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# Koopman Operator Approach to Vital Sign Detection

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**Abstract**—Koopman operator approach has attracted much attention in the field of nonlinear time series analysis, because this approach enables us to define nonlinear dynamic modes based on dynamical systems theory only from mixed-mode time series data. Here, we propose a new method for detecting vital signs by the Koopman operator approach, and demonstrate the validity of our method.

## 1. Introduction

The detection of vital signs, such as heart beats, respiration and body motion, has become a hot topic, because of the recent progress in sensor technology. In these days, we can measure vital signs without touching patients by a novel sensor technology, Doppler sensor [1]. The Doppler sensor utilizes the Doppler effect of microwave signals to measure the motion of a patient's body including respiratory and cardiac oscillations.

When vital signs are measured by the Doppler sensor, the measured signal is a mixture of several vital signs. Thus, it is necessary to decompose the mixed signal into several kinds of vital signs such as heart beats, respiration and body motion. In clinical applications, signal processing techniques, such as anomaly detection, are used to each vital sign.

In this study, with the aim of establishing a unified framework of the signal decomposition and anomaly detection of vital sign signals, we employ a theoretical approach based on the Koopman operator [2]. By the dynamic mode decomposition (DMD) [3, 4] based on the Koopman operator, we can decompose a mixed signal of vital signs and extract essential dynamical variables, e.g. an oscillation phase of each vital sign. From the Koopman operator reconstructed from measured signals, we can obtain the conditional probability that describes the dynamics of each dynamical variable, which can be used for developing methods of anomaly detection.

## 2. Signal decomposition and anomaly detection

We introduce a linear operator called the Koopman operator. We assume the existence of a latent stationary Markov process underlying vital sign signals:

$$\boldsymbol{x}_{t+1} \sim p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t), \tag{1}$$

where $\boldsymbol{x}_t \in \Omega$ is the state of the system at time $t$, and $p(\boldsymbol{x}|\boldsymbol{x}')$ is a conditional probability that describes the dynamics of this system. The Koopman operator $\mathcal{K}$ corresponding to Eq. (1) is defined as a conditional expectation:

$$\mathcal{K}f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')p(\boldsymbol{x}'|\boldsymbol{x})d\boldsymbol{x}', \tag{2}$$

for an arbitrary function of $\boldsymbol{x} \in \Omega$, $f(\boldsymbol{x})$, in an appropriate functional space.

Since the Koopman operator is a linear operator, we can consider the spectral decomposition of this operator, and the eigenvalues and eigenfunctions of the Koopman operator can be computed by the DMD. Let $\theta_t = \Theta(\boldsymbol{x}_t)$ be an oscillation phase as defined in Ref. [5]. The oscillation phase $\theta_t$ can be estimated from a mixed signal of vital signs by the DMD, because $e^{i\Theta(\boldsymbol{x}_t)}$ is an eigenfunction of the Koopman operator. In addition, we can reconstruct a conditional probability $q(\theta_{t+\tau}|\theta_t)$ from time series data. From this conditional probability, it is expected that anomalies in the data can be detected by thresholding, because such rare events have much lower transition probability than the normal dynamics. In the presentation, we will demonstrate the validity of our method.

## References

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