# Application of the Complex Source Point Method for Analyzing the Diffraction of an Electromagnetic Gaussian Beam by a Cylinder Using UTD Concepts

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## 1. Introduction

The complex source point technique (CSP) is combined with the ray based Uniform Geometrical Theory of Diffraction (UTD) to analyze the scattering by a cylinder illuminated by a sequence of electromagnetic Gaussian beams (GBs) which can describe a relatively general antenna illumination. GBs and Gaussian pulses have been successfully used for treating a variety of wave propagation problems. GBs have been utilized in optics to simulate laser outputs. GB concept has been used in a different sense as wave packets in Quantum mechanics as well as in EM pulse propagation studies (Gaussian Wave Packet). GBs can serve as EM field basis functions for aperture fields or currents as well as wave propagators. Furthermore, it can be used in representing field at ray caustics in EM and in geophysical wave application. The GBs have also been used for analysis of large reflector systems, large apertures and large antenna-radome structures. More recently, the GBs technique is used to accelerate the computation time of the Method of Moments. The works by J. B. Keller, W. Streifer and G. A. Deschamps in [1-2] independently showed that a CSP generates a GB field. Also L. B. Felsen has written extensively about the propagation and diffraction of GBs via the CSP. One of his earliest but most comprehensive papers on CSP model for GB is in [3]. Some others previous works on CSP-GB reflection and diffraction were published in [4-6]. The work on the UTD-GB diffraction by an edge has been done in [7] to provide a Physical Optic (PO) based edge diffraction of GB. The PO-GB is more general in that it can handle astigmatic GB illumination of an edge. The CSP method is more accurate since it yields a UTD-GB; however it is limited to rotationally symmetric GBs. However the PO-GB can be augmented by PTD correction if desired. The CSP-GB via UTD method can be used to analyze the diffraction by a wedge illuminated in [8].

The UTD solution in [9] has been widely used to analyze the diffraction of electromagnetic waves by a smooth convex surface. The CSP-UTD is also applied to analyze the scattered fields from the cylindrical structures for the multi-path propagation and the path loss prediction in various aspects. The work in [10] showed the possibility of CSP-UTD for analyzing such a problem in 2D case of a cylinder.

This work presents the extension of the well established UTD ray analysis of the diffraction by a cylinder illuminated by a ray optical spherical wave generated by a point current (3D) to treat the diffraction of an incident GB by the same structure via the CSP technique. In particular, in the CSP method, a point source in real space becomes a CSP by replacing its real position coordinates by appropriate complex values; in so doing, this CSP generates a rotationally symmetric GB field in its paraxial region. The diffraction of the GB incident on the conducting cylinder is then found via the CSP technique by analytically continuing the corresponding UTD ray solution to include the complex point source coordinates. Since the point source is in complex space, it is noted that the points of reflection and diffraction in general turn out to be in complex space as well. The CSP method thus allows one to directly obtain a closed form analytically continued UTD solution for the 3D electromagnetic diffraction by cylinders illuminated by a rotationally symmetric GB. As mentioned earlier, a relatively general antenna illumination can be easily represented by a sequence of GBs. Therefore, by superposing the diffraction fields of each GB in the sequence, it is possible to find the total scattered field in an efficient manner from a large structure containing curved surfaces when the latter is illuminated by a complicated antenna pattern. This work will be very useful for the path loss prediction in a Thai commercial orchard [11].

## 2. Gaussian Beam Field

In optics, a GB, named in honor of Johann Carl Friendrich Gauss (1777-1855), is a beam of light whose electric field intensity distribution is a Gaussian function [12]. The work in [3] showed that the field is described in terms of local plane waves with complex phase moving along complex ray trajectories and the observable field, given by the real-space intersections of the complex rays, defines a GB. GBs are paraxial wave solutions of Maxwell's equations. GBs are valid in near and far zone of their waist. There are four types of the GB field, i.e. rotationally symmetric GB, elliptical GB, astigmatic GB and general astigmatic GB. Here in this work the CSP technique is applied to obtain the GB field. The CSP technique can provide only the rotationally GB field in the paraxial region [3]. Thus the 3D rotationally symmetric GB is obtained by a CSP method. It is of interest in this work to investigate the scattering of a rotationally GB from a conducting cylinder.

## 3. Numerical results

The configuration of the PEC cylindrical problem with Gaussian beam illumination is shown in Fig.1. Here the radius of the cylinder is *a*. The *P* denotes the distance from the origin to an observation point and the *S* represents the source distance from the origin to the source location. The distance parameter of observation and source points are related to the angle parameters $\phi$ and $\phi'$, and the distance parameters $\rho$ and $\rho'$, respectively. Thus, the source location can be written in the form of $S(x',y',z') = \rho'\cos(\phi')\hat{x} + \rho'\sin(\phi')\hat{y} + z'\hat{z}$ and the observation point can be written by $P(x,y,z) = \rho\cos(\phi)\hat{x} + \rho\sin(\phi)\hat{y} + z\hat{z}$. The beam waist of the GB $w_0$ is used to control the beam width of the GB. The beam direction of the GB can be defined by $\phi_b$ and $\theta_b$. To verify the accuracy of the proposed idea in the work, the comparisons between this work and the exact solution are performed. The exact solution can be obtained by applying the analytic continuation of the Eigen solution [13] to deal with the GB excitation scattered by a PEC cylinder which is called Exact solution-CSP in the plots of Figs. 2 and 3. The observation is along the Keller's cone of diffraction as shown in the Fig. 1. The result shows good agreement between the CSP-UTD and the Exact solution-CSP. It is verified that the CSP-UTD solution is still valid by employing the analytic continuation of the source to be complex location for the case of the circular cylinder.

## 4. Conclusion

The diffraction analysis of the 3D electromagnetic GBs by a PEC cylinder based on the UTD and the CSP techniques is performed in this work. The UTD from [10] can deal with the GB excitation by simply applying the analytic continuation of the source to be complex location. When the source is in the complex location, it becomes a rotationally GB within the paraxial region. This allows ones to come up with the idea of treating the diffraction analysis by the GB excitation from the known UTD solution. This approach is called CSP-UTD here in this work. The agreement between the proposed method (CSP-UTD) and the exact Eigen solution is shown in the numerical results. A sequence of electromagnetic GBs can describe a relatively general antenna illumination. This will allow ones to employ the sequence of GBs to model the source or antenna located in the complicated environment for predicting the path loss in the commercial orchard.

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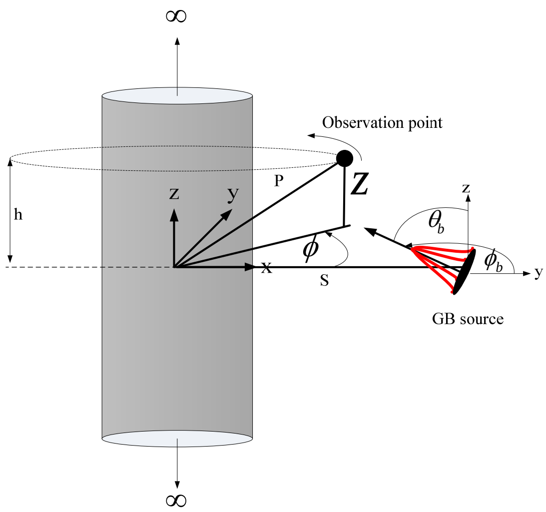


Figure 1: Circular cylinder with a GB

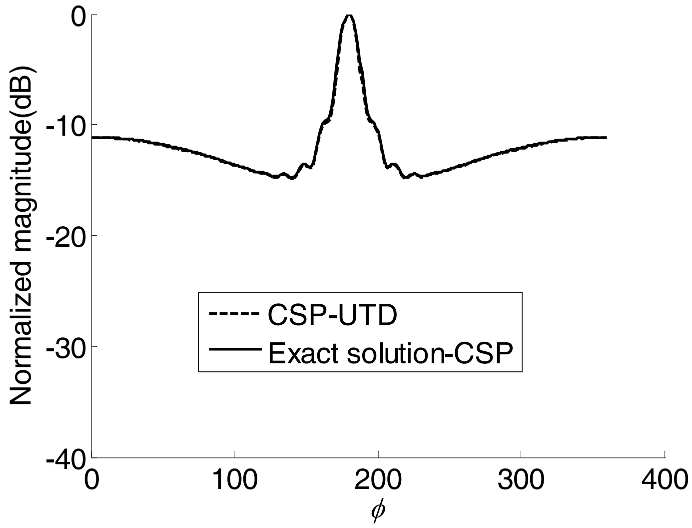


Fig. 2 Scattered field $TE^z$ case: source location $\rho' = 20\lambda$, $\phi' = 0^\circ$, $z' = 0\lambda$ and observation point $\rho = 50\lambda$, $z = 0\lambda$. GB parameter $w_0 = 1\lambda$, $\phi_b = 180^0$ and $\theta_b = 90^0$.

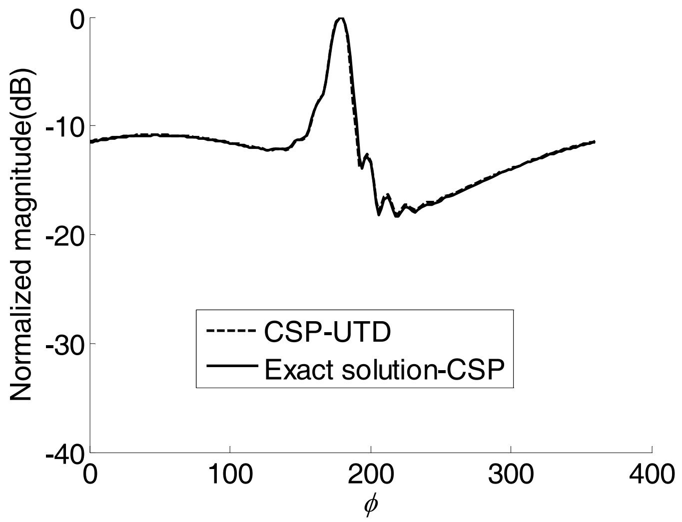


Fig. 3 Scattered field $TE^z$ case: source location $\rho' = 20\lambda$, $\phi' = 0^\circ$, $z' = 0\lambda$ and observation point $\rho = 50\lambda$, $z = 0\lambda$. GB parameter $w_0 = 1\lambda$, $\phi_b = 170^0$ and $\theta_b = 90^0$.