2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

# Robust performance of two-wheeled mobile robot circular formations controlled by coupled oscillators

Tadashi Nakamura†, Koki Yoshida†, Keiji Konishi†, and Naoyuki Hara†

†Graduate School of Engineering, Osaka Prefecture University
1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531 JAPAN
Email: konishi@eis.osakafu-u.ac.jp

**Abstract**—Formation control has recently received considerable attention in the field of robotics and control theory. The present paper deals with two-wheeled mobile robots which achieve circular formations based on dynamics of coupled oscillators. The main advantage of this control is that circular formations can be obtained by a simple control law based on nonlinear dynamics. The purpose of this paper is to show an another advantage, robustness of formation: even if some robots on formations stop due to trouble, the remaining robots make up for the troubled robots and keep to form the circular pattern without changing control law. It is shown on numerical simulations that the robustness of formations depends on how we choose robots to be removed from formed robots and to be added to them.

## 1. Introduction

Considerable attention has been paid to the research of complex network science in a variety of fields. Many subjects on this research are based on dynamics of coupled oscillators [1]. In the past decades, the collective phenomena in coupled oscillators have been received more and more attention because they provide us useful information on mechanism of various nonlinear behaviors in complex networks [2, 3]. Nowadays, their potential applications in engineering field have been demonstrated in many fields: Fukunaga *et al.* proposed a new control system for reducing peak power in energy storages on the basis of collective behavior in oscillators coupled by delayed power price [4]; Okuda *et al.* used synchronization of pulse-coupled oscillators to synchronize wireless sensor networks [5]; Zhou and Low employed collective behavior of coupled oscillators for locomotion control of an underwater vehicle [6].

In recent years, formation control, which manages behavior of multiple robots such that they form a specific pattern, has been one of the hot topics in the field of robotics and control theory. Hara *et al.* showed that a simple control law based on dynamics of coupled oscillators achieves mobile robot circular formations [7]. Tsukiji *et al.* experimentally demonstrated that the control law works well for two-wheeled mobile robots [8]. Very recently, Nakamura *et al.* [9] provided a simple approach for analyzing stability of mobile robot circular formations controlled by the simple law [7,8]. This approach gave us a simple design procedure of control parameters stabilizing a desired formation.

Indeed, from a practical viewpoint, we should consider a natural situation where some of robots stop due to trouble. Even for such situation, it is expected that the remaining robots make up for the troubled robots and keep to form the circular pattern. Although the previous studies [7–9] dealt with two-wheeled mobile robot circular formations for a given number of robots, it remains an unsettled question how robots cope with such situation. The purpose of the present paper is to investigate behavior of two-wheeled mobile robots in a situation where some robots are removed from and added to circular formations during their work. It is shown that the circular formations are robust against small remove/add disturbances and large *balanced* remove/add disturbances. However, we notice that large *unbalanced* remove/add disturbances may destroy the circular formations, and induce other pattern formations.

## 2. Two-wheeled mobile robots [9]

As illustrated in Fig. 1, dynamics of two-wheeled mobile robot $i \in \{1, \ldots, N\}$ can be expressed as

$$\begin{bmatrix} \dot{r}_i \\ r_i \kappa_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}. \quad (1)$$

The robot $i$ moves with the angular velocity of robot $i$ around the origin, $\kappa_i \in \mathbb{R}$, and is located distance $r_i > 0$ from the origin. The robot $i$ moves with the angle $\theta_i \in \mathbb{R}$ to the radial direction. $\psi_i \in \mathbb{R}$ defines the angle between the radial direction of robot $i$ and that of $i + 1$. Here the robot $i$ is forced by the control signals, the heading-direction component of velocity $v_i \in \mathbb{R}$ and the angular velocity around its center $\omega_i \in \mathbb{R}$. We obtained a desired control signals by adjusting the rotational velocity of its two wheels.

Let us move on to reference dynamics of one-way coupled oscillators,

$$\dot{r}_i = f(r_i, \hat{r}_i) := a r_i \left(1 - \frac{r_i^2}{\hat{r}_i^2}\right), \quad (2a)$$

$$\kappa_i = g(\psi_i) := \Omega + \varepsilon \sin\psi_i. \quad (2b)$$

Equations (2a) and (2b) describe dynamics of $i$-th distance $r_i$ and dynamics of one-way coupled phase oscillators. As

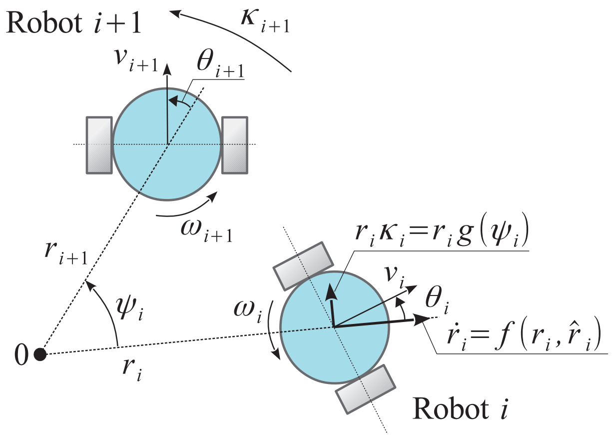


Figure 1: Sketch of two-wheeled mobile robots.

the angle between the radial directions for robots $N$ and 1 is denoted by $\psi_N$, one-way coupled phase oscillators (2b) has the periodic boundary. The coupled oscillators (2) have the following parameters: $\hat{r}_i > 0$, $a \in \mathbb{R}$, $\Omega \geq 0$, and $\varepsilon \in \mathbb{R}$.

Our previous study [9] proposed control law for $v_i$ and $\omega_i$, which leads the two-wheeled mobile robots (1) to behaver of the reference dynamics,

$$v_i = \overline{v}_i := k_v \{f(r_i, \hat{r}_i) \cos\theta_i + r_i g(\psi_i) \sin\theta_i\}, \quad (3a)$$
$$\omega_i = \overline{\omega}_i := k_\omega \{r_i g(\psi_i) \cos\theta_i - f(r_i, \hat{r}_i) \sin\theta_i\}. \quad (3b)$$

The feedback gains, $k_v \in \mathbb{R}$ and $k_\omega \in \mathbb{R}$, can be set as one wants.

The control law (3) suggests that the robot $i$ always measures the three real-time data, $r_i$, $\theta_i$, and $\psi_i$. Since every robot is supposed to be able to measure the distance to a target and the angle between targets in real time, they can autonomously move in accordance with control law (3).

The state space model of $N$ robots (1) controlled by law (3) with $\dot{\psi}_i := \kappa_{i+1} - \kappa_i$ is given by

$$\begin{cases} \dot{r}_i = \overline{v}_i \cos\theta_i \\ \dot{\theta}_i = \overline{\omega}_i \end{cases}, \ (i = 1, \ldots, N) \quad (4a)$$
$$\dot{\psi}_i = \frac{1}{r_{i+1}} \overline{v}_{i+1} \sin\theta_{i+1} - \frac{1}{r_i} \overline{v}_i \sin\theta_i, \ (i = 1, \ldots, N-1). \quad (4b)$$

This model has equilibrium points,

$$r_i = \hat{r}_i, \ \theta_i = \frac{\pi}{2}, \ \psi_i = 2\pi \frac{l}{N}, \quad (5)$$

where $l \in \{0, \ldots, N\}$ denotes a type of formations. The equilibrium point with formation $l$ indicates that robots $i \in \{1, \ldots, N\}$ move at equally spaced intervals on circles with radius of $\hat{r}_i$. Our previous study provided a procedure for design of control parameters.

Table 1: Design of control law (3).

| $\frac{l}{N} \in \left[0, \frac{1}{4}\right)$ | $\frac{l}{N} \in \left(\frac{1}{4}, \frac{1}{2}\right]$ | $\frac{l}{N} \in \left[\frac{1}{2}, \frac{3}{4}\right)$ | $\frac{l}{N} \in \left(\frac{3}{4}, 1\right]$ |
|---|---|---|---|
| $k_\omega > 0$ | | | |
| $\varepsilon > 0$ | | $\varepsilon < 0$ | |
| $k_v > 0$ | $k_v < 0$ | $k_v > 0$ | $k_v < 0$ |
| $a > 0$ | $a < 0$ | $a > 0$ | $a < 0$ |

**Fact 1** ( [9] ). *Formation number (l), total number of robots (N), and angular velocity ($\Omega \geq 0$), are assumed to be given. If the parameters in control law (3) (i.e., $\varepsilon$, a, $k_v$, $k_\omega$) are designed in accordance with Table 1, then formation l (i.e., equilibrium point (5)with formation l) is stable*

It should be noted that, for the designed parameters, the other formations might be stabilized because Table 1 is derived on the basis of sufficient condition for equilibrium point (5) with formation $l$ to be stable.

## 3. Removing/Adding of robots

This section investigates behavior of circular formation $l = 1$ when some robots are removed or added. From Fact 1, we can obtain the following result.

**Corollary 1.** *If the parameters in control law (3) are set to*

$$k_\omega > 0, \ \varepsilon > 0, \ k_v > 0, \ a > 0, \quad (6)$$

*then formation $l = 1$ (i.e., equilibrium point (5) with formation $l = 1$) with counterclockwise direction is stable for any $N \geq 5$.*

*Proof.* It is clear from Table 1 that formation $l = 1$ on $N$ robots with designed parameters (6) is stable for

$$\frac{1}{N} \in \left[0, \frac{1}{4}\right) \leftrightarrow 4 < N. \quad (7)$$

This fact indicates that if the number of robots is equal to or greater than five, formation $l = 1$ is stable independent of the number. In addition, the angular velocity at formation $l = 1$ with the designed parameters (6) is given by

$$\kappa_i = \frac{\overline{v}_i}{\hat{r}_i} \sin\frac{\pi}{2} = k_v \left\{\Omega + \varepsilon \sin\left(2\pi \frac{1}{N}\right)\right\} > 0, \quad (8)$$

for all $i \in \{1, \ldots, N\}$. The positive $\kappa_i$ indicates that robot $i$ runs with counterclockwise direction. □

This corollary implies that formation $l = 1$ remains stable even if the number of robots changes with time under condition $N \geq 5$. In other words, the robots can remain to form $l = 1$ if some robots are removed or added under this condition.

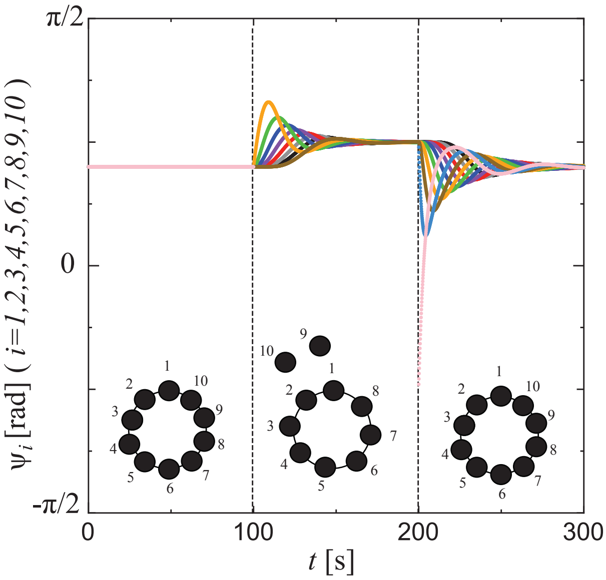


Figure 2: Time series data of angles $\psi_i$ ($i = 1, \ldots, 10$) in the case that two robots (i.e., robots 9 and 10) are removed at $t = 100$ and are added at $t = 200$.

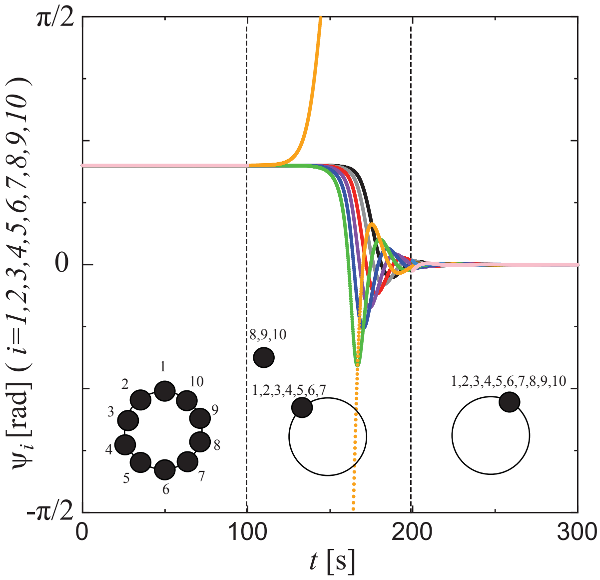


Figure 3: Time series data of angles $\psi_i$ ($i = 1, \ldots, 10$) in the case that three robots (i.e., robots 8, 9, and 10) are removed at $t = 100$ and are added at $t = 200$.

## 4. Numerical examples

This section will provide some numerical examples to verify the analytical results. In our numerical examples, small uniformly distributed random signals with amplitude $[-1.0 \times 10^{-4}, 1.0 \times 10^{-4}]$ are added to the right-hand side of control signals (3a) (3b) in order to confirm the local stability of formations.

We consider the following situation: $N = 10$ robots runs with formation $l = 1$; then, two of them are removed (i.e., $N : 10 \to 8$); finally, two robots are added (i.e., $N : 8 \to 10$). Corollary 1 guarantees that formation $l = 1$ remains stable for such removing/adding of robots if the parameters in control law (3) are set in accordance with Eq. (6). Figure 2 shows time series data of angles $\psi_i$ ($i = 1, \ldots, 10$). It can be seen that the robots 9 and 10 are removed at $t = 100$, and then the eight remaining robots keep formation $l = 1$ with $\psi_i = \pi/4$ ($i = 1, \ldots, 8$) through transient behavior. The two robots are added at $t = 200$, and then the ten robots keep formation $l = 1$ with $\psi_i = \pi/5$ ($i = 1, \ldots, 10$) through transient behavior.

Corollary 1 states that, for $N = 10$, formation $l = 1$ remains stable if we remove and add up to five robots. Now we remove and add *three* robots. The time series data of angles $\psi_i$ ($i = 1, \ldots, 10$) are shown in Fig. 4. The robots 8, 9, and 10 are removed at $t = 100$. However, the seven remaining robots change their formation from $l : 1 \to 0$ through transient behavior. Remark that Fact 1 suggests the coexistence of formations $l = 0$ and $l = 1$. Further, the three robots are added at $t = 200$, but the ten robots cannot turn back to $l = 1$.

In order to avoid destroying formation $l = 1$, we choose the three robots (3, 6, and 9) *uniformly* from ten robots. The chosen robots are removed at $t = 100$ as shown in Fig. 4. Although the removing of robots 8, 9, and 10 destroys formation $l = 1$ (see Fig. 3), the uniform choice can keep formation $l = 1$. The chosen robots are added at $t = 200$, then the ten robots keep formation $l = 1$.

It should be summarized, from what have been seen above, that stable formation $l = 1$ exists if the parameters in control law (3) are set in accordance with Eq. (6). In addition, this formation can remain if uniformly chosen robots are removed and added, but can be destroyed by non-uniformly chosen robots.

## 5. Conclusion

This paper dealt with the practical situation where some of two-wheeled mobile robots forming a circular pattern stop due to trouble and the remaining robots make up for the troubled robots. The main result of this paper is that our control law [9] based on coupled oscillators work well even in such situation without change of the law. In addition, we have shown that the circular formations are robust against small removing/adding and large balanced removing/adding of robots. Our robots form a circle, since they are based on dynamics of coupled oscillators which have a circular limit cycle. Thus, we guess that if these oscillators are replaced by other type of oscillators, our robots might form other shapes. We should confirm our guess for future work.

propulsion," IEEE/ASME Transactions on Mechatronics, vol.17, pp.25–35, 2012.

[7] N. Hara, Y. Takarada, and K. Konishi, "Coupled nonlinear oscillators with application to mobile robot formation control," Proc. of SICE Annual Conference, pp.2174–2177, 2013.

[8] M. Tsukiji, N. Hara, and K. Konishi, "Experimental evaluation of formation control of mobile robot group by coupled oscillators," Proc. of SICE Annual Conference, pp.301–306, 2014.

[9] T. Nakamura, M. Tsukiji, N. Hara, and K. Konishi, "Stability analysis of mobile robot formations based on synchronization of coupled oscillators," Proc. of IFAC Workshop on Distributed Estimation and Control in Networked Systems, 2016 (submitted).

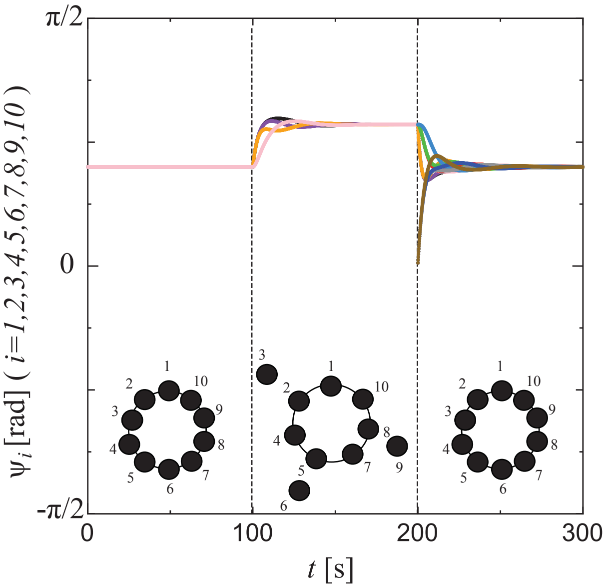


Figure 4: Time series data of angles $\psi_i$ ($i = 1, \ldots, 10$) in the case that three robots (i.e., robots 3, 6, and 9) are removed at $t = 100$ and are added at $t = 200$.

**Acknowledgments**

This research was partially supported by JSPS KAKENHI (26289131).

## References

[1] A. Pikovsky, M. Rosenblum, and J. Kurths, Synchronization, Cambridge University Press, 2001.

[2] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," Physics Reports, vol.424, pp.175–308, 2006.

[3] A. Arenas, A. Diaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," Physics Reports, vol.469, pp.93–153, 2008.

[4] T. Fukunaga, T. Imasaka, A. Ito, Y. Sugitani, K. Konishi, and N. Hara, "Dynamical behavior and peak power reduction in a pair of energy storage oscillators coupled by delayed power price," Phys. Rev. E, vol.93, p.022220, 2016.

[5] T. Okuda, K. Konishi, and N. Hara, "Experimental verification of synchronization in pulse-coupled oscillators with a refractory period and frequency distribution," Chaos, vol.21, p.023105, 2011.

[6] C. Zhou and K.H. Low, "Design and locomotion control of a biomimetic underwater vehicle with fin