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# Mobility of Discrete Breather in Truncated Pairwise Interaction Symmetric Lattices

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**Abstract**—Traveling discrete breathers (DBs) in an approximate version of the pairwise interaction symmetric lattice (PISL) are investigated. The approximated PISL is constructed by truncating the pairwise interactions up to a small number of nearest neighbor particles. Even in the truncated PISL, traveling DBs have good mobility. An effective periodic barrier for the traveling DBs becomes smaller than that of the original Fermi-Pasta-Ulam $\beta$ lattice.

## 1. Introduction

Mobility of discrete breather (DB) is one of the most important problems in nonlinear lattice dynamics since traveling DBs can transport energy in discrete systems such as crystals, metamaterials and MEMS. The traveling DBs are highly affected by the discreteness of the system for example, the discreteness causes reduction and variation of velocity of DB. We have proposed a new lattice model called a pairwise interaction symmetric lattice (PISL) that counteracts the effects of discreteness. The PISL has all-to-all connection between particles. However, it is difficult to realize this all-to-all connection in practical physical systems. In this study, we propose an approximated PISL that may reduce this difficulty.

## 2. Model

We consider an nonlinear lattice model with $N$ particles. Hamiltonian of the system is

$$H = \sum_{n=1}^{N} \frac{p_n^2}{2} + \sum_{n=1}^{N} \frac{1}{2}(q_{n+1} - q_n)^2 + \sum_{n=1}^{N} \frac{\beta}{4}(q_{n+1} - q_n)^4 + \sum_{n=1}^{N}\sum_{d=2}^{M} \frac{\beta\alpha_d}{4}(q_{n+d} - q_n)^4, \quad (1)$$

where $p_n$ and $q_n$ are the momentum and displacement of $n$th particle, respectively, $\beta$ is the coefficient for nonlinear interaction between nearest neighbor particles, $\alpha_d$ is the coefficient for nonlinear interaction between $d$th nearest neighbor particles which is calculated according to the procedure for constructing the PISL [2]. In Eq.(1), the interactions up to $M$th nearest particles are considered. The system reduces to the Fermi-Pasta-Ulam (FPU) $\beta$ lattice when $\alpha_d = 0$ for all $d \geq 2$. The system becomes the original PISL when $M = N/2$.

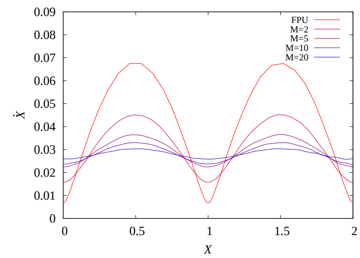


Figure 1: Trajectory of traveling DBs in $(X,\dot{X})$ space.

## 3. Numerical Results

Figure 1 shows the trajectory of the traveling DBs in $(X,\dot{X})$ space with $M = 1, 2, 5, 10$ and 20. The parameters $X$ and $\dot{X}$ are the short time averaging position and velocity of the DB, respectively.

In the case of the original FPU-$\beta$ lattice, a large variation in the velocity of DB is observed. By introducing the long range interactions, the variation of velocity decreases. The variation of velocity indicates that the existence of an effective potential barrier for the traveling DB. Figure 1 shows that the effective potential barrier can be reduced by considering the truncated PISL. This result indicates that the truncated PISL supports the smooth mobility of traveling DB and is useful for constructing the practical physical systems that supports the traveling DBs.

## References

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