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Modeling and Simulation of Motion of an Underwater Robot

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Abstract—This paper presents how system dynamics and system control equations for an underwater robot were derived using an Arnold-type operator to control the OpenROV. Typical behavior of the OpenROV on MATLAB numerical simulations is illustrated.

1. Introduction

Although there are several designs, control system equations, and dynamic equations for underwater robots, such as [1], unified methods to describe the dynamic equations for the rigid body kinetics of an underwater robot have yet to be established. Using the mathematical foundation of rigid body dynamics provided by V. I. Arnold [2], we apply Arnold's operator in order to facilitate the derivation of dynamics for the rigid body kinetics of an underwater robot. Secondly, OpenROV (Fig 1) (open-source remotely operated vehicle) projects [3] have recently been promoted to examine the sea bottom. Since simulations of the behavior of OpenROV are of value, we describe the equations of motion for real OpenROV. Finally, we illustrate the typical behavior of OpenROV on MATLAB numerical simulations. In the following, let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^n$ be the set of real number vectors.

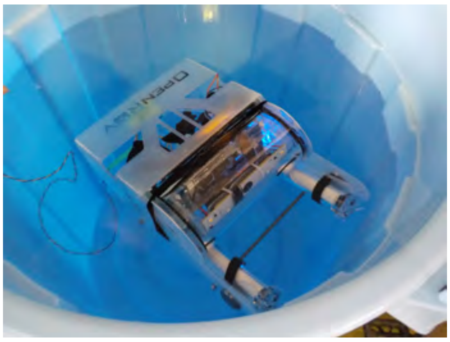

Fig 1: OpenROV version 2.7

2. Motion in a Moving Coordinate System

In this section, we detail the mathematical foundation for describing the motion of fundamental rigid body kinetics [1] of an underwater robot based on [2], [4]. The time parameter t for all the stated variables, such as $\boldsymbol{r}(t)$, or $\boldsymbol{\Omega}(t)$, etc., is omitted for convenience. We use the following notation as [2] (Fig 2):
$\boldsymbol{e}_i \in w$ $(i = 1, 2, 3)$ are the base vectors of a right-handed Cartesian stationary coordinate system at the origin $\boldsymbol{O}$;
$\boldsymbol{E}_i \in W$ $(i = 1, 2, 3)$ are the base vectors of a right moving coordinate system connected to the body at the center of the mass $\boldsymbol{O}_c$.

Definition 1 Let w and W be oriented euclidean spaces (i.e. orthogonal spaces). A motion of W relative to w is a smooth mapping on t:

$$\boldsymbol{B} : W \to w, \tag{1}$$

which preserves the metric and the orientation (Fig 2).

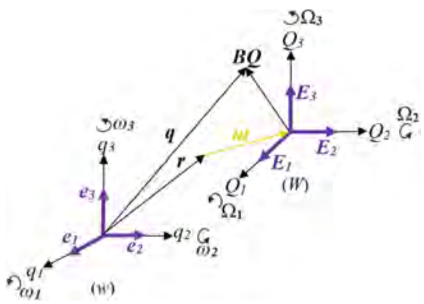

Fig 2: Radius vector of a point with respect to stationary ($\boldsymbol{q}$) and moving ($\boldsymbol{Q}$) coordinate systems

Definition 2 A motion $\boldsymbol{B}$ is called a rotation if it takes the origin of W to the origin of w (i.e. if $\boldsymbol{B}$ is a linear operator).

Definition 3 w is called a stationary coordinate system, W a moving one, and $\boldsymbol{q} \in w$ the radius-vector of a point moving relative to the stationary system; if

$$\boldsymbol{q} = \boldsymbol{r} + \boldsymbol{u}t + \boldsymbol{BQ} \tag{2}$$

There exists a flow of fluid velocity vector $\boldsymbol{u}$ in w. $\boldsymbol{Q}$ is called the radius vector of the point relative to the moving system (Fig 2). We express the "absolute velocity" $\dot{\boldsymbol{q}}$ in terms of the relative motion $\boldsymbol{Q}$ and the motion of the coordinate system $\boldsymbol{B}$. By differentiating with respect to t in Eq. (2), we arrive at Eq. (3) for the addition of velocities.

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{u}}t + \boldsymbol{u} + \frac{d}{dt}(\boldsymbol{BQ}). \tag{3}$$

In order to carry the stationary frame $\boldsymbol{e}_i$ $(i = 1, 2, 3)$ into the moving frame $\boldsymbol{E}_i$ $(i = 1, 2, 3)$, we perform three rotations (Fig 3):

1. Given an angle ψ around the $\boldsymbol{e}_3$ axis, under this rotation, $\boldsymbol{e}_3$ remains fixed and $\boldsymbol{e}_2$ goes to $\boldsymbol{E}_2^{-2}$ by means of Eq. (5).
2. Given an angle θ around the $\boldsymbol{E}_2^{-2}$ axis, under this rotation, $\boldsymbol{E}_2^{-2}$ remains fixed and $\boldsymbol{E}_1^{-2}$ goes to $\boldsymbol{E}_1^{-1}$ by means of Eq. (6).
3. Given an angle ϕ around the $\boldsymbol{E}_1^{-1}$ axis, under this rotation, $\boldsymbol{E}_1^{-1}$ remains fixed and $\boldsymbol{E}_3^{-1}$ goes to $\boldsymbol{E}_3$ by means of Eq. (7).

After all three rotations are completed, $\boldsymbol{e}_1$ has moved to $\boldsymbol{E}_1$, and $\boldsymbol{e}_2$ to $\boldsymbol{E}_2$; therefore, $\boldsymbol{e}_3$ moves to $\boldsymbol{E}_3$. The angles ψ, θ, and ϕ are called the Tait-Bryan angles (one of the Euler angle systems).

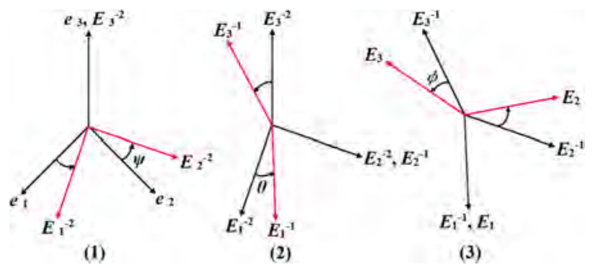

Fig 3: Rotations defining the Tait-Bryan angles

Here we describe an operator $\boldsymbol{B}$, as follows:

$$\begin{aligned}\boldsymbol{B} &= \boldsymbol{R}_\psi \boldsymbol{R}_\theta \boldsymbol{R}_\phi \\ &= \begin{pmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix},\end{aligned} \tag{4}$$

$$\boldsymbol{R}_\psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

$$\boldsymbol{R}_\theta = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{6}$$

$$\boldsymbol{R}_\phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}. \tag{7}$$

$c\psi$, $s\psi$, $c\theta$, $s\theta$, $c\phi$ and $s\phi$ denote $\cos\psi$, $\sin\psi$, $\cos\theta$, $\sin\theta$, $\cos\phi$ and $\sin\phi$, respectively. Hence, the base vectors of the moving coordinate system generated by means of the operator $\boldsymbol{B}$ are expressed as:

$$\boldsymbol{BE}_1 = \cos\psi\cos\theta\boldsymbol{e}_1 + \sin\psi\cos\theta\boldsymbol{e}_2 - \sin\theta\boldsymbol{e}_3, \tag{8}$$

$$\begin{aligned}\boldsymbol{BE}_2 &= (\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi)\boldsymbol{e}_1 \\ &+ (\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi)\boldsymbol{e}_2 \\ &+ \cos\theta\sin\phi\boldsymbol{e}_3,\end{aligned} \tag{9}$$

$$\begin{aligned}\boldsymbol{BE}_3 &= (\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)\boldsymbol{e}_1 \\ &+ (\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)\boldsymbol{e}_2 \\ &+ \cos\theta\cos\phi\boldsymbol{e}_3.\end{aligned} \tag{10}$$

Since W is a moving coordinate system connected to the body of an underwater robot, $\boldsymbol{Q}$ is at rest in W (i.e., $\dot{\boldsymbol{Q}} = 0$) and the coordinate system W rotates (i.e., $\boldsymbol{r} = 0$). In this case, the motion of the point $\boldsymbol{q}$ is a transferred rotation given by Eq. (11).

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{u}}t + \boldsymbol{u} + \dot{\boldsymbol{B}}\boldsymbol{Q} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{u}}t + \boldsymbol{u} + [\boldsymbol{B\Omega}, \boldsymbol{BQ}], \tag{11}$$

where $[\,\cdot\,,\,\cdot\,]$: the vector product.

The vector $\boldsymbol{\Omega} \in W$ is called the vector of angular velocity in the underwater robot. In this case, $\boldsymbol{\Omega}$ is expressed by:

$$\boldsymbol{\Omega} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{\omega}. \tag{12}$$

The vector $\boldsymbol{\omega} \in w$ is called the instantaneous angular velocity given by Eq. (13).

$$\boldsymbol{\omega} = \dot{\psi}\boldsymbol{e}_3 + \dot{\theta}\boldsymbol{E}_2^{-2} + \dot{\phi}\boldsymbol{E}_1^{-1}. \tag{13}$$

In numerous studies and texts, the angular velocity vector $(\dot{\psi}, \dot{\theta}, \dot{\phi})$ of the Tait-Bryan angles [5], [6] is often referred to as coordinate components $\boldsymbol{E}_i, (i = 1, 2, 3)$ on the base vectors of the moving coordinate system. It must be stressed, however, that the description given in Eq. (13) is correct. Using the angular velocity vector of the Tait-Bryan angles of Eq. (13), we can rewrite Eq. (11), as follows:

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{r}} + \dot{\boldsymbol{u}}t + \boldsymbol{u} + \dot{\psi}\frac{\partial}{\partial\psi}(\boldsymbol{BQ}) + \dot{\theta}\frac{\partial}{\partial\theta}(\boldsymbol{BQ}) + \dot{\phi}\frac{\partial}{\partial\phi}(\boldsymbol{BQ}). \tag{14}$$

Here, let $\boldsymbol{h} \in w$ be the angular momentum of the underwater robot in the stationary inertia coordinate system, $\boldsymbol{H} \in W$ be the angular momentum of the underwater robot in the moving coordinate system, and $\hat{\boldsymbol{I}}$ be the moment of inertia of the underwater robot. Using operator $\boldsymbol{B}$, we obtained the following equations:

$$\boldsymbol{h} = \hat{\boldsymbol{I}}\boldsymbol{\omega} = \boldsymbol{BH} \in \mathrm{w}, \tag{15}$$

$$\boldsymbol{H} = \hat{\boldsymbol{I}}\boldsymbol{\Omega} \in \mathrm{W}. \tag{16}$$

In addition, let $\boldsymbol{\tau} \in w$ be the torque of the underwater robot. We obtain the time derivative of an angular momentum which is equal to the moment, as follows:

$$\frac{d}{dt}\boldsymbol{h} = \boldsymbol{\tau} = \frac{d}{dt}\boldsymbol{BH} = \boldsymbol{BT},\ \hat{\boldsymbol{I}}\dot{\boldsymbol{\Omega}} + [\boldsymbol{\Omega}, \boldsymbol{H}] - \boldsymbol{T} = \boldsymbol{0}. \tag{17}$$

Then, $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$ can be expressed in concrete terms by:

$$\begin{aligned}\boldsymbol{\Omega} &= (-\dot{\psi}\sin\theta + \dot{\phi})\boldsymbol{E}_1 + (\dot{\psi}\cos\theta\sin\phi + \dot{\theta}\cos\phi)\boldsymbol{E}_2 \\ &+ (\dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi)\boldsymbol{E}_3,\end{aligned} \tag{18}$$

and

$$\begin{aligned}\dot{\boldsymbol{\Omega}} &= (-\ddot{\psi}s\theta - \dot{\psi}\dot{\theta}c\theta + \ddot{\phi})\boldsymbol{E}_1 \\ &+ (\ddot{\psi}c\theta s\phi - \dot{\psi}\dot{\theta}s\theta s\phi + \dot{\psi}\dot{\phi}c\theta c\phi + \ddot{\theta}c\phi - \dot{\theta}\dot{\phi}s\phi)\boldsymbol{E}_2 \\ &+ (\ddot{\psi}c\theta c\phi - \dot{\psi}\dot{\theta}s\theta c\phi - \dot{\psi}\dot{\phi}c\theta s\phi - \ddot{\theta}s\phi - \dot{\theta}\dot{\phi}c\phi)\boldsymbol{E}_3,\end{aligned} \tag{19}$$

respectively.

A torque $\boldsymbol{\tau}$ of Eq. (17) is also given as:

$$\boldsymbol{\tau} = [\boldsymbol{r}, \boldsymbol{f}]. \tag{20}$$

Here, $\boldsymbol{r}$ and $\boldsymbol{f}$ denote the position vector on which external forces act and a vector of external forces, respectively.

The coordinate components of an angular momentum of the underwater robot are given as Eq. (21).

$$\boldsymbol{H}_A = \sum_{i=1}^{3} H_{iA}\boldsymbol{E}_i. \tag{21}$$

Then, we have the following equation:

$$(H_1, H_2, H_3)^{\mathrm{T}} = \hat{\boldsymbol{I}}(\Omega_1, \Omega_2, \Omega_3)^{\mathrm{T}}. \tag{22}$$

The moment of inertia $\hat{\boldsymbol{I}}$ is also defined by

$$\hat{\boldsymbol{I}} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}. \tag{23}$$

In this paper, for $\hat{\boldsymbol{I}}$, we assume that $I_{kl} = 0$, and for $k \neq l$.

3. Dynamic Model of OpenROV

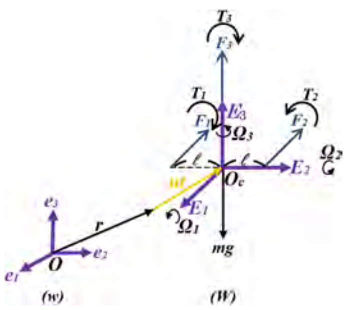

Fig 4: Coordinate systems and forces/moments acting on OpenROV ($m = 2.4\,[kg]$, $g = 9.80665\,[m/s^2]$, $I_{11} = 0.01355\,[kg \cdot m^2]$, $I_{22} = 0.00480\,[kg \cdot m^2]$, $I_{33} = 0.00593\,[kg \cdot m^2]$, $\ell = 0.165\,[m]$)

The coordinate systems and free body diagram for OpenROV are shown in Figure 4. Based on the preceding mathematical foundation, we can describe the dynamic model of OpenROV (Fig 4). $F_i, (i = 1, 2, 3)$ and $T_i, (i = 1, 2, 3)$ of Figure 4, represent vertical forces and moment, respectively. $F_i, (i = 1, 2, 3)$ and $T_i, (i = 1, 2, 3)$ are defined in a similar manner [7]. Each motor of OpenROV has an angular speed ω_i and produces a vertical force F_i according to:

$$F_i = k_{Fi}\omega_{Mi}^2, \quad i = 1, 2, 3, \tag{24}$$

Experimentation with a fixed motor in a steady state shows that $k_F \approx 7.3 \times 10^{-6} \frac{N}{rpm^2}$. The motors also produce a moment according to:

$$T_i = k_{Ti}\omega_{Mi}^2, \quad i = 1, 2, 3, \tag{25}$$

The constant, k_T, is determined to be approximately $1.5 \approx 10^{-8} \frac{Nm}{rpm^2}$ by matching the performance of the simulation to the real system. For the rigid body kinetics of Eq.(3) and Eq.(17), taking into consideration gravitational or buoyancy terms, system inertia matrix (including added mass), system moment matrix (including added moment), viscous damping, current loads, and an assumption of $\ddot{\boldsymbol{u}} = 0$, we arrive at the equations of motion for OpenROV, as follows:

$$\begin{aligned}
&(\dot{\boldsymbol{M}} + \boldsymbol{M}_{add})(\ddot{\boldsymbol{r}} + 2\dot{\boldsymbol{u}}) \\
&= (\rho V_{vol} - m)g\boldsymbol{e}_3 - k_D|\dot{\boldsymbol{r}} - \boldsymbol{u}|(\dot{\boldsymbol{r}} - \boldsymbol{u}) \\
&+ k_L|\dot{\boldsymbol{r}} - \boldsymbol{u}|^2\Big(\sum_{i=1}^{3} \boldsymbol{B}\boldsymbol{E}_i - (\dot{\boldsymbol{r}} - \boldsymbol{u}, \boldsymbol{B}\boldsymbol{E}_i)\frac{\dot{\boldsymbol{r}} - \boldsymbol{u}}{|\dot{\boldsymbol{r}} - \boldsymbol{u}|}\Big) \\
&- \boldsymbol{\Gamma}_t(\dot{\boldsymbol{r}} - \boldsymbol{u}) + \boldsymbol{B}(-(-F_1\boldsymbol{E}_1 - F_2\boldsymbol{E}_1) + F_3\boldsymbol{E}_3),
\end{aligned} \tag{26}$$

$$\begin{aligned}
(\hat{\boldsymbol{I}} + \hat{\boldsymbol{I}}_{add})\dot{\boldsymbol{\Omega}} + [\boldsymbol{\Omega}, (\hat{\boldsymbol{I}} + \hat{\boldsymbol{I}}_{add})\boldsymbol{\Omega}] &= -\boldsymbol{\Gamma}_r\dot{\boldsymbol{\Omega}} + (T_1 - T_2)\boldsymbol{E}_1 \\
&+ [\ell\boldsymbol{E}_2, F_2\boldsymbol{E}_1] + [-\ell\boldsymbol{E}_2, F_1\boldsymbol{E}_1].
\end{aligned} \tag{27}$$

m: the gross weight of OpenROV, $\boldsymbol{M} = m \times Unitmatrix$, ρ: fluid density, V_{vol}:the volume of the fluid displaced by OpenROV, g: the gravitational acceleration, $\rho V_{vol} g$: buoyancy, k_D: drift force coefficient function, k_L: lift force coefficient function, Γ_t, Γ_r: viscous damping coefficient functions, $\boldsymbol{M}_{add}$: added mass matrix, $\hat{\boldsymbol{I}}_{add}$: added moment of inertia, $k_D = k_D(\boldsymbol{u}, \ddot{\boldsymbol{r}}, \psi, \theta, \phi)$, and $k_L = k_L(\boldsymbol{u}, \ddot{\boldsymbol{r}}, \psi, \theta, \phi)$. Notice that M_{add}, $\hat{I}_{add}$, Γ_t and Γ_r are diagonal matrices.

Since the operator $\boldsymbol{B}$ states variables $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$ are expressed as the functions of $(\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi)$, the state equation of OpenROV can be rewritten as equations of the function of $(\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi)$. Note that OpenROV can shift the direction of rotation of its motors using $s_i, (i = 1, 2, 3)$. When OpenROV moves forward, however, Motor 1 and Motor 2 rotate at differential directions to maintain the balance of OpenROV. Furthermore, the propeller pitch of Motor 1 is different from the propeller pitch of Motor 2.

$$\dot{\boldsymbol{x}} = F(\boldsymbol{x}) + G(\boldsymbol{x}, \boldsymbol{u}), \tag{28}$$

$$\boldsymbol{x}^{\mathrm{T}} = (\dot{\psi}, \dot{\theta}, \dot{\phi}, \psi, \theta, \phi), \tag{29}$$

$$\delta\boldsymbol{x}^{\mathrm{T}} = (\delta\dot{\psi}, \delta\dot{\theta}, \delta\dot{\phi}, \delta\psi, \delta\theta, \delta\phi,), \tag{30}$$

$$\boldsymbol{u}^{\mathrm{T}} = ((-1)^{s_1}\omega_{M1}^2, (-1)^{s_2}\omega_{M2}^2, (-1)^{s_3}\omega_{M3}^2), \tag{31}$$

$$F(\boldsymbol{x}) = \begin{bmatrix} f_\psi(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ f_\theta(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ f_\phi(\dot{\psi}, \dot{\theta}, \dot{\phi}, \theta, \phi) \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}, \tag{32}$$

$$\boldsymbol{B}_G = \begin{bmatrix} [\frac{1}{(\sin^2\theta - 1)(I_{33} + I_{a33})}]\boldsymbol{B}_{G\psi}^{\mathrm{T}} \\ [\frac{1}{I_{33}I_{a33}}]\boldsymbol{B}_{G\theta}^{\mathrm{T}} \\ [\frac{1}{(\sin^2\theta - 1)(I_{11} + I_{11a})(I_{33} + I_{33a})}]\boldsymbol{B}_{G\phi}^{\mathrm{T}} \\ [0^{\mathrm{T}}] \\ [0^{\mathrm{T}}] \\ [0^{\mathrm{T}}] \end{bmatrix}. \tag{33}$$

The variational equation (34) that can be used to control the OpenROV is described by

$$\frac{d}{dt}\delta\boldsymbol{x} = DF(\boldsymbol{x}_0)\cdot\delta\boldsymbol{x} + \boldsymbol{B}_G\boldsymbol{u}, \qquad (34)$$

where $DF(\boldsymbol{x}_0)$ and $\boldsymbol{x}_0$ denote the Jacobian of $F(\boldsymbol{x})$ and the driving point of $\boldsymbol{x}$. In particular, f_ψ, f_θ, and f_ϕ in $F(\boldsymbol{x})$ are easily obtained using Maple symbolic computations on $\boldsymbol{B}_{G\psi}^{\mathrm{T}}$, $\boldsymbol{B}_{G\theta}^{\mathrm{T}}$, and $\boldsymbol{B}_{G\phi}^{\mathrm{T}}$, as follows:

$$\boldsymbol{B}_{G\psi}^{\mathrm{T}} = [\ell\cos\phi\cos\theta k_{F1}, -\ell\cos\phi\cos\theta k_{F2}, -\cos\phi\cos\theta k_{M3}], \quad (35)$$

$$\boldsymbol{B}_{G\theta}^{\mathrm{T}} = [\ell\sin\phi k_{F1}, -\ell\sin\phi k_{F2}, \sin\phi k_{M3}], \qquad (36)$$

$$\begin{aligned}\boldsymbol{B}_{G\phi}^{\mathrm{T}} = [&(I_{33}+I_{a33})\sin^2\theta k_{M1} \\ &+(I_{11}+I_{a11})\ell\cos\phi\cos\theta\sin\theta k_{F1} - (I_{33}+I_{a33})k_{M1}, \\ &(I_{33}+I_{a33})\sin^2\theta k_{M2} - (I_{11}+I_{a11})\ell\cos\phi\cos\theta\sin\theta k_{F2} \\ &-(I_{33}+I_{a33})k_{M2}, -(I_{11}+I_{a11})\cos\phi\cos\theta\sin\theta k_{M3}]. \quad (37)\end{aligned}$$

4. Simulation of Motion of OpenROV

By means of numerical computations on MATLAB with ode45 solver applied to Eq. (26) and Eq. (28), the simulation results illustrated in Figure 5 and Figure 6 are obtained. The initial values are set as $\rho V_{vol} g = 23.53596\,[N]$, $k_D = 0.015$, $k_L = 0.5$, $\Gamma_t = \Gamma_r = 2$, $\boldsymbol{M}_{add} = 4$, $\hat{\boldsymbol{I}}_{add} = 0$, $\dot\psi = \dot\theta = \dot\phi = 0\,[rad/s]$, $\psi = \theta = \phi = 0\,[rad]$, $r_1 = r_2 = 0\,[m]$, $r_3 = -1\,[m]$, $\dot r_1 = \dot r_2 = \dot r_3 = 0\,[m/s]$, $u_1 = 0.1\,[m/s]$, $u_2 = -0.2\,[m/s]$, $u_3 = -0.1\,[m/s]$, $\omega_{M1} = 1000\,[rpm]$, $\omega_{M2} = 900\,[rpm]$, $\omega_{M3} = 0\,[rpm]$, and $s_1 = s_2 = s_3 = 0$.

5. Concluding remarks

The following results are obtained:

(1) We have derived the system dynamics and system control equations for an underwater robot by using the operator $\boldsymbol{B}$ together with two coordinate systems, W and w, where simple ocean currents exist. In addition, we have described the equations of motion for a real OpenROV version 2.7.

(2) We have reliably illustrated the typical behavior of the OpenROV by numerical simulations on MATLAB with an ode45 solver; however, appropriate numerical methods for reliable simulations should be investigated further.

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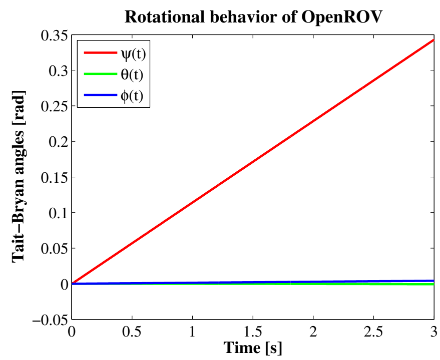

Fig 5: Simulation results of Tait-Bryan angles

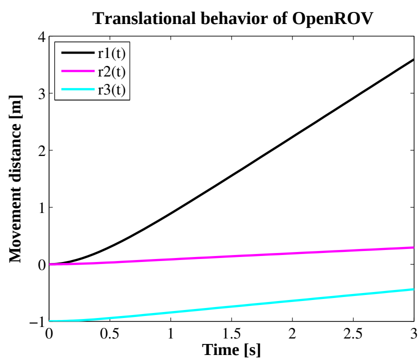

Fig 6: Simulation results for position of motion