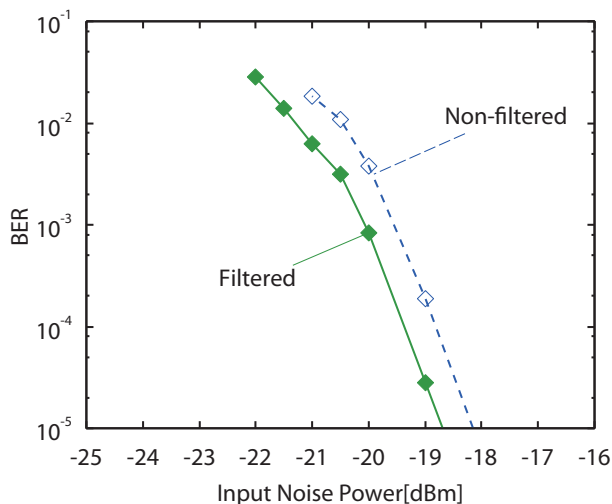


Figure 6: BER performance in a lower input noise power of Fig. 5

that of the non-filtered one. The signal enhancement by noise is stochastic, but the amplitude fluctuation by filtering is not stochastic. This contributes to signal detection by SR. Owing to the amplitude fluctuation, in the SR receiver, when filtered and non-filtered BPSK signals have the same power, filtering leads to better BER performance as filtering causes the amplitude fluctuation.

#### 4. Conclusion

In this paper, we experimentally evaluated the BER performance of filtered BPSK on an SR receiver. The result shows that filtering improves the BER performance of the SR receiver because amplitude fluctuation contributes to improving the SR effect. This result indicates the applicability of the SR receiver to bandlimited BPSK signals.

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