2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

Dynamic Behavior of a Bouncing Ball

Shu Karube†, Takuji Kousaka†† and Yuya Kawazu†††

†National Institute of Technology, Oita College, 1666 Maki, Oita, 870-0152, Japan
††Oita University, Faculty of Engineering, 700 Dannoharu, Oita, 870-1192, Japan
†††Mechanical-Environmental System Engineering Major, National Institute of Technology, Oita College, 1666 Maki, Oita, 870-0152, Japan
Email: karube@oita-ct.ac.jp, takuji@oita-u.ac.jp, amc15084@ga.oita-ct.ac.jp

Abstract– Bouncing ball system is often used for describing collision dynamics of various mechanical systems. We produced the experimental bouncing ball equipment using a ping-pong ball, a table tennis paddle with rubber and a shaker for vibrating the paddle. In this study, we investigate frequency responses of the bouncing ball system under constant paddle amplitude condition and constant paddle acceleration amplitude condition experimentally, and compare our experimental results with simulations. Under constant amplitude condition, we found that the maximum height of the ball increases stepwise as varying the paddle frequency by both experiment and simulation. Under constant acceleration amplitude condition, we found the maximum height of the ball decreases exponentially as varying the vibrated plate frequency by both experiment and simulation. Summarizing the above, we observed the nonlinear characteristics that the height of ball increases stepwise in simple bouncing ball system.

1. Introduction

Generally, most mechanical systems have gaps, such as a pair of gears with backlash. In these systems, nonlinear vibrations by collision phenomenon often occur. A bouncing ball system is effective to explain these nonlinear vibrations on collision systems.

The bouncing ball system can be applied to various mechanical systems. For example, M. Paskota analyzed and controlled collision vibration of a molding box during in aluminum production [1]. K.Sakai et al. analyzed dynamics of the tractor on the rigid road using a bouncing ball model [2]. As experimental studies, A.Kini et al studied the impact vibration when a super ball collided with a fixed table with rubber [3].

We produced the bouncing ball equipment using a ball and a paddle for table tennis [4]. Our equipment had the advantage that frictional resistance for the ball was small. However, the vibration direction of the paddle caused a slight error, hence the unbalanced support of the paddle. The paddle is fixed to a shaker at its handle and vibrated. This way distorted the vertical direction of the surface of the racket collided with the ball.

In this study, we improve our previous bouncing ball equipment. The surface of paddle is fixed at the center of shaker, in order to keep precise vertical vibration of the paddle. Next, we carry out the frequency response experiments under both constant amplitude condition and constant acceleration amplitude condition, then we compare these experimental results with our simulation results.

2. Experimental setup

Fig.1 shows the photograph of our experimental setup we produced. Fig.2 shows the schematic illustration of the equipment. The device consists of four parts mainly, i.e. a ping-pong ball, a paddle for table tennis, a ball guide and a shaker. The shaker vibrates the paddle periodically to the vertical direction. The ball is vibrated by the paddle, and jumps on the paddle. The displacement of the ball is measured by a sensor 1, which is a laser displacement sensor LK-G405 produced by Keyence corporation. The ball guide consists of a pair of top and bottom plates and four stainless steel poles fixed between the plates. The ball guide limits the horizontal movement of the ball by the method surrounding with four stainless steel poles whose diameter is 3mm in order to keep the ball at the same position of the paddle. There is a hole to let the ball go through, whose diameter is 40mm, at the center of the top plate and the bottom plate. The four stainless steel poles are set around the hole at intervals of 90degrees. The distance between the top and the bottom plate is 300mm, they are supported with four support pillars whose length are 767mm. The displacement of the paddle is measured by a sensor 2, which is a laser displacement sensor LK-G155 produced by Keyence corporation. The positive direction of the sensors is upward. When the ball stops, the distance of the ball and the sensor 1 is 400mm. The mass of the ball is each 2.8g. The diameter of the ball is 39.8mm.

Fig.3 shows the paddle vibration control system. The system is feedback controlled by vibration controller K2 sprint, which is produced by IMV corporation. The vibration controller is able to control the acceleration, velocity, and amplitude of the paddle by using the signal from the acceleration pickup fixed on the paddle.

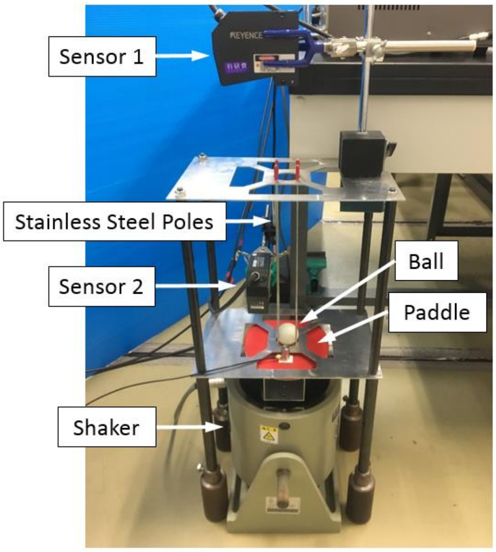


Fig.1 Photograph of experimental setup

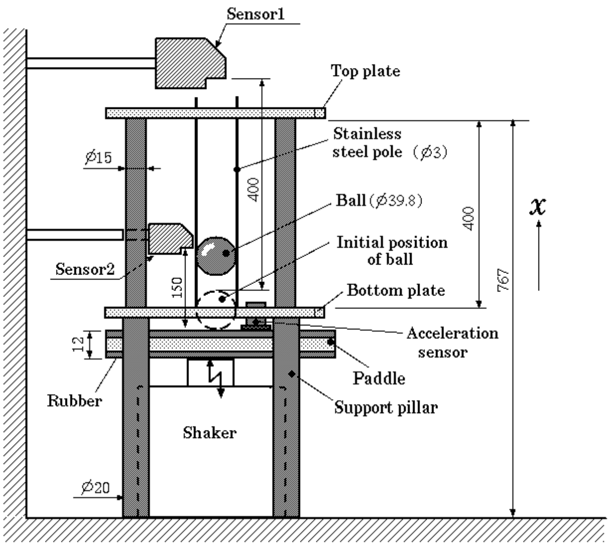


Fig.2 Illustration of experimental setup

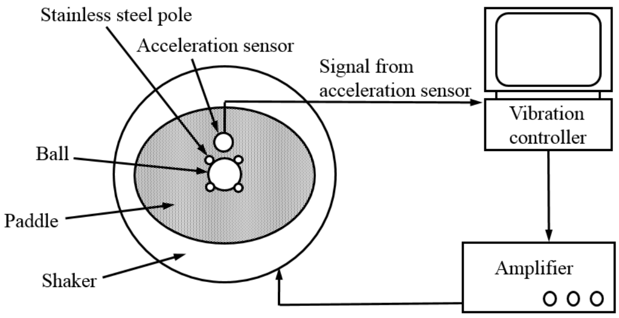


Fig.3 Paddle vibration control system

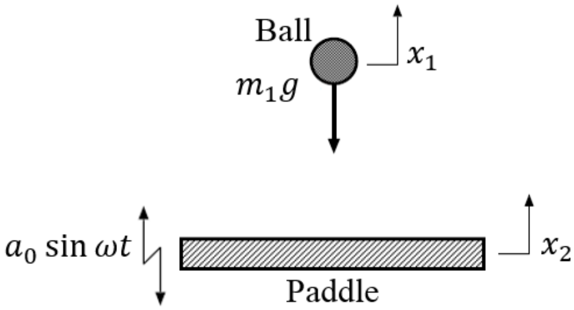


Fig.4 Bouncing ball model

3. Simulation

3.1. Bouncing ball model

Fig.4 shows physical model of the bouncing ball system that we use in this study. m_1 denotes the mass of the ball. x_1 and x_2 denote each the displacement of the ball and the paddle. The motion equations are defined by Eq.1.

$$\begin{cases} m_1 \ddot{x}_1 = -m_1 g \\ x_2 = a_0 \sin \omega t \end{cases} \tag{1}$$

The parameters in Eq.1 are shown as follows: g is the gravitational acceleration, ω is the angle frequency of the paddle, a_0 is amplitude of the paddle. When $x_1 = x_2$ the ball collides with the paddle. Suppose $m_2 >> m_1$, where m_2 is the mass of the paddle. Therefore, the change of the velocity of the ball $\dot{x}_1$ is given by

$$\dot{x}_{1+} = -e\dot{x}_{1-} + (1+e)\dot{x}_2 \tag{2}$$

where $\dot{x}_{1-}$ and $\dot{x}_{1+}$ is the velocity of the ball before and after impact.

The parameters used in the simulation are as follows: e =0.68, m_1 =2.8 g, a_0 =0.69 mm, g =9.8m/s^2.

3.2. Simulation condition

In this paper, we use Runge-Kutta-Gill 4th order Method with time step $(128f)^{-1}$, transient number 100,000 and data number 100,000.

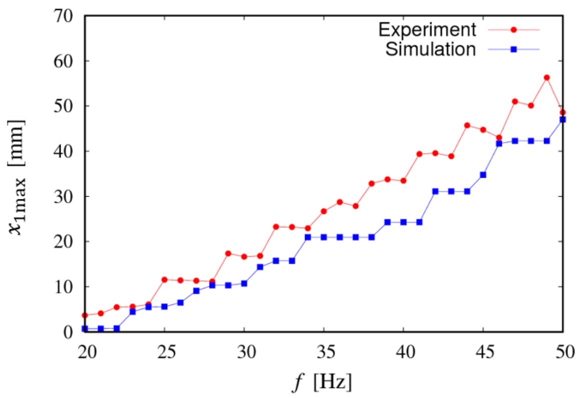
The velocity and acceleration of the paddle are defined by Eq.3 and 4.

$$\dot{x}_2 = a_0 \omega \cos \omega t \tag{3}$$

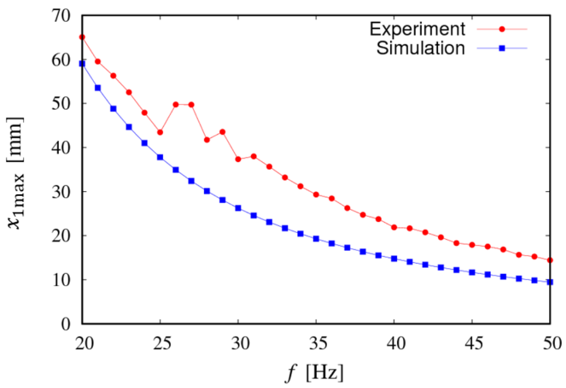
$$\ddot{x}_2 = -a_0 \omega^2 \sin \omega t \tag{4}$$

By defined the acceleration amplitude $A_0 \equiv |\ddot{x}_2|_{\max}$ as Eq.4 can be rearranged to Eq.5.

$$A_0 = a_0 \omega^2 \tag{5}$$



(a) Constant amplitude



(b) Constant acceleration amplitude

Fig.5 Experimental and simulation results

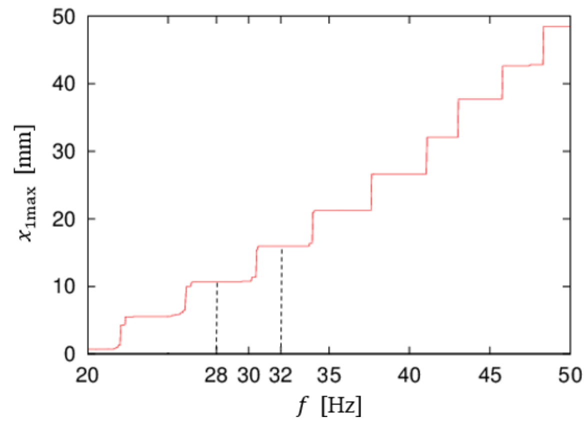
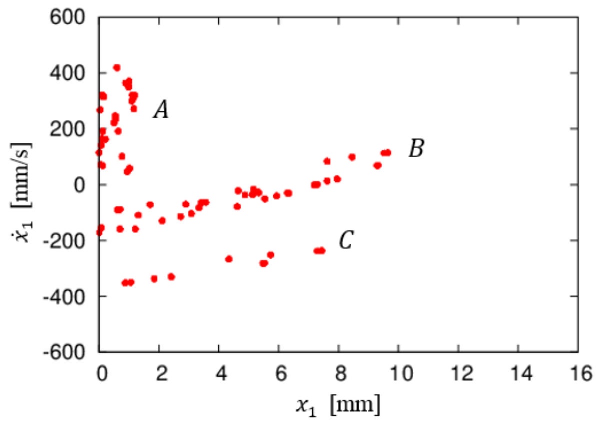
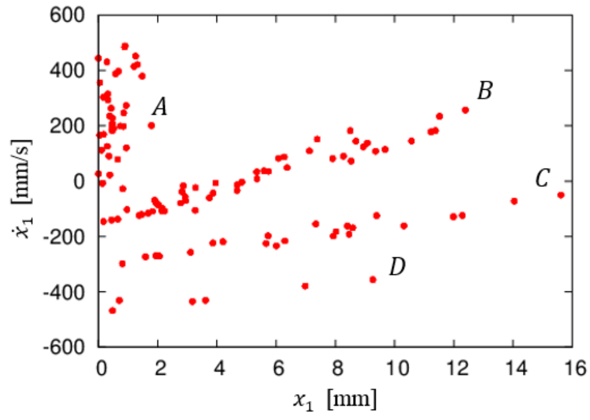


Fig.6 Bifurcation diagram under constant amplitude



(a) f =28Hz



(b) f =32Hz

Fig.7 Poincare map

4. Experimental and Simulation result

In this section, we investigate frequency responses of our bouncing ball system. Increasing frequency of the paddle f from 20Hz to 50Hz by 1Hz, we measure the maximum value of the displacement of the ball $x_{1\max}$ in each frequency. We carry out two types of the frequency response experiment. One is the experiment while keeping the paddle amplitude a_0 uniformity, which is called as constant amplitude condition. The other is this while keeping the acceleration amplitude A_0 uniformity, which is called as constant acceleration. We determine a_0 =0.69mm under constant amplitude condition, A_0 =35m/s^2 under constant acceleration.

Fig.5(a) shows experimental and simulation results under constant amplitude condition. The vertical axis indicates the maximum displacement of the ball $x_{1\max}$, and the horizontal axis indicates the frequency of the paddle f . Both experimental and simulation result, the graphs show stepwise increase of $x_{1\max}$ as varying the paddle frequency f .

Fig.5(b) shows experimental and simulation result under acceleration displacement constant. Both experimental and simulation result, the graphs show exponential decrease of $x_{1\max}$ as varying the paddle frequency f . The trend of increasing $x_{1\max}$ under constant amplitude condition in Fig.5(a) can be explained by Eq.6.

$$\left|\dot{x}_2\right|_{\max}\Big|_{a_0=\text{const.}} = a_0\omega \propto f \qquad (6)$$

Eq.6 indicates that maximum velocity of the paddle $\left|\dot{x}_2\right|_{\max}$ is proportional to f . Similarly, the trend of decreasing $x_{1\max}$ under constant acceleration amplitude condition in Fig.5(b) can be explained by Eq.7

$$\left|\dot{x}_2\right|_{\max}\Big|_{A_0=\text{const.}} = a_0\omega = A_0/\omega \propto 1/f \qquad (7)$$

Eq.7 indicates that $\left|\dot{x}_2\right|_{\max}$ is inversely proportional to f. Bouncing height of the ball depends on the velocity of the paddle when the ball colliding the paddle. Hence, under constant amplitude condition, according to Eq.6, $x_{1\max}$ monotonic increases, and constant acceleration amplitude condition, according to Eq.7, $x_{1\max}$ monotonic decreases. However the stepwise increasing of $x_{1\max}$ under constant amplitude condition in Fig.5(a) cannot be explained by Eq.6. We suppose that the trend of stepwise increasing is caused by nonlinear characteristics in our simple bouncing ball system.

5. Poincare map in constant amplitude

In this section, we detailed simulation for previous results in Fig.5(a).

Fig.6 is the bifurcation diagram under constant amplitude condition, in the case when the paddle frequency f is increased from 20Hz to 50Hz by 0.05Hz. In Fig.6, the trend of stepwise increase of $x_{1\max}$ is clearer than the graph of the simulation result in Fig.5(a). We suppose that the bifurcations are occurred at the intermittent points like stepwise. Next, we investigate the dynamics of our system when the frequency is before and after bifurcation, f =28Hz and 32Hz.

Fig.7 shows Poincare maps under constant amplitude condition shown by Fig.6. Fig.7(a) and (b) show the Poincare map at 28Hz and 32Hz, i.e. before and after bifurcation. The Poincare map in Fig.7(a) shows three regions of Poincare plots, A, B and C. In contrast, Fig.7(b) shows four regions, A, B, C, and D. The region D is occurred by the bifurcation. Therefore, we suppose that there are the bifurcations which keep the dynamics and add new region.

6. Summary

In this study, we investigated the dynamics of a bouncing ball system, which was consisted of a ball and a paddle for table tennis, experimentally and numerically. The paddle was vibrated by a shaker.

First, we improved our previous bouncing ball equipment and was capable of more precision experiments.

Second, we carried out frequency response experiments under two types of condition. One was the constant amplitude condition, which was keeping the amplitude of the paddle as varying the frequency of the paddle by vibration control system. The other condition was the constant acceleration amplitude condition, which was keeping the acceleration amplitude of the paddle. Under the constant amplitude condition, our experiment showed the maximum height of ball increased stepwise. Same trend was shown by our numerical simulation. Calculated Poincare maps showed occurrence of new region of Poincare plots surrounding stepwise increasing points.

As a result, we supposed that there was the bifurcations which kept the conventional structure of the dynamics and added new structure in the simple bouncing ball system.

7. Acknowledgement

This work was supported by JSPS KAKENHI Grant Numbers JP16H02876.

References

[1] M.Paskota, "On Modelling and the Control for Vibroformers in Alumimium Production" *Chaos, Solitons & Fractals*, pp.323-335, 1998.

[2] K. Sakai, "NONLINEAR VIBRATIONS IN AN AGRICULTURAL IMPLEMENT SYSTEM" *International Journal of Bifurcation and Chaos*, Vol.4, No. 2, pp.465-470, 1994.

[3] A.Kini, Thomas L. Vincent and Brad Paden, "The Bouncing Ball Apparatus as an Experimental Tool" *J. Dynamics Systems, Measurement and Control*, Vol. 128, pp.330-340, 2006.

[4] K.Yuki and S. Karube, "Bouncing Dynamics of a Ping-Pong Ball on a Periodic Oscillating Elastic Plate" *International Symposium on Technology for Sustainability 2014 (ISTS2014), Taipei, Taiwan, D5 Mechanical,* No.281, 2014 (CD-ROM).