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Investigation of Phase Itinerancy of Complex Waves on A Ring Constructed by Van Der Pol Oscillators

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Abstract—In our previous study, we analyzed synchronization phenomena and confirmed the waves which intricately behave on van der Pol oscillators coupled by inductors as a ring. In this study, we investigate the details of this complex waves by using phase states, phase differences between adjacent oscillators and instantaneous electric powers in changing the number of van der Pol oscillators.

1. Introduction

In this world, there is a synchronization phenomenon which is one of important phenomena. Synchronization phenomena are shown as flashing groups of fireflies in south-east Asian, as motion among pacemaker cells, as a relationship of the rotation and the revolution of the moon, as a generating phenomena of laser on semiconductor, and so on[1]-[4]. Especially, the synchronization phenomena can be easily observed as one of quickly and clearly phenomena in electric circuits.

In our previous study, we investigated and analyzed synchronization phenomena on van der Pol oscillators coupled by inductors as a ring. We discovered and observed continuously propagating wave motions with switching phase states between adjacent oscillators. The wave motions are named phase-inversion waves[5]. We also observed special waves which propagate a phase difference between adjacent oscillators and change to the phase-inversion waves or disappear. The waves are called phase-waves[6]. Furthermore, complex waves, which are not phase-inversion waves and phase-waves and continuously propagate, were discovered. The complex waves can be classified to two types. One of the complex waves is waves like mixed phase-inversion waves and phase-waves, and the other one is winding waves.

In this paper, we investigate these complex waves. Relationships between the complex waves and itinerancies of phase states are analyzed, and the details of complex waves are investigated by using phase differences between adjacent oscillators, instantaneous electric powers, and so on, in changing the number of oscillators.

2. Circuit model

Our circuit model is shown in Fig. 1. N van der Pol oscillators are coupled by inductors L_c as a ring. Each van der Pol oscillator is constructed by using a inductor L, a capacitor C and a nonlinear negative resistor $f(v)$. The $f(v_k)$ of k-th oscillator(Oscillator k) is assumed as Eq. (1). The Oscillator k is written as OSC_k in this paper.

$$f(v_k) = -g_1 v_k + g_3 v_k^3 \qquad (1 \le k \le N) \tag{1}$$

Circuit equations of this circuit are normalized by using Eq. (2). The normalized equations are shown in Eq. (3).

$$\begin{aligned} &i_k = \sqrt{\frac{Cg_1}{3Lg_3}}x_k, \quad v_k = \sqrt{\frac{g_1}{3g_3}}y_k, \quad t = \sqrt{LC}\tau, \\ &\alpha = \frac{L}{L_c}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}, \quad \delta = \frac{g_1^2}{3g_3}. \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{dx_k}{d\tau} &= y_k, \\ \frac{dy_k}{d\tau} &= -x_k + \alpha\left(x_a - 2x_k + x_b\right) \\ &\qquad + \varepsilon\left(y_k - \frac{1}{3}y_k^3\right). \end{aligned} \tag{3}$$

{If $k = 1,\ a = N$ and $b = 2$. If $k = N,\ a = N-1$ and $b = 1$. If $2 \le k \le N-1,\ a = k-1$ and $b = k+1$.}
The instantaneous electric powers are calculated by using Eqs. (4)–(5). The P_k shows an instantaneous electric power of OSC_k. The $P_{Lc(k-1,k)}$ shows an instantaneous electric power of coupling inductor L_c between OSC_{k-1} and OSC_k.

$$P_k = \frac{\alpha\delta}{\varepsilon}y_k\left(x_a - 2x_k + x_b\right) \tag{4}$$

$$P_{Lc(a,k)} = \frac{\alpha\delta}{\varepsilon}\left(x_k - x_a\right)\left(y_k - y_a\right) \tag{5}$$

{If $k = 1,\ a = N$ and $b = 2$. If $k = N,\ a = N-1$ and $b = 1$. If $2 \le k \le N-1,\ a = k-1$ and $b = k+1$.}

Normalized circuit equations of this circuit model are simulated by using fourth order Runge-Kutta method.

3. Phase Itinerancy of Complex Waves

Complex waves are investigated on the ring. We set 5 observation conditions as follows.

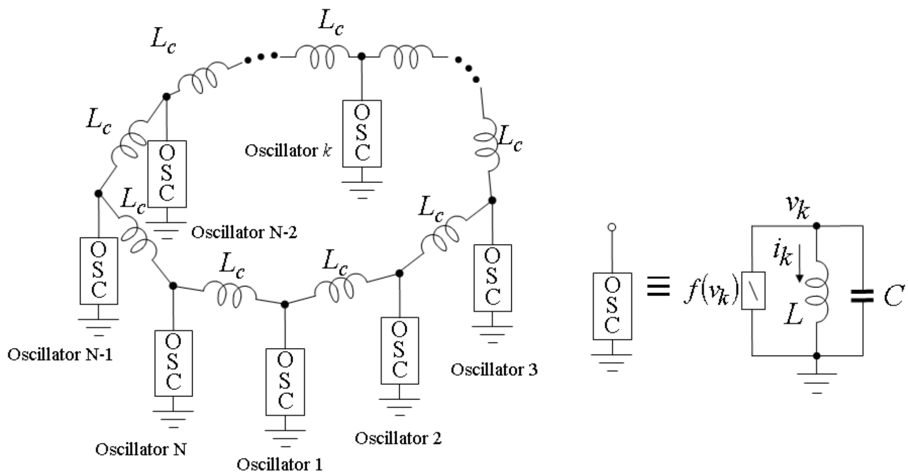

Figure 1: Circuit model

1. N is changed from 9 to 16.
2. δ is fixed as 1.
3. α is fixed as 0.50.
4. ε is fixed as 0.35.
5. The phase-inversion waves are generated in the in-phase synchronizations and initial values which a set of two phase-inversion waves propagates are set.

Some differences between phase differences itinerancies of two types of complex waves are investigated.

3.1. Changing the number of oscillator

The itinerancies of phase differences are shown in Figs. 2–7. The Figs. 2–4 are constructed by stacking long and thin boxes. In each box, sum of voltages of adjacent oscillators is shown along time. A phase state between OSC_1 and OSC_2 is shown in the top box, and a phase state between OSC_N and OSC_1 is shown in the bottom box. Therefore, black regions express the almost in-phase synchronization and white regions express the almost anti-phase synchronization. In the Fig. 2, we can confirm many waves propagate and disappear. The winding complex waves can be observed in the Figs. 3 and 4. When the number of oscillator is increased, the width of black regions and white regions are increased too(see Figs. 3–4). The Figs. 5–7 show itinerancies of phase differences of adjacent oscillators. When the Figs. 2–4 are observed, we can understand that itinerancies are very complex.

3.2. Two types of complex waves

The complex waves can be classified to two types.

Type A The waves look like mixed phase-inversion waves and phase-waves.

Type B The waves is winding and propagating.

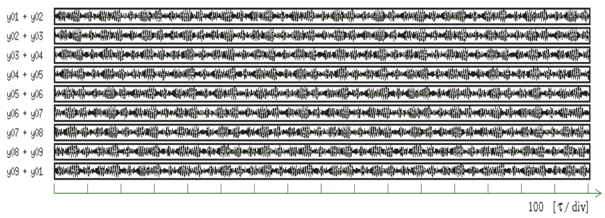

Figure 2: Complex waves(N = 9, α = 0.50, and ε = 0.35)

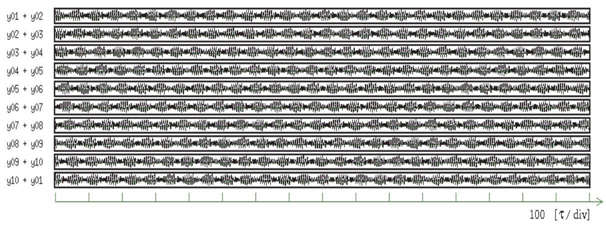

Figure 3: Winding complex waves(N = 10, α = 0.50, and ε = 0.35)

The types of the complex waves are shown in Table 1 when the number of oscillators is changed from 9 to 16. We investigate relationships between itinerancies of phase differences and the complex wave of the each type. Itineracies of phase differences of the Type A and Type B are shown in Figs. 8 and 9, respectively. We can observe phase differences which are continuously expanding along time in the Fig. 8 when the Type A complex waves are propagating. However, in the Fig. 9, we can confirm phase differences which do not continuously expands when the complex waves of Type B are propagating. Characteristics of Type A complex waves differ from characteristics of Type B complex waves.

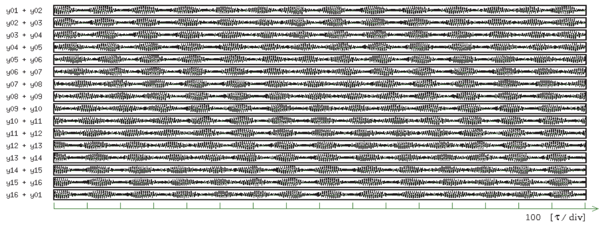

Figure 4: Winding complex waves(N = 16, α = 0.50, and ε = 0.35)

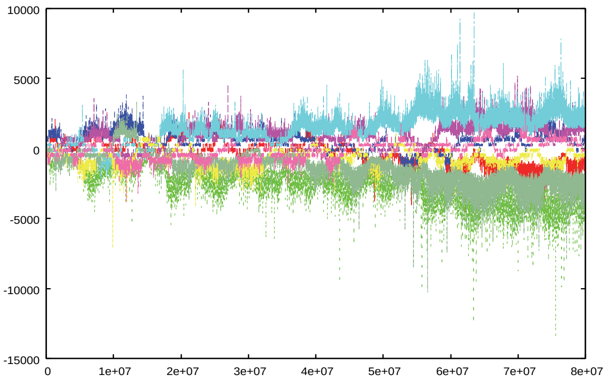

Figure 5: Itinerancies of phase differences of complex waves(N = 9, α = 0.50, and ε = 0.35)

3.3. Detail investigation by using instantaneous electric powers

The relationship between complex waves and instantaneous electric powers are investigated. The instantaneous electric powers are shown in Figs. 10–12. The Fig. 10 shows instantaneous electric powers of OSC_2 and OSC_3 of the Type A complex wave on the nine oscillators array, and the Figs. 11 and 12 show instantaneous electric powers of OSC_2 and OSC_3 of the Type B complex wave on the 10 oscillators array and the 16 oscillators array. The itinerancy of the instantaneous electric power of the Type A complex wave differ from the itinerancy of the instantaneous electric power of the Type B. The itinerancy of instantaneous electric power of the Type A is an irregular pattern and very complex(see the Fig. 10). However, the itinerancy of the Type B is not the irregular pattern(see the Figs. 11 and 12). Characteristics of Type A complex waves differ from characteristics of Type B complex waves.

In the Figs. 11 and 12, a period of the large amplitude shows the white regions of the Figs. 3 and 4, and a period of the small amplitude shows the black regions of the Figs. 3 and 4. When phase-inversion waves are propagating, the widths of white regions are fixed. However, when complex waves are propagating, the width of white regions are fluctuated. We can understand that the rate of black and white regions are almost not different when the number of oscillator is changed(see the Figs. 11 and 12).

Table 1: Type of complex waves

9	10	11	12	13	14	15	16
A	B	B	B	B	B	B	B

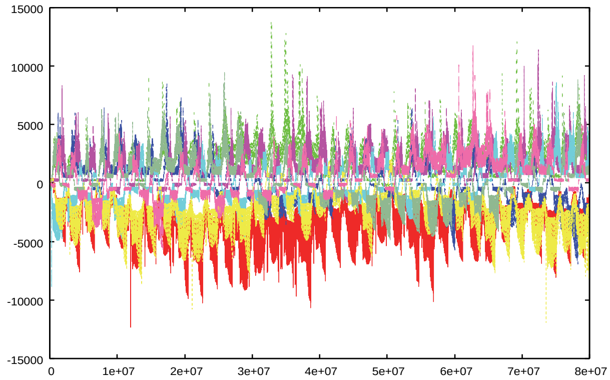

Figure 6: Itinerancies of phase difference of winding complex waves(N = 10, α = 0.50, and ε = 0.35)

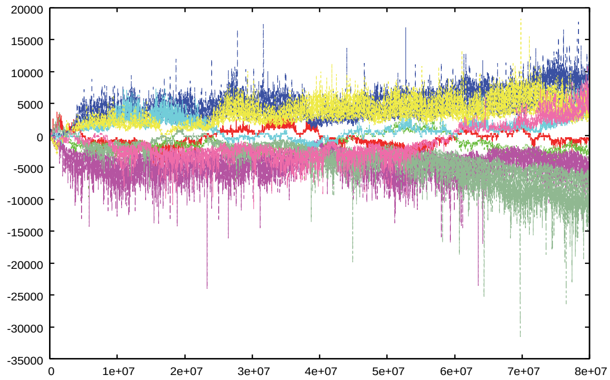

Figure 7: Itinerancies of phase difference of winding complex waves(N = 16, α = 0.50, and ε = 0.35)

4. Conclusions

We made clear that complex waves are effected by the number of oscillators and the complex waves can be classified to two types. When the Type A complex waves were propagating, the phase differences of each oscillators expanded along time. However, we made clear that the phase differences of each oscillators do not expand when the Type B complex waves propagate, Moreover, an itinerancy instantaneous electric power of Type A differed from an itinerancy instantaneous electric power of Type B. The itinerancies of instantaneous electric powers of Type B was almost regular, but itinerancies of instantaneous electric powers of Type A was not regular. We clarified that character-

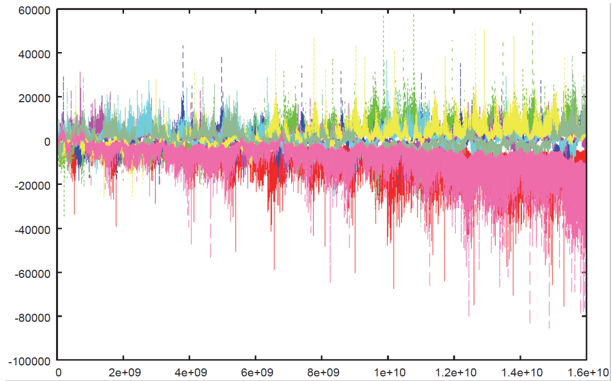

Figure 8: Itinerancy of phase difference between adjacent oscillators in Type A(N = 9, α = 0.50, and ε = 0.35)

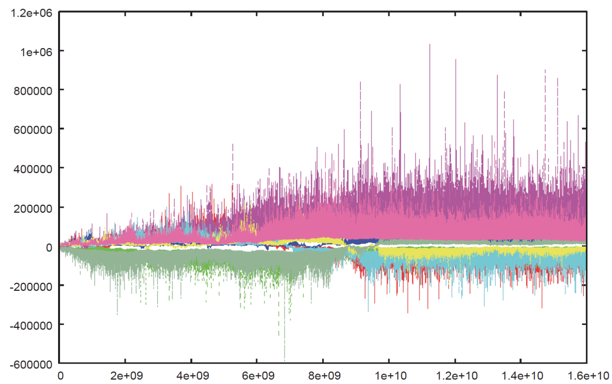

Figure 9: Itinerancy of phase difference between adjacent oscillators in Type B(N = 16, α = 0.50, and ε = 0.35)

istics of Type A complex waves differ from characteristics of Type B complex waves. Furthermore, we observed that the rate of black and white regions are almost not different when the number of oscillators is changed and Type B complex waves can be observed.

Acknowledgments

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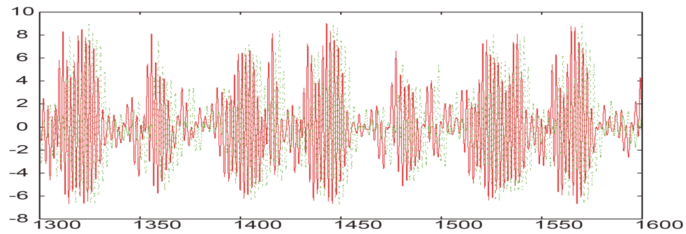

Figure 10: Instantaneous electric powers of OSC_2 and OSC_3 of Type A(N = 9, α = 0.50, and ε = 0.35)

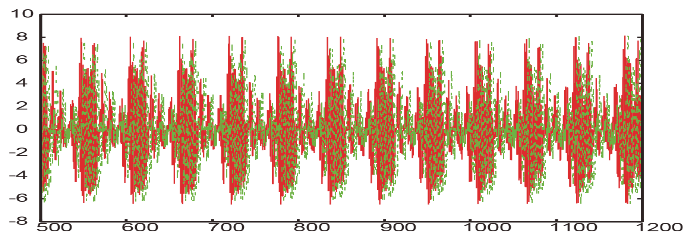

Figure 11: Instantaneous electric powers of OSC_2 and OSC_3 of Type B(N = 10, α = 0.50, and ε = 0.35)

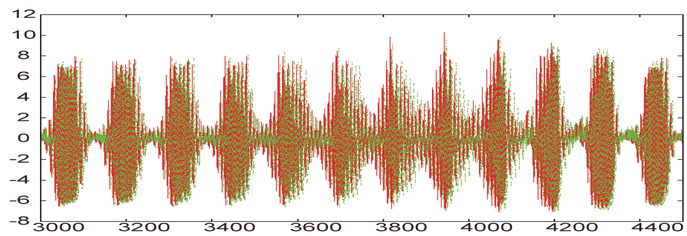

Figure 12: Instantaneous electric powers of OSC_2 and OSC_3 of Type B(N = 16, α = 0.50, and ε = 0.35)

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