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Investigation of Attracting Force to Synchronization States on Coupled Oscillator System by Using Electric Power

Ryoji Fukumasa† , Masayuki Yamauchi† and Yoshifumi Nishio‡

†Hiroshima Institute of Technology
2-1-1, Miyake, Saeki-ku, Hiroshima, 731–5193, Japan
Phone/FAX:+81-82-921-4248
E-mail: ma16009@cc.it-hiroshima.ac.jp

‡Tokushima University
2-1, Minamijyosanjima-cho, Tokushima, 770–8506, Japan
E-mail: nishio@ee.tokushima-u.ac.jp

Abstract

In this study, the attracting forces to a steady states are theoretically analyzed by using reactive powers in coupled oscillator system, and are obtained by using a simulation. The attracting forces are investigated in changing a coupling parameter by using our theoretical method.

1. Introduction

We can observe synchronization phenomena in our body. There are a synchronization of among pacemaker cells in our heart, and the pulse synchronization phenomenon can be observed in other nervus system, too. Therefore, we can say that the synchronization phenomenon is one of most important phenomena in this world. The synchronization phenomena can be observed on electric circuits. Many kinds of synchronization phenomena and characteristics of the phenomena were reported by many researchers[2]. The synchronization phenomena have been analyzed by many methods. The averaging method is often used when the synchronization phenomena are theoretically analyzed[2]. The method is hard to be used for transient state.

In this study, an attracting force, which two oscillators are attracted to a synchronization state, are investigated by using reactive powers of the system and simulations. Furthermore, the attracting force is investigated in changing a coupling parameter by using our theoretical method.

2. Circuit Model

Our circuit model is shown in Fig. 1. The van der Pol oscillators are coupled by inductor L_C. An inductor and a capacitor of each van der Pol oscillator are shown as "L" and "C" respectively. Characteristic of nonlinear negative resistor of k-th oscillator is shown as $f(v_k)$ in Eq. (1).

$$f(v_k) = -g_1 v_k + g_3 v_k^3 \quad (k = 1 \text{ or } 2) \tag{1}$$

Circuit equations of this circuit are normalized by using Eq. (2). The normalized equations are shown in Eq. (3).

$$i = \sqrt{\frac{Cg_1}{3Lg_3}}x_k,\ \ v = \sqrt{\frac{g_1}{3g_3}}y_k,\ \ t = \sqrt{LC}\tau,$$
$$\alpha = \frac{L}{L_c},\ \ \varepsilon = g_1\sqrt{\frac{L}{C}},\ \ \delta = \frac{g_1^2}{3g_3}. \tag{2}$$

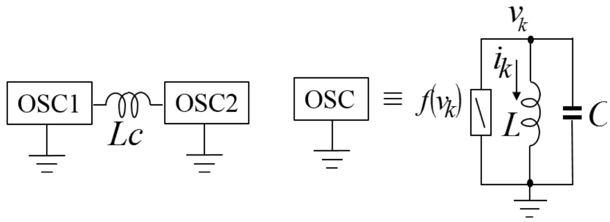

Figure 1: Circuit model.

$$\frac{dx_k}{d\tau} = y_k,$$
$$\frac{dy_k}{d\tau} = -x_k + \alpha(x_a - 2x_k + x_b) + \varepsilon(y_k - \frac{1}{3}y_k^3). \tag{3}$$

(If $k = 1,\ a = N$ and $b = 2$.
If $k = N,\ a = N - 1$ and $b = 1$.
If $2 \le k \le N - 1,\ a = k - 1$ and $b = k + 1$.)

Where α expresses a coupling parameter and ε shows nonlinearity of each oscillator.

2.1. Calculation of reactive power

An instantaneous electric power of each oscillators and an instantaneous electric power of each inductor between adjacent oscillators are calculated by which each oscillation wave shape is assumed as a sinusoidal wave. Each current x_k ($k = 1$ or 2) and ezch voltage y_k are assumed as Eq. (4) Amplitudes of two oscillators are assumed as same value. Angular frequencies of two oscillators are assumed as same value, too.

$$x_k = Xsin(\omega\tau + \theta_k)$$
$$y_k = \omega Xsin(\omega\tau + \theta_k) \tag{4}$$

<Instantaneous electric power of the inductor in each oscillator>

$$P_{Lk} = \frac{\delta}{\varepsilon}x_k y_k \tag{5}$$

<Instantaneous electric power of the capacitor in each oscillator>

$$P_{Ck} = \frac{\delta}{\varepsilon}y_k\frac{dy_k}{d\tau} \tag{6}$$

<Instantaneous electric power of the coupling inductor>

$$P_{Lc(2,1)} = \frac{\alpha\delta}{\varepsilon}(y_1 - y_2)(x_1 - x_2) \tag{7}$$

A normalized equation of a total instantaneous reactive power are assumed as Eq. (8).

$$P_{rall} = \sum_{k=1}^{2}\left(\frac{\delta}{\varepsilon}x_k y_k + \frac{\delta}{\varepsilon}y_k\frac{dy_k}{d\tau}\right) + \frac{\alpha\delta}{\varepsilon}(y_1 - y_2)(x_1 - x_2) \quad (8)$$

2.2. Derivation of angular frequency

In this system, we can consider that a power effect is best when a reactive power, which can be assumed as sum of instantaneous electric powers of L, C and L_C, is zero. In other words, we can guess that this system is a steady state when the reactive power is zero. In this study, angular frequencies of the in-phase synchronizations and the anti-phase synchronizations are obtained when the reactive powers are zero.

$$P_{rall} = 0 \quad (9)$$

We want to calculate Eq. (8), but we can't calculate the equation without an oscillation angular frequency ω and an amplitude X . However, when θ_1 and θ_2 are zero or θ_1 is 0 and θ_2 is π, ω can be obtained by using Eq. (9).
<Phase angles of the in-phase synchronization>

$$(\theta_k = 0 \qquad (k = 1 \text{ or } 2)) \quad (10)$$

<Phase angles of the anti-phase synchronization>

$$\begin{pmatrix} \theta_1 & = 0 \\ \theta_2 & = \pi \end{pmatrix} \quad (11)$$

Firstly, an in-phase synchronization frequency ω_{in} and an anti-phase synchronization frequency ω_{anti} of this circuit are calculated by using the Eq. (9). A phase of each oscillator is set as θ_k.
<Phase angles of the in-phase synchronization>

$$(\theta_k = 0 \qquad (k = 1 \text{ or } 2)) \quad (12)$$

<Phase angles of the anti-phase synchronization>

$$\begin{pmatrix} \theta_1 & = 0 \\ \theta_2 & = \pi \end{pmatrix} \quad (13)$$

The angular frequency of the in-phase synchronization is achieved by which Eq. (12) is applied to Eq. (9).
<Anglular frequency of the in-phase synchronization>

$$\omega_{in} = 1 \quad (14)$$

The angular frequency of the anti-phase synchronization is achieved by which Eq. (13) is applied to Eq. (9).
<Anglular frequency of the anti-phase synchronization>

$$\omega_{anti} = \sqrt{1 + 2\alpha} \quad (15)$$

We assume that each angular frequency ω of each phase difference $(\theta_2 - \theta_1)$[degree] exists between ω_{in} and ω_{anti} and linearly vary between ω_{in} and ω_{anti}. Therefore, ω is calculated by the following equation.

$$\omega = \frac{\omega_{anti} - \omega_{in}}{180} \times (\theta_2 - \theta_1) + \omega_{in} \quad (16)$$

Next, amplitude X is calculated. A total active power of this circuit is obtained as a sum of powers of a nonlinear negative resistor in each oscillator(see Eq. (17)).

$$P_{aall} = \sum_{k=1}^{2}(-\delta\frac{d^2x_k}{d\tau^2} + \frac{\delta}{3}\frac{d^4x_k}{d\tau^4}) \quad (17)$$

An amplitude X of each oscillator is calculated, when P_{aall} is integrated in a period and the result is assumed zero as follows.

$$\int_0^{\tau} P_{aall}d\tau = 0, \qquad X = \frac{2}{\omega} \quad (18)$$

We calculate P_{rall} of each phase difference by using the ω and the X.

3. Attracting Force

The attracting force is considered by using reactive powers and simulations.

3.1. Analyzing method by using reactive powers

We set $\theta = \theta_2 - \theta_1$. Waveforms of reactive powers are shown in Figs. (2)–(6) in changing the phase difference θ. We can understand that the amplitude is zero when the θ is 0 degrees or 180 degrees. In other words, if a condition of the system is a steady state, the amplitude of the reactive power is zero, and if the condition is not the steady state, the amplitude is not zero. Therefore, we think that the system becomes unstable when a value, which the reactive power is squared and integrated in a period, become large. The value is expressed as P_{rT}.

$$P_{rT} = \int_0^{\tau} Prall^2 d\tau \quad (19)$$

The P_{rT} is calculated in changing phase difference θ and shown in Figs. (7) and (8). The Fig. (7) shows results of when the α is set as 0.05 and the ε is 0.1, and the Fig. (8) shows results of when the α is set as 0.1 and the ε is 0.1, We can understand that P_{rT} becomes a maximum value when $\theta = 90$ degrees. In under 90 degrees, when the θ becomes small, P_{rT} becomes small. In over 90 degrees, when the θ becomes large, P_{rT} becomes small. We assume that θ changes so that the P_{rT} becomes small. In other words, the θ is attracted to zero under 90 degrees, and θ is attracted to 180 degrees over 90 degrees. If change of the P_{rT} is large when the θ is a little changed, we think that the attracting force is large. Therefore, we calculate gradient values of the graph of the P_{rT}. The gradient values are multiplied by -1, because we want to show the results as negative values in a domain of which the θ decreases to zero, and as positive values in a domain of which the θ increases to 180 degrees. The inversion gradient values g are shown in Figs. (9) and (10)(see Eq. (20)).

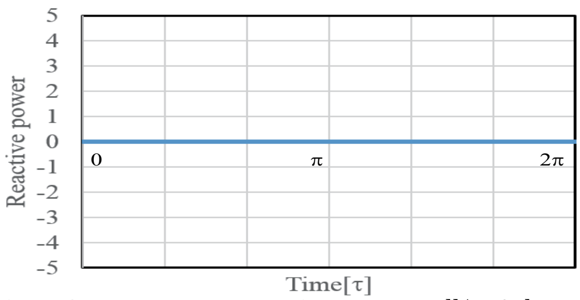

Figure 2: Instantaneous reactive power Prall(θ=0 degrees).

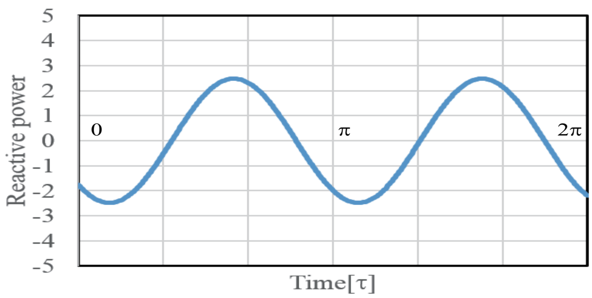

Figure 3: Instantaneous reactive power Prall(θ=45 degrees).

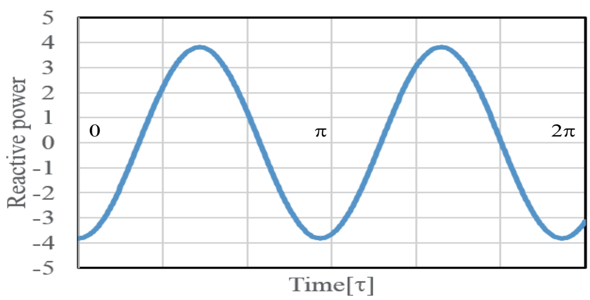

Figure 4: Instantaneous reactive power Prall(θ=90 degrees).

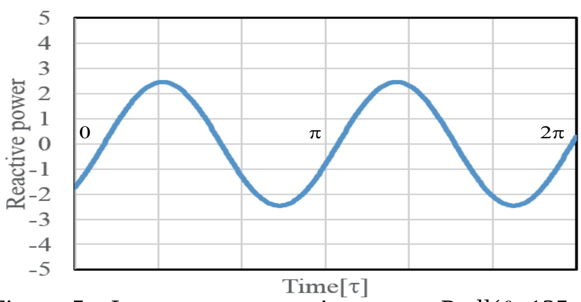

Figure 5: Instantaneous reactive power Prall(θ=135 degrees).

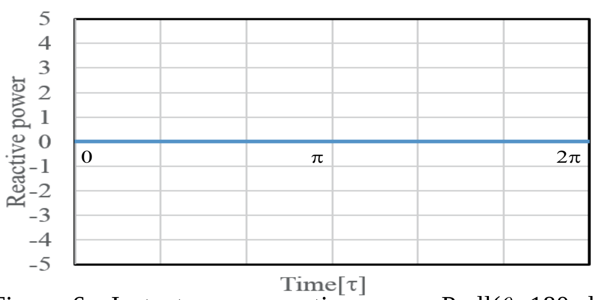

Figure 6: Instantaneous reactive power Prall(θ=180 degrees).

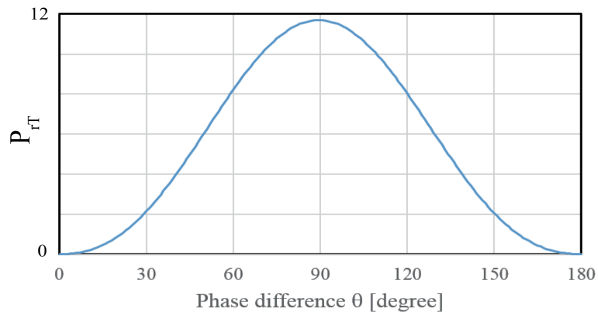

Figure 7: Theoretical results of P_{rT}(α = 0.05 and ε = 0.10).

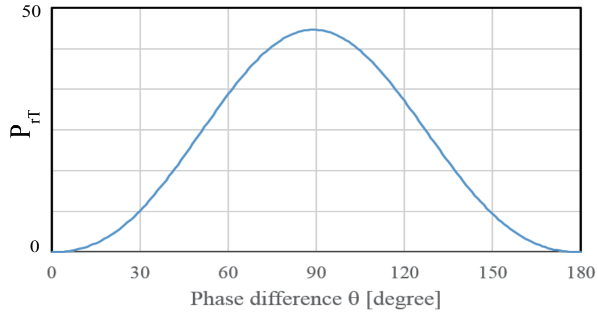

Figure 8: Theoretical results of P_{rT}(α=0.10 ε=0.10).

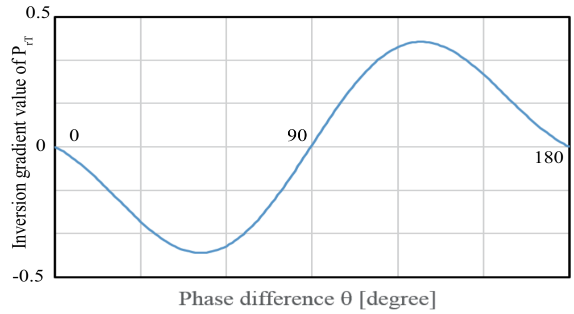

Figure 9: Theoretical results of g(α=0.05 ε=0.10).

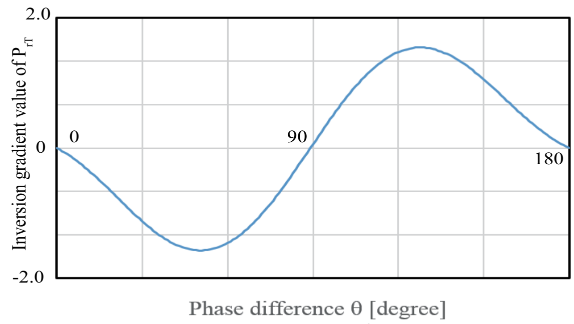

Figure 10: Theoretical results of g(α=0.10 ε=0.10).

$$g = -\frac{dP_{rT}}{d\theta} \tag{20}$$

We can think that the attracting force to stable state is strongest when an absolute value of the g is a maximum value. Therefore, we can understand that when θ=45 degrees and 135 degrees attracting force become strongest.

3.2. Analyzing method by using simulations

An attracting force of each θ is investigated by using a simulator. An initial phase difference between two oscillators is changed by which initial values of a voltage and a current of each oscillator are changed. A calculation method of phase difference is shown as follows(see Fig. 11). In the Fig. 11, two sinusoidal waves show voltages of two oscillators. The a_1 shows the first positive peak of a oscillation waveform of an oscillator after $\tau = 0$, and the a_2 expresses the second positive peak. The b_1 shows the first positive peak of a oscillation waveform of another oscillator. Time of a_1, a_2 and b_2 are expressed τ_{a1}, τ_{a2} and τ_{b2}. The θ is calculated by using Eq. (21).

$$\theta = \frac{\tau_{b2} - \tau_{a2}}{\tau_{a2} - \tau_{a1}} \times 360[\text{degree}] \tag{21}$$

In this paper, we investigate how much the θ is changed during 1τ for each the initial phase difference. A changing value of θ is shown as $\Delta\theta$ and calculated by Eq. (22).

$$\Delta\theta = \left(\frac{\tau_{b3} - \tau_{a3}}{\tau_{a3} - \tau_{a2}} - \frac{\tau_{b2} - \tau_{a2}}{\tau_{a2} - \tau_{a1}}\right) \times \frac{1}{\tau_{a3} - \tau_{a2}} \times 360[\text{degree}] \tag{22}$$

The simulation results are shown in Figs. 12 and 13. The horizontal axes show the θ, and the vertical axes show the $\Delta\theta$ in Figs. 12 and 13. Therefore, this graph means an attracting force g which is strong when a absolute value of $\Delta\theta$ is large value.

We can observe that the shapes of graphs of simulation results are same shape with theoretical graphs, basically.

4. Comparison

Attracting forces are investigated in changing the coupling parameter α by our using theoretical method. The nonlinearity ε is fixed as 0.1 and the α is set as 0.05, 0.10, 0.15 or 0.20. The theoretical results are shown in Fig. 14. We can understand that the attracting force g becomes large as the coupling parameter α becomes large.

5. Conclusions

In this study, the attracting forces to a steady states were theoretically analyzed by using reactive powers and were obtained by a simulation in a coupled oscillator system. The theoretical results and the simulation results were observed same results, basically. Furthermore, we investigated that the attracting force becomes strong as coupling parameter is increased by using our theoretical method.

Acknowledgment

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References

[1] Masayuki Yamauchi, Masahiro Wada, Yoshifumi Nishio and Akio Ushida, “Wave Propagation Phenomena of Phase States in Oscillators Coupled by Inductors as a Ladder,” *IEICE Trans. Fundamentals*, vol.E82-A, no.11, pp.2592-2598, Nov. 1999.

[2] Tetsuro Endo and Sinsaku Mori, “Mode Analysis of a Ring of a Large Number of Mutually coupled van der Pol Oscillators,” IEEE Trans. on Circuits And Systems, vol. cas-25, no. 1, pp.7-18, Jan. 1978.

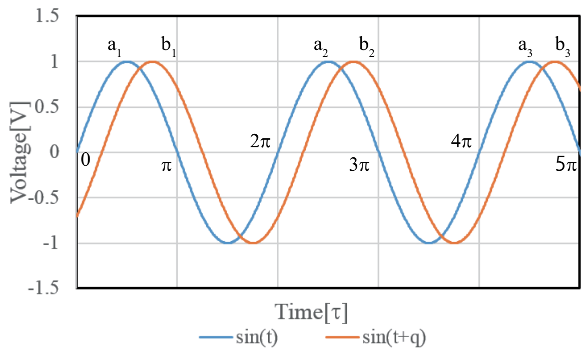

Figure 11: A calculation method of a phase difference.

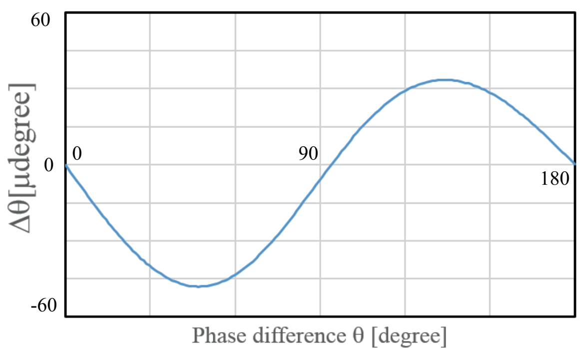

Figure 12: Simulation result of $g(\alpha = 0.05$ and $\varepsilon = 0.10)$.

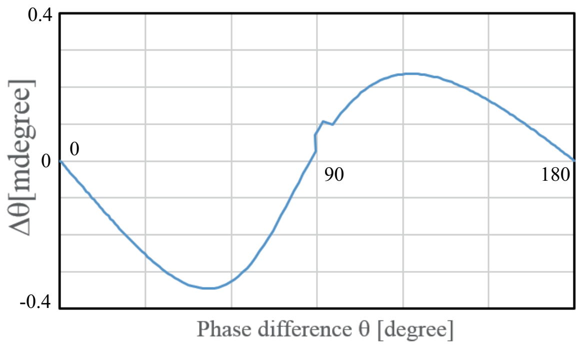

Figure 13: Simulation result of $g(\alpha = 0.10$ and $\varepsilon = 0.10)$.

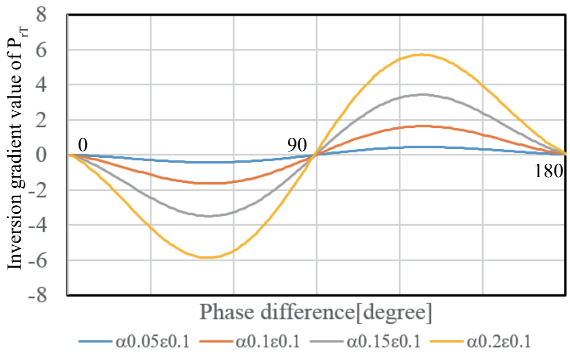

Figure 14: Theoretical results in changing a coupling parametar from 0.05 to 0.20 every 0.05.