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Modeling and Simulation of Motion of a Quadcopter

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Abstract—This paper presents how system dynamics and system control equations for a quadcopter were derived using an Arnold-type operator together with a moving coordinate system and a stationary inertia coordinate system. The Lagrangian Method is also discussed and simulation results are illustrated.

1. Introduction

Although there are several designs, control system equations, and dynamic equations for quadcopters, such as [1], unified methods to describe the dynamic equations of quadcopters have yet to be established. Using the mathematical foundation of rigid body dynamics provided by V. I. Arnold [2], we apply Arnold's operator in order to facilitate the derivation of the dynamics of a quadcopter. Secondly, we clarify the relationship between the Euler angles, the stationary inertia coordinate system and the moving coordinate system for engineering problems. Finally, we illustrate the typical behavior of the quadcopter on MATLAB numerical simulations. In the following, let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^n$ be the set of real number vectors.

2. Motion in a Moving Coordinate System

In this section, we detail the mathematical foundation for describing the motion of a quadcopter based on [2]. It should be noted that the method applied to describe the motion in Inohara *et al*. [3] is used for this study. The time parameter t for all the stated variables, such as $\boldsymbol{r}(t)$, or $\boldsymbol{\Omega}(t)$, etc., is omitted for convenience. We use the following notation as [2] (Fig 1):
$\boldsymbol{e}_i \in w$ $(i = 1, 2, 3)$ are the base vectors of a right-handed Cartesian stationary coordinate system at the origin $\boldsymbol{O}$;
$\boldsymbol{E}_i \in W$ $(i = 1, 2, 3)$ are the base vectors of a right moving coordinate system connected to the body at the center of the mass $\boldsymbol{O}_c$.

Definition 1 Let w and W be oriented euclidean spaces (i.e. orthogonal spaces). A motion of W relative to w is a smooth mapping depending on t:

$$\boldsymbol{B} : W \to w, \tag{1}$$

which preserves the metric and the orientation (Fig 1).

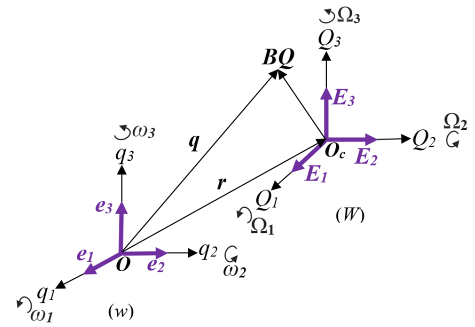

Fig 1: Radius vector of a point with respect to stationary ($\boldsymbol{q}$) and moving ($\boldsymbol{Q}$) coordinate systems

Definition 2 A motion $\boldsymbol{B}$ is called a rotation if it takes the origin of W to the origin of w (i.e. if $\boldsymbol{B}$ is a linear operator).

Definition 3 w is called a stationary coordinate system, W a moving one, and $\boldsymbol{q} \in w$ the radius-vector of a point moving relative to the stationary system; if

$$\boldsymbol{q} = \boldsymbol{r} + \boldsymbol{BQ} \tag{2}$$

$\boldsymbol{Q}$ is called the radius vector of the point relative to the moving system (Fig 1).

We express the "absolute velocity" $\dot{\boldsymbol{q}}$ in terms of the relative motion $\boldsymbol{Q}$ and the motion of the coordinate system $\boldsymbol{B}$. By differentiating with respect to t in Eq. (2), we arrive at Eq. (3) for the addition of velocities.

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{B}}\boldsymbol{Q} + \boldsymbol{B}\dot{\boldsymbol{Q}}. \tag{3}$$

In order to carry the stationary frame $\boldsymbol{e}_i$ $(i = 1, 2, 3)$ into the moving frame $\boldsymbol{E}_i$ $(i = 1, 2, 3)$, we perform three rotations (Fig 2):

1. Given an angle ψ around the $\boldsymbol{e}_3$ axis, under this rotation, $\boldsymbol{e}_3$ remains fixed and $\boldsymbol{e}_2$ goes to $\boldsymbol{E}_2^{-2}$ by means of Eq. (5).

2. Given an angle θ around the $\boldsymbol{E}_2^{-2}$ axis, under this rotation, $\boldsymbol{E}_2^{-2}$ remains fixed and $\boldsymbol{E}_1^{-2}$ goes to $\boldsymbol{E}_1^{-1}$ by means of Eq. (6).

3. Given an angle ϕ around the $\boldsymbol{E}_1^{-1}$ axis, under this rotation, $\boldsymbol{E}_1^{-1}$ remains fixed and $\boldsymbol{E}_3^{-1}$ goes to $\boldsymbol{E}_3$ by means of Eq. (7).

After all three rotations are completed, $\boldsymbol{e}_1$ has moved to $\boldsymbol{E}_1$, and $\boldsymbol{e}_2$ to $\boldsymbol{E}_2$; therefore, $\boldsymbol{e}_3$ moves to $\boldsymbol{E}_3$. The angles ψ, θ, and ϕ are called the Tait-Bryan angles (one of the Euler angle systems).

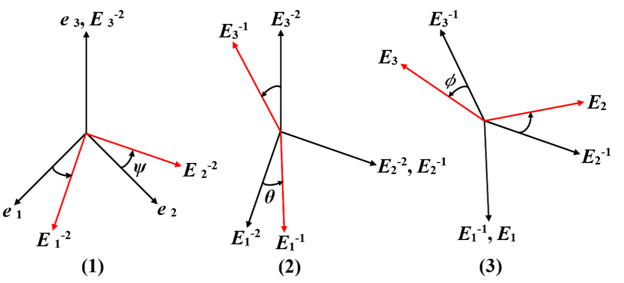

Fig 2: Rotations defining the Tait-Bryan angles

Here we describe an operator $\boldsymbol{B}$, as follows:

$$\begin{aligned}\boldsymbol{B} &= \boldsymbol{R}_\psi \boldsymbol{R}_\theta \boldsymbol{R}_\phi \\ &= \begin{pmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix}, \end{aligned} \tag{4}$$

$$\boldsymbol{R}_\psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

$$\boldsymbol{R}_\theta = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{6}$$

$$\boldsymbol{R}_\phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}. \tag{7}$$

$c\psi$, $s\psi$, $c\theta$, $s\theta$, $c\phi$ and $s\phi$ denote $\cos\psi$, $\sin\psi$, $\cos\theta$, $\sin\theta$, $\cos\phi$ and $\sin\phi$, respectively. Hence, the base vectors of the moving coordinate system generated by means of the operator $\boldsymbol{B}$ are expressed as:

$$\boldsymbol{B}\boldsymbol{E}_1 = \cos\psi\cos\theta\boldsymbol{e}_1 + \sin\psi\cos\theta\boldsymbol{e}_2 - \sin\theta\boldsymbol{e}_3, \tag{8}$$

$$\begin{aligned}\boldsymbol{B}\boldsymbol{E}_2 &= (\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi)\boldsymbol{e}_1 \\ &+ (\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi)\boldsymbol{e}_2 \\ &+ \cos\theta\sin\phi\boldsymbol{e}_3, \end{aligned} \tag{9}$$

$$\begin{aligned}\boldsymbol{B}\boldsymbol{E}_3 &= (\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)\boldsymbol{e}_1 \\ &+ (\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)\boldsymbol{e}_2 \\ &+ \cos\theta\cos\phi\boldsymbol{e}_3. \end{aligned} \tag{10}$$

Since W is a moving coordinate system connected to the body of a quadcopter, $\boldsymbol{Q}$ is at rest in W (i.e., $\dot{\boldsymbol{Q}} = 0$) and the coordinate system W rotates (i.e., $\boldsymbol{r} = 0$). In this case, the motion of the point $\boldsymbol{q}$ is a transferred rotation given by Eq. (11).

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{B}}\boldsymbol{Q} = \dot{\boldsymbol{r}} + \boldsymbol{B}[\boldsymbol{\Omega}, \boldsymbol{Q}] = \dot{\boldsymbol{r}} + [\boldsymbol{B}\boldsymbol{\Omega}, \boldsymbol{B}\boldsymbol{Q}], \tag{11}$$

where $[\,\cdot\,,\,\cdot\,]$: the vector product.

The vector $\boldsymbol{\Omega} \in W$ is called the vector of angular velocity in the quadcopter. In this case, $\boldsymbol{\Omega}$ is expressed by:

$$\boldsymbol{\Omega} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{\omega}. \tag{12}$$

The vector $\boldsymbol{\omega} \in w$ is called the instantaneous angular velocity given by Eq. (13).

$$\boldsymbol{\omega} = \dot{\psi}\boldsymbol{e}_3 + \dot{\theta}\boldsymbol{E}_2^{-2} + \dot{\phi}\boldsymbol{E}_1^{-1}. \tag{13}$$

In numerous studies and texts, the angular velocity vector $(\dot{\psi}, \dot{\theta}, \dot{\phi})$ of the Tait-Bryan angles [4], [5] is often referred to as coordinate components $\boldsymbol{E}_i, (i = 1, 2, 3)$ on the base vectors of the moving coordinate system. It must be stressed, however, that the description given in Eq. (13) is correct. Using the angular velocity vector of the Tait-Bryan angles of Eq. (13), we can rewrite Eq. (11), as follows:

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\psi}\frac{\partial}{\partial\psi}(\boldsymbol{B}\boldsymbol{Q}) + \dot{\theta}\frac{\partial}{\partial\theta}(\boldsymbol{B}\boldsymbol{Q}) + \dot{\phi}\frac{\partial}{\partial\phi}(\boldsymbol{B}\boldsymbol{Q}). \tag{14}$$

Here, let $\boldsymbol{h} \in w$ be the angular momentum of the quadcopter in the stationary inertia coordinate system, $\boldsymbol{H} \in W$ be the angular momentum of the quadcopter in the moving coordinate system, and $\hat{\boldsymbol{I}}$ be the moment of inertia of the quadcopter. Using operator $\boldsymbol{B}$, we obtained the following equations:

$$\boldsymbol{h} = \hat{\boldsymbol{I}}\boldsymbol{\omega} = \boldsymbol{B}\boldsymbol{H} \in \mathrm{w}, \tag{15}$$

$$\boldsymbol{H} = \hat{\boldsymbol{I}}\boldsymbol{\Omega} \in \mathrm{W}. \tag{16}$$

In addition, let $\boldsymbol{\tau} \in w$ be the torque of the quadcopter. We obtain the time derivative of an angular momentum which is equal to the moment, as follows:

$$\frac{d}{dt}\boldsymbol{h} = \boldsymbol{\tau} = \frac{d}{dt}\boldsymbol{B}\boldsymbol{H} = \boldsymbol{B}\boldsymbol{T},\ \hat{\boldsymbol{I}}\dot{\boldsymbol{\Omega}} + [\boldsymbol{\Omega}, \boldsymbol{H}] - \boldsymbol{T} = \boldsymbol{0}. \tag{17}$$

Then, $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$ can be expressed in concrete terms by:

$$\begin{aligned}\boldsymbol{\Omega} &= (-\dot{\psi}\sin\theta + \dot{\phi})\boldsymbol{E}_1 + (\dot{\psi}\cos\theta\sin\phi + \dot{\theta}\cos\phi)\boldsymbol{E}_2 \\ &+ (\dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi)\boldsymbol{E}_3, \end{aligned} \tag{18}$$

and

$$\begin{aligned}\dot{\boldsymbol{\Omega}} &= (-\ddot{\psi}s\theta - \dot{\psi}\dot{\theta}c\theta + \ddot{\phi})\boldsymbol{E}_1 \\ &+ (\ddot{\psi}c\theta s\phi - \dot{\psi}\dot{\theta}s\theta s\phi + \dot{\psi}\dot{\phi}c\theta c\phi + \ddot{\theta}c\phi - \dot{\theta}\dot{\phi}s\phi)\boldsymbol{E}_2 \\ &+ (\ddot{\psi}c\theta c\phi - \dot{\psi}\dot{\theta}s\theta c\phi - \dot{\psi}\dot{\phi}c\theta s\phi - \ddot{\theta}s\phi - \dot{\theta}\dot{\phi}c\phi)\boldsymbol{E}_3, \end{aligned} \tag{19}$$

respectively.

A torque $\boldsymbol{\tau}$ of Eq. (17) is also given as:

$$\boldsymbol{\tau} = [\boldsymbol{r}, \boldsymbol{f}]. \tag{20}$$

Here, $\boldsymbol{r}$ and $\boldsymbol{f}$ denote the position vector on which external forces act and a vector of the external forces, respectively. The coordinate components of an angular momentum of the quadcopter are given as Eq. (21).

$$\boldsymbol{H}_A = \sum_{i=1}^{3} H_{iA}\boldsymbol{E}_i. \tag{21}$$

Then, we have the following equation:

$$(H_1, H_2, H_3)^{\mathrm{T}} = \hat{\boldsymbol{I}}(\Omega_1, \Omega_2, \Omega_3)^{\mathrm{T}}. \tag{22}$$

The moment of inertia $\hat{\boldsymbol{I}}$ is also defined by

$$\hat{\boldsymbol{I}} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}. \tag{23}$$

In this paper, for $\hat{\boldsymbol{I}}$, we assume that $I_{kl} = 0$, and for $k \neq l$.

3. Dynamic Model of a Quadcopter

The coordinate systems and free body diagram for the quadcopter are shown in Figure 3. Based on the preceding mathematical foundation, we can describe the dynamic model of the quadcopter (Fig 3).

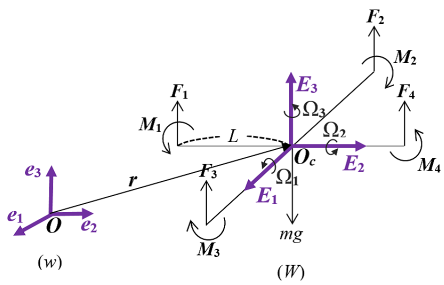

Fig 3: Coordinate systems and forces/moments acting on the quadcopter ($m = 1.656\,[kg]$, $g = 9.80665\,[m/s^2]$, $I_{11} = 0.01982\,[kg \cdot m^2]$, $I_{22} = 0.01954\,[kg \cdot m^2]$, $I_{33} = 0.03221\,[kg \cdot m^2]$, $L = 0.365\,[m]$)

$F_i, (i = 1, 2, 3, 4)$ and $M_i, (i = 1, 2, 3, 4)$ of Figure 3, represent vertical forces and moment, respectively. $F_i, (i = 1, 2, 3, 4)$ and $M_i, (i = 1, 2, 3, 4)$ are defined in a similar manner [1]. Each motor of the quadcopter has an angular speed ω_i and produces a vertical force F_i according to:

$$F_i = k_{Fi}\omega_{Mi}^2, \quad i = 1, 2, 3, 4, \tag{24}$$

Experimentation with a fixed motor in a steady state shows that $k_F \approx 1.79 \times 10^{-7} \frac{N}{rpm^2}$. The motors also produce a moment according to:

$$M_i = k_{Mi}\omega_{Mi}^2, \quad i = 1, 2, 3, 4, \tag{25}$$

The constant, k_M, is determined to be approximately $4.38 \approx 10^{-9} \frac{Nm}{rpm^2}$ by matching the performance of the simulation to the real system.

We establish the Lagrangian of the quadcopter Lag, as follows:

$$Lag = \frac{1}{2}m(\dot{\boldsymbol{r}}, \dot{\boldsymbol{r}}) + \frac{1}{2}(\hat{\boldsymbol{I}}\boldsymbol{\Omega}, \boldsymbol{\Omega}) - mgr_3, \tag{26}$$

where $(\,\cdot\,,\,\cdot\,)$: the scalar product, $\boldsymbol{r} = (r_1, r_2, r_3)$.
We also identify Lagrange's equations for the quadcopter ($k = 1, 2, 3$), as follows:

$$\begin{aligned} \frac{d}{dt}\frac{\partial Lag}{\partial \omega_k} &= (\boldsymbol{B}L(F_4 - F_1)\boldsymbol{E}_1, \boldsymbol{B}\boldsymbol{E}_k) \\ &+ (\boldsymbol{B}L(F_2 - F_3)\boldsymbol{E}_2, \boldsymbol{B}\boldsymbol{E}_k) \\ &+ (\boldsymbol{B}(M_1 - M_2 - M_3 + M_4)\boldsymbol{E}_3, \boldsymbol{B}\boldsymbol{E}_k), \end{aligned} \tag{27}$$

$$\frac{d}{dt}\frac{\partial Lag}{\partial \dot{\boldsymbol{r}}_k} - \frac{\partial Lag}{\partial \boldsymbol{r}_k} = (\boldsymbol{B}(\sum_{i=1}^{4}\boldsymbol{F}_i), \boldsymbol{e}_k). \tag{28}$$

We summarize the vector equations of the quadcopter, as follows:

$$\begin{aligned} \hat{\boldsymbol{I}}\dot{\boldsymbol{\Omega}} + [\boldsymbol{\Omega}, \hat{\boldsymbol{I}}\boldsymbol{\Omega}] &= L(F_4 - F_1)\boldsymbol{E}_1 + L(F_2 - F_3)\boldsymbol{E}_2 \\ &+ (M_1 - M_2 - M_3 + M_4)\boldsymbol{E}_3, \end{aligned} \tag{29}$$

$$m\ddot{\boldsymbol{r}} = -mg\boldsymbol{e}_3 + \boldsymbol{B}(\sum_{i=1}^{4}\boldsymbol{F}_i), \tag{30}$$

$$\hat{\boldsymbol{I}}\boldsymbol{E}_k = I_{1k}\boldsymbol{E}_1 + I_{2k}\boldsymbol{E}_2 + I_{3k}\boldsymbol{E}_3, \tag{31}$$

where m: the gross weight of the quadcopter.

Since the operator $\boldsymbol{B}$ states variables $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$ are expressed as the functions of $(\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi)$, the state equation of the quadcopter can be rewritten as equations of the function of $(\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi)$.

$$\dot{\boldsymbol{x}} = F(\boldsymbol{x}) + G(\boldsymbol{x}, \boldsymbol{u}), \tag{32}$$

$$\boldsymbol{x}^{\mathrm{T}} = (\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi), \tag{33}$$

$$\delta\boldsymbol{x}^{\mathrm{T}} = (\delta\dot{\psi}, \delta\dot{\theta}, \delta\dot{\phi}, \delta\psi, \delta\theta, \delta\phi,), \tag{34}$$

$$\boldsymbol{u}^{\mathrm{T}} = (\omega_{M1}^2, \omega_{M2}^2, \omega_{M3}^2, \omega_{M4}^2), \tag{35}$$

$$F(\boldsymbol{x}) = \begin{bmatrix} f_{\psi}(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ f_{\theta}(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ f_{\phi}(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}, \tag{36}$$

$$
\boldsymbol{B}_G = \begin{bmatrix} [\frac{1}{I_{22}I_{33}(\sin^2\theta-1)}]\boldsymbol{B}_{G\psi}^{\mathrm{T}} \\ [\frac{1}{I_{33}I_{22}}]\boldsymbol{B}_{G\theta}^{\mathrm{T}} \\ [\frac{1}{I_{11}I_{22}I_{33}(\sin^2\theta-1)}]\boldsymbol{B}_{G\phi}^{\mathrm{T}} \\ [0^{\mathrm{T}}] \\ [0^{\mathrm{T}}] \\ [0^{\mathrm{T}}] \end{bmatrix}. \tag{37}
$$

The variational equation (38) that can be used to control the quadcopter is described by

$$
\frac{d}{dt}\delta\boldsymbol{x} = DF(\boldsymbol{x}_0)\cdot\delta\boldsymbol{x} + \boldsymbol{B}_G\boldsymbol{u}, \tag{38}
$$

where $DF(\boldsymbol{x}_0)$ and $\boldsymbol{x}_0$ denote the Jacobian of $F(\boldsymbol{x})$ and a driving point of $\boldsymbol{x}$. In particular, f_ψ, f_θ, and f_ϕ in $F(\boldsymbol{x})$ are easily obtained using Maple symbolic computations on $\boldsymbol{B}_{G\psi}^{\mathrm{T}}$, $\boldsymbol{B}_{G\theta}^{\mathrm{T}}$, and $\boldsymbol{B}_{G\phi}^{\mathrm{T}}$, as follows:

$$
\begin{aligned}
\boldsymbol{B}_{G\psi}^{\mathrm{T}} = {} & [-I_{22}\cos\phi\cos\theta k_{M1}, -I_{33}L\cos\theta\sin\phi k_{F2} + I_{22}\cos\phi\cos\theta k_{M2}, \\
& I_{33}L\cos\theta\sin\phi k_{F3} + I_{22}\cos\phi\cos\theta k_{M3}, -I_{22}\cos\phi\cos\theta k_{M4}],
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
\boldsymbol{B}_{G\theta}^{\mathrm{T}} = {} & [-I_{22}\sin\phi k_{M1}, I_{22}\sin\phi k_{M2} + I_{33}L\cos\phi k_{F2}, \\
& I_{22}\sin\phi k_{M3} - I_{33}L\cos\phi k_{F3}, -I_{22}\sin\phi k_{M4}],
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
\boldsymbol{B}_{G\phi}^{\mathrm{T}} = {} & [-I_{22}I_{33}L\sin^2\theta k_{F1} - I_{11}I_{22}\cos\phi\cos\theta\sin\theta k_{M1} + I_{22}I_{33}Lk_{F1}, \\
& I_{11}I_{22}\cos\phi\cos\theta\sin\theta k_{M2} - I_{11}I_{33}L\sin\phi\cos\theta\sin\theta k_{F2}, \\
& I_{11}I_{22}\cos\phi\cos\theta\sin\theta k_{M3} + I_{11}I_{33}L\sin\phi\cos\theta\sin\theta k_{F3}, \\
& I_{22}I_{33}L\sin^2\theta k_{F4} - I_{11}I_{22}\cos\phi\cos\theta\sin\theta k_{M4} - I_{22}I_{33}Lk_{F4}].
\end{aligned} \tag{41}
$$

4. Simulation of Motion of a Quadcopter

By means of numerical computations on MATLAB with ode45 solver $t\,[s] \in [0\ 3]$ applied to Eq. (30) and Eq. (32), an example of typical quadcopter flight behavior without any controls is obtained. This behavior includes $3m$ hovering, forward moving and descending, and crashing. The simulation results illustrated in Figure 4 and Figure 5 are obtained. The initial values are set as $\dot{\psi} = \dot{\theta} = \dot{\phi} = 0\,[rad/s]$, $\psi = \theta = \phi = 0\,[rad]$, $r_1 = r_2 = 0\,[m]$, $\dot{r_1} = \dot{r_2} = \dot{r_3} = 0\,[m/s]$, $r_3 = 3\,[m]$, $\omega_{M1} = \omega_{M3} = \omega_{M4} = 4760\,[rpm]$, and $\omega_{M2} = 4770\,[rpm]$. Notice that the numerical computation of Eq. (30) is carried out by using the computation results of Eq. (32), together with interpolations of $\psi(t)$, $\theta(t)$ and $\phi(t)$.

5. Conclusion

The following results are obtained:

(1) We have derived the system dynamics and system control equations for the quadcopter by using the operator $\boldsymbol{B}$ together with two coordinate systems, w and W. In addition, we have described Lagrange's equations for the quadcopter.

(2) We have reliably illustrated the typical behavior of the quadcopter using numerical simulations on MATLAB with an ode45 solver.

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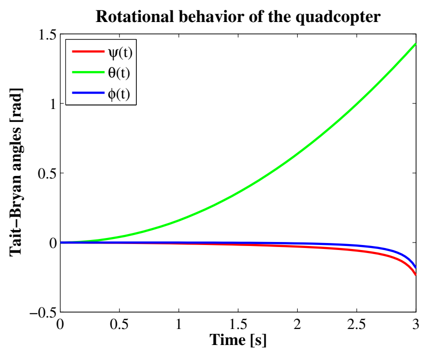

Fig 4: Simulation results of Tait-Bryan angles

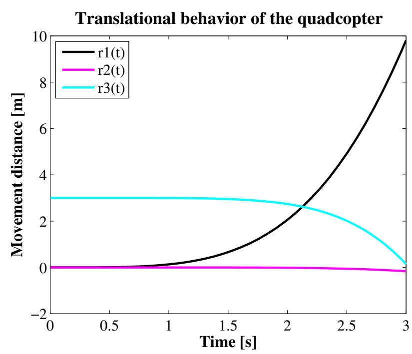

Fig 5: Simulation results for position of motion