2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

Fast information processing by using fast transient response in a semiconductor laser with strong optical injection

Kazutaka Kanno† and Masatoshi Bunsen†

†Department of Electronics Engineering and Computer Science, Fukuoka University
8–19–1 Nanakuma, Johnan-ku, Fukuoka, Japan
Email: kkanno@fukuoka-u.ac.jp, bunsen@fukuoka-u.ac.jp

Abstract—Reservoir computing (RC) is a machine learning paradigm based on information processing in the human brain. Recently, RC based on a semiconductor laser with time-delayed optical feedback has been proposed. In this scheme, fast transient response (a fast relaxation time) of the laser is necessary for fast information processing. By using numerical and linear stability analysis, in this study, we show that a semiconductor laser with strong optical injection can produce fast transient response. We also numerically demonstrate that RC based on a semiconductor laser with optical feedback and strong optical injection enables fast information processing.

1. Introduction

Delay-based Reservoir Computing (RC) has been proposed as an information processing method using time-delayed dynamical systems [1–4]. RC is a machine-learning paradigm that can process empirical data and is inspired by the way that the brain processes information [5]. A time-delayed dynamical system is treated as a virtual network, where nodes are considered by temporally dividing feedback delay with a small interval θ, which is called a node interval. When an input signal is injected into the time-delayed dynamical system, the system produces a transient response and virtual node states can be obtained from the response. The output of RC is given by a weighed linear combination of the virtual node states, where the weights are decided in a training procedure.

The node interval $\theta = 0.2 \cdot T_{ro}$ has been selected in some literatures [1, 3], where T_{ro} is the characteristic time scale of the system's relaxation oscillations (ROs). For smaller θ, dynamical systems cannot respond to an input signal. For larger θ, the connectivity among the virtual nodes is lost. We can increase the information processing speed in RC using a small θ because a time for processing an input data point corresponds to $N \cdot \theta$, where N is the number of nodes. Therefore, time-delayed dynamical systems with fast ROs are required to enable a smaller θ. In RC using a semiconductor laser with time-delayed optical feedback, $\theta = 0.2$ ns has been achieved [3]. The laser's RO frequency corresponding to a few GHz enables the small θ.

It has been studied that strong optical injection can enhance the RO frequency of an optically injected semiconductor laser [6]. In this study, we show that the node interval θ can be reduced in RC based on a semiconductor laser with time-delayed optical feedback and optical injection when the optical injection strength is strong.

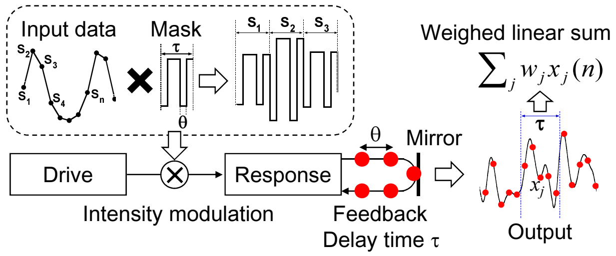

Figure 1: Schematic diagram of our RC based on coupled semiconductor lasers.

In addition, we investigate the dependence of the performance of our RC on the injection strength and the initial optical frequency detuning between the laser with optical injection and the injected light. It has been known that the initial detuning affects synchronization properties and consistency in coupled semiconductor lasers [7,8]. It is expected that the initial detuning also affects transient dynamics in the laser and the information processing performance of RC.

2. Numerical simulation method for RC with semiconductor lasers

2.1. Delay-based RC method

Figure 1 shows the schematic diagram of our RC system. The system is composed of an input layer, a reservoir, and an output layer. The reservoir is a semiconductor laser (called the response) with time-delayed optical feedback and optical injection from another laser (called the drive). N virtual nodes are considered by temporally dividing the feedback delay time τ. A time interval for the division is θ and is called the node interval.

In the input layer, time-discrete input data s_n (n is the discrete time) are multiplexed by a temporal mask signal. The mask signal is a step waveform which has a period τ. The step interval of the mask signal is equal to the node interval θ. The mask values are randomly selected from the values $\{-1, -0.6, 0.6, 1.0\}$.

A weighed linear combination of virtual node states is calculated in the output layer and is the output of the reservoir. The output $y(n)$ for the n-th input data is given by the following equation,

$$y(n) = \sum_{j=1}^{N} w_j x_j(n), \quad (1)$$

where x_j are the node states and w_j are the weights for j-th node. The node states x_j are sampled at the center of the node interval θ in the temporal output of the response laser. The weights w_j are trained by minimizing the mean-square error between the target function $\bar{y}(n)$ and the reservoir output $y(n)$ as follows,

$$\frac{1}{N_{tr}} \sum_{j=1}^{N_{tr}} (y(n) - \bar{y}(n))^2 \to min, \quad (2)$$

where N_{tr} is the number of input data for training.

To evaluate the performance of our RC scheme, we use the Santa-Fe time-series prediction task [1–4]. The aim of the task is to perform single-point-prediction of chaotic time-series. The time-series is generated from a far-infrared laser. We use 3,000 points for training and 1,000 points for testing.

The performance of the prediction task is quantitatively evaluated by using the normalized mean-square error (NMSE) as follows,

$$NMSE = \frac{1}{N_{te}} \frac{\sum_{j=1}^{N_{te}} (y(n) - \bar{y}(n))^2}{\sigma^2}, \quad (3)$$

where N_{te} is the number of input data in the test procedure. σ is the standard deviation of $\bar{y}(n)$. The NMSE represents the difference between the target $\bar{y}(n)$ and the output $y(n)$ of RC, and a NMSE close to zero indicates a low prediction error.

2.2. Numerical model of the response laser

We consider two unidirectionally coupled semiconductor lasers, which are called drive and response. The drive laser solitarily operates so that the laser shows temporally constant intensity. The constant output of the drive laser is unidirectionally injected into the response laser, which also have time-delayed optical feedback. We consider the dynamics of the response laser since the output of the drive laser is constant. The rate equations for the response laser are written as follows [9]:

$$\begin{aligned}\frac{dE_r(t)}{dt} &= \frac{1+i\alpha}{2}\left[\frac{G_N(N_r(t)-N_0)}{1+\epsilon|E_r(t)|^2} - \frac{1}{\tau_p}\right]E_r(t) \\ &+ \kappa E_r(t-\tau)\exp[-i\varphi] \\ &+ \sigma(1+S(t))A_d \exp[i\Delta\omega t] + \xi(t), \quad (4)\end{aligned}$$

$$\frac{dN_r(t)}{dt} = J_r - \frac{N_r(t)}{\tau_s} - G_N(N_r(t)-N_0)|E_r(t)|^2, \quad (5)$$

where E is the slowly varying complex electric field amplitude and N is the carrier density. The subscripts d and r represent the drive and response lasers, respectively. G_N is the gain coefficient, N_0 is the carrier density at transparency, α is the linewidth enhancement factor, τ_p is the photon lifetime, τ_s is the carrier lifetime, and J_r is the injection current of the response laser. The injection current is given by $J_r = 1.05J_{th}$, where J_{th} is the injection current at the lasing threshold. ω_r is the angular optical frequency of the response laser. These parameter values are set to the same as in [8].

Optical feedback is related to the second term in the right hand side of Eq. (4). κ and τ in the term are the feedback strength and the feedback delay time, respectively. The delay time is given by the product of the number of nodes N and the node interval θ in RC. In this study, the number of nodes is two hundreds and the node interval is varied, which results in $\tau = 200 \cdot \theta$. φ is the feedback phase and fixed at zero for simplicity.

The third term in the right hand side of Eq. (4) represents optical injection, through which an input signal is injected. σ is the optical injection strength. $\Delta\omega$ is the angular optical frequency detuning between the drive and response lasers and given by $\Delta\omega = 2\pi\Delta f_{ini}$, where Δf_{ini} is the initial optical frequency detuning and is changed in our study. A_d is the constant electric field amplitude of the drive laser and is calculated from steady state solutions of a solitary laser. In the calculation of the solutions, the injection current J_d of the drive laser is $1.30J_{th}$ and other parameter values are the same as the response laser.

3. Numerical results on RC for chaotic time-series prediction task

3.1. Dependence of RC performance on node interval

We numerically show that strong optical injection induces fast oscillations in the transient response of the laser, which results in a broad probability distribution in node states. We also investigate the dependence of the NMSE on the node interval θ for weak and strong injection strengths. It is shown that a minimum NMSE is obtained at a smaller θ for strong optical injection comparing with weak one.

We firstly show the temporal waveforms of the intensity $I(t) = |E_r(t)|^2$ in the response laser when the input signal shown in Fig. 2(a) is injected. Figures 2(b) and 2(c) show the temporal waveforms when the node interval θ is 0.03 ns. The optical injection strengths are 5 ns^{-1} and 40 ns^{-1} for Figs. 2(b) and 2(c), respectively. The red circles represent the node states. Since the number of nodes is two hundreds, the feedback delay time is $\tau = 200\dot{\theta} = 6$ ns. The initial optical frequency detuning Δf_{ini} is fixed at 0 GHz. The feedback strengths κ are 1 ns^{-1} and 22 ns^{-1} for (b) and (c), respectively.

For the weak injection strength in Fig. 2(b), oscillations in the waveform are slow in comparison with the input sig-

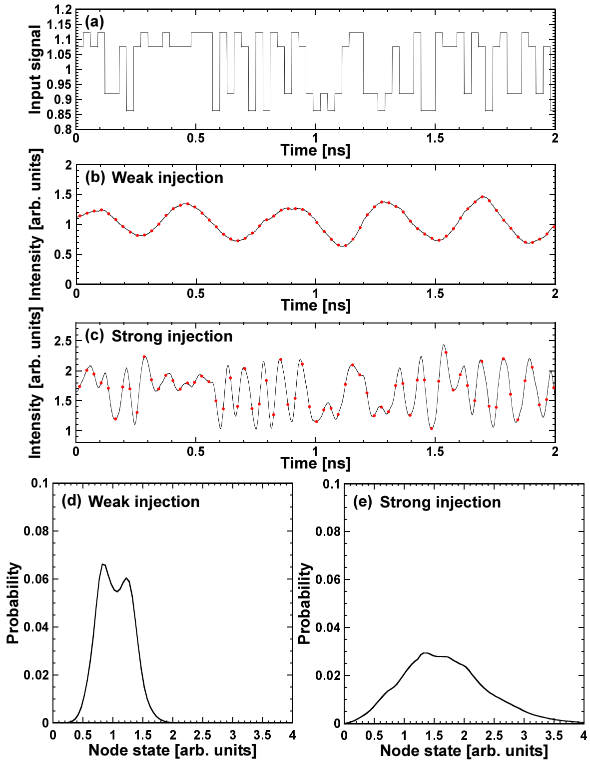

Figure 2: (a) A temporal waveform of a masked input signal. (b), (c) The temporal waveforms of the response intensity. The injection strengths σ are 5 ns^{-1} and 40 ns^{-1} for (b) and (c), respectively. The red circles represent node states. The node interval θ is 0.03 ns. (d), (c) Probability distributions of the node states. The distributions (d) and (c) correspond to (b) and (c), respectively.

nal. On the other hand, fast oscillations corresponding to the input signal are observed in the waveform shown in Fig. 2(c) for the strong optical injection. The fast oscillations result from fast ROs due to the strong optical injection [6]. When the injection strength is weak, the laser cannot respond to the input signal since the RO frequency of the laser is slower than the modulation frequency of the input signal.

When oscillations in the transient response is slow, as shown in Fig. 2(b), the width of the probability distribution of node states becomes narrow. Figures 2(d) and 2(e) show the probability distributions of node states corresponding to Figs. 2(b) and 2(d), respectively. It is found that the narrow distribution is obtained for the weak injection strength in comparison with the strong one. The narrow distribution indicates that the variety of node states is not rich, which causes reduction of the RC performance. The NMSEs for the weak and strong injection strength are 0.148 and 0.038, respectively. Thus, the better performance is obtained when node states have a broad distribution.

We show the dependences of the NMSE on the node interval θ for strong and weak optical injection and investigate the interval at which the minimum value of the NMSE is obtained. Figures 3(a) and 3(b) show the dependences when the optical injection strengths are weak (σ = 5 ns^{-1}) and strong (σ = 40 ns^{-1}), respectively. The minimum NMSEs are obtained at θ = 0.18 ns and 0.03 ns for the weak and strong injection strength, respectively. This result indicates that strong optical injection enables us to use a small θ.

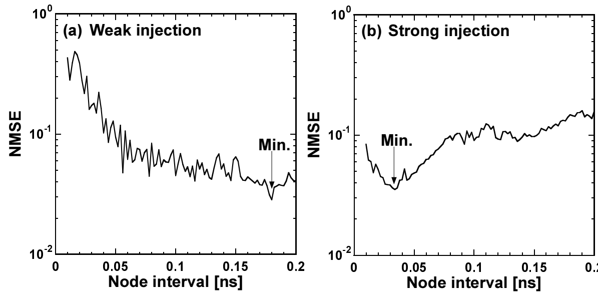

Figure 3: NMSE as a function of the node interval θ. The injection strengths σ are 5 ns^{-1} and 40 ns^{-1} for (a) and (b), respectively. The delay time is also varied with the change of θ through the relationship of $\tau = N \cdot \theta$.

3.2. Dependence of RC performance on coupling parameters

Strong optical injection can induce injection locking in coupled semiconductor lasers. In RC with the laser, injection locking is necessary for consistency and keeping the laser stable when the injected light into the laser is not modulated. The injection strength σ and the initial optical frequency detuning Δf_{ini} are important parameters for injection locking. In this section, we show injection locking region in the two-dimensional parameter space and investigate the dependence of the NMSE on the two parameters.

Injection locking can be identified by the optical frequency detuning between the drive and response lasers under coupling. The detuning Δf_c is given by the following equation,

$$\Delta f_c = \Delta f_{ini} + \frac{\Delta\phi(t) - \Delta\phi(t - T_a)}{2\pi T_a}, \quad (6)$$

where $\Delta\phi(t)$ is the phase difference between the drive and response lasers and is given by $\phi_d(t) - \phi_r(t)$. $\phi(t)$ is the phase of the complex electric field amplitude $E(t)$ and the subscripts d and r represent the drive and response lasers, respectively.. Since the optical output of the drive laser is temporally constant, $\phi_d(t)$ is also temporally constant, which results in $\Delta\phi(t) = -\phi_r(t)$. T_a is a time for the convergence of Δf_c and T_a = 50000 ns is used in our numerical simulation. $\Delta f_c = 0$ indicates the match of the optical frequencies of the drive and response lasers, that is, injection locking.

Figure 4(a) shows injection locking region on the two-dimensional space of the injection strength σ and the ini-

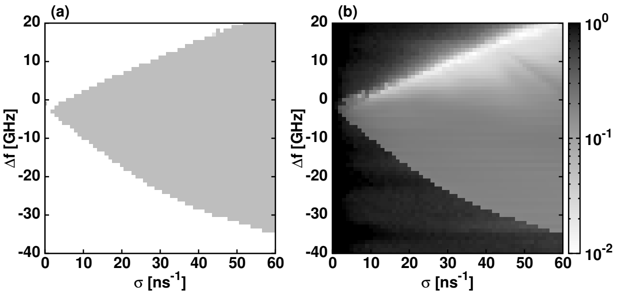

Figure 4: (a) Two-dimensional map of $|\Delta f_c| \leq 0.1$ GHz on the parameter space of the injection strength σ and the initial optical frequency detuning Δf_{ini}. A region with $|\Delta f_c| \leq 0.1$ GHz is shown by the gray color. (b) NMSE corresponding to (a).

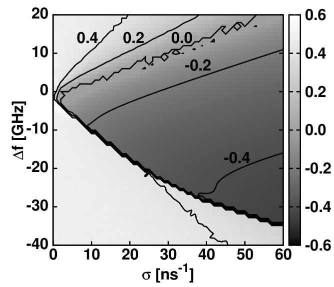

Figure 5: Two-dimensional map of the conditional Lyapunov exponent λ_c corresponding to Fig. 4.

tial optical frequency detuning Δf_{ini} when the injected light into the response laser is not modulated. In the gray region, $|\Delta f_c| \leq 0.1$ GHz is obtained and Δf_c close to zero results from injection locking. An asymmetric property for Δf_{ini} is observed and negative detuning makes it easy to achieve injection locking. This property results from a large α of the laser.

The two-dimensional map of the NMSE corresponding to Fig. 4(a) is shown in Fig. 4(b). White and black colors in the map represent small and large NMSEs, respectively. A large NMSE represented by the black color is obtained outside the gray region shown in Fig. 4(a). It indicates that the performance of RC deteriorates when injection locking does not occur. A small NMSE represented by the white color is obtained near the boundary between the gray and white regions shown in Fig. 4(a) inside the gray region. It's worth noting that a small NMSE is not obtained at the side of negative detunings but is obtained at the side of positive ones.

The dependence of the NMSE on Δf_{ini} can be explained using the conditional Lyapunov exponent [8] for a synchronized solution between the response laser and its auxiliary system [10]. The conditional Lyapunov exponent is an exponential convergence (growth) rate of perturbations to a synchronized solution and a negative exponent indicates that the synchronized solution is stable. A negative value of the exponent is necessary for consistency. The exponent close to zero is also required.

Figure 5 shows the conditional Lyapunov exponent λ_c corresponding to the two-dimensional map of σ and Δf_{ini} when the injected light into the response laser is not modulated. From comparing to Fig. 4(a), the region with $\lambda_c \leq 0.0$ corresponds to the injection locking region shown by the gray region. λ_c around the boundary with $\lambda_c = 0.0$ shows gradual changes at the side of positive detunings and sudden changes at the side of negative ones. Thus, a negative λ_c close to zero is obtained at the side of positive detunings, which results in a small NMSE shwon in Fig. 4(b).

4. Conclusion

In this study, we numerically demonstrated RC with a semiconductor laser with time-delayed optical feedback and strong optical injection. The performance of RC was quantitatively evaluated using the chaotic time-series prediction task. It was shown that strong optical injection induces fast oscillations in the transient response of the laser, which enables a small node interval in RC. We also investigated the dependence of the RC performance on the injection strength and the initial optical frequency detuning. Strong optical injection induces injection locking in the laser and high performance of RC can be obtained in an injection locking region on the two-dimensional parameter space. A positive detuning near the boundary of injection locking is also required for high performance of RC.

References

[1] L. Appeltant, M.C. Soriano, G.V. derSande, J. Danckaert, S. Massar, J. Dambre, B. Schrauwen, C.R. Mirasso, and I. Fischer, Nat. Commun., vol. 2, pp. 468–1–6 (2011).

[2] L. Larger, M.C. Soriano, D. Brunner, L. Appeltant, J.M. Gutierrez, L. Pesquera, C.R. Mirasso, and I. Fischer, Opt. Express, vol. 20, pp. 3241–3249 (2012).

[3] D. Brunner, M.C. Soriano, C.R. Mirasso, and I. Fischer, Nat. Commun., vol. 4, pp. 1364–1–7 (2013).

[4] J. Nakayama, K. Kanno, and A. Uchida, Opt. Express, vol. 24, pp. 8679–8692 (2016).

[5] M. Rabinovich, R. Huerta, and G. Laurent, Science, vol. 321, pp. 48–50 (2008).

[6] A. Murakami, Phys. Rev. E, vol. 65, pp. 056617–1–8 (2002).

[7] A. Murakami and J. Ohtsubo, Phys. Rev. A, vol. 65, pp. 033826–1–7 (2002).

[8] K. Kanno and A. Uchida, Phys. Rev. E, vol. 86, pp. 066202–1–9 (2012).

[9] R. Lang and K. Kobayashi, IEEE Journal of Quantum Electronics, vol. 16, pp. 347–355 (1980).

[10] H.D.I. Abarbanel, N.F. Rulkov, and M.M. Sushchik, Phys. Rev. E, vol. 53, pp. 4528–4535 (1996).