A Mesh-Tearing Sub-Entire Domain Basis Function Method for Improved Electromagnetic Analysis of Strong-Coupled Cube Array

Lin LIU, Xiaoxiang HE, Chen LIU, Yang YANG
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College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics
29 Yudao ST., Baixia Dist., Nanjing 210016, China

***Abstract*- A mesh-tearing sub-entire domain (MTSED) basis function method for improved electromagnetic (EM) scattering analysis of strong-coupled finite periodic structures is proposed in this paper. By tearing the coarse mesh of classic SED basis function into several smaller ones, the modeling precision for strong-coupled arrays can be improved more than 13dB maximally with less than four times computational time increasing compared to classic SED method. The mesh-tearing technique based on both simplified SED (MTSSED) basis function and accurate SED (MTASED) basis function are also investigated in detail. The detailed algorithm is presented and the numerical results demonstrate that the proposed MTSED method is an accurate method for electromagnetic scattering analysis of strong-coupled periodic structures.**

I. INTRODUCTION

Nowadays, finite periodic structures such as photonic band-gap (PBG) crystals [1] and phased-array antennas [2], frequency selective surfaces (FSS) [3] are applied widely in electromagnetic engineering. And accurate and efficient techniques for analysis of periodic structures are always based on periodic boundary condition (PBC), which is either applied in the frequency domain in the context of the method of moments (MoM) [4], and the finite element method (FEM) [5], or implemented by using an equivalent delay condition in the time domain analyses, such as the finite difference time domain (FDTD) [6].

Among these full-wave analysis techniques, the MoM is a robust approach to deal with electromagnetic scattering of periodic structures. However, the conventional MoM requires $O(N^2)$ memory and $O(N^3)$ computational complexity, which are unaffordable for large-scale strong-coupled problems with desired accuracy. Several physical-based entire-domain basis functions have been developed to reduce the number of unknowns. For example, a macro basis function (MBF) was employed to analyze finite printed antenna arrays [7]. The synthetic basis function (SBF) is similar to MBF, which was applied in analyzing large-scale non-periodic arrays [8]. Both MBF and SBF are time consuming because they consider the mutual coupling effects in an iterative way. The characteristic basis function (CBF) is another kind of physical-based entire-domain basis function [9]. The CBF method uses a new type of high-level (secondary) basis function to calculate the mutual coupling, so the CBF can be obtained directly. For each single cell, N^2 CBFs should be considered, where N represents the number of cells. Some other techniques have also been presented to analyze the planar circuits and antenna arrays [10].

Recently, to decrease the memory requirement, an accurate sub-entire domain (ASED) basis function method [11] is proposed by Cui et al. Different from MBF and CBF, the mutual coupling effects are considered by using dummy cells. And all the elements are divided into 9 kinds according to the ASED. Consider one periodic structure with N elements and M discrete edges on each unit cell and the large-scale problem involves $N_0=NM$ unknowns. The ASED basis function decomposed the original problem into two smaller-size problems, one part contains $9M$ unknowns and the other part contains only N unknowns. To further simplify the calculation procedure, a simplified sub-entire domain (SSED) basis function method is proposed [12] which can reduce the number of unknowns. When computing the basis function, the mutual coupling among the elements is neglected directly so that it is shared by all the elements. We compared and discussed the efficiency and accuracy of the ASED and SSED methods in [13].

In this paper, a mesh-tearing sub-entire domain basis function method (MTSED) is proposed to analyze the scattering of strong-coupled finite periodic structures. And with slight computational complexity increasing, the modeling accuracy can be improved significantly.

II. THEORY

a. MTSED basis function method

Consider a two-dimensional periodic structure with N perfectly electric conducting (PEC) cells in free space is illuminated by a plane wave, the magnetic field integral equation (MFIE) can be written as:

$$\frac{1}{2}\mathbf{J}(\mathbf{r})-\frac{1}{4\pi}\mathbf{n}(\mathbf{r})\times\nabla\times\int_S G(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')d\mathbf{r}'=\mathbf{n}(\mathbf{r})\times\mathbf{H}^{inc}(\mathbf{r}) \quad (1)$$

Where $\mathbf{H}^{inc}(\mathbf{r})$ represents the incident wave, $\mathbf{n}(\mathbf{r})$ is the unit normal vector, $G(\mathbf{r},\mathbf{r}')$ is the Green's function in free space,

$\mathbf{J}(\mathbf{r})$ represents the electric current distribution of the whole periodic structure, which can be written as:

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N} \beta_n \mathbf{f}_n(\mathbf{r}) \tag{2}$$

where $\mathbf{f}_n(\mathbf{r})$ represents the SED basis function on the nth cell which can be computed by conventional MoM with dense mesh (usually about $\lambda/10$), β_n denotes the corresponding coefficient. After using the Galerkin's procedure and the SED basis function, (1) can be written in a matrix form:

$$[\mathbf{Z}]\cdot[\beta] = [\mathbf{V}] \tag{3}$$

Where $[\beta]=(\beta_1,\beta_2,\beta_3\cdots\beta_N)^T$ is the expansion coefficient vector and the elements of $[\mathbf{V}]$ and $[\mathbf{Z}]$ can be written as:

$$V_m = \iint_{f_m} \mathbf{f}_m(\mathbf{r})\cdot[\mathbf{n}(\mathbf{r})]\times\mathbf{H}^{inc}(\mathbf{r})d\mathbf{r} \tag{4}$$

$$Z_{mn} = \frac{1}{2}\int_{f_m=f_n} \mathbf{f}_m(\mathbf{r})\cdot\mathbf{f}_n(\mathbf{r})d\mathbf{r}' + \frac{1}{4\pi}\int_{f_m} \mathbf{f}_m(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})\times\int_{f_n} \mathbf{f}_n(\mathbf{r})\nabla' G(\mathbf{r},\mathbf{r}')d\mathbf{r}'d\mathbf{r} \tag{5}$$

In the SED method, the current distribution (SED basis function) on each cell that is very important to the final solution is computed with approximate (ASED) or even no (SSED) mutual coupling consideration. As a result, the error caused by the approximation is inevitable. Furthermore, the size of the coarse mesh for the whole array analysis is chosen the same as the unit cell for the whole array analysis, which is much larger than $\lambda/10$. For weakly-coupled arrays, SED method in [11], [12] can approach acceptable accuracy. However, for strong-coupled arrays, SED method even can not make the matrix solver converge to the real solution.

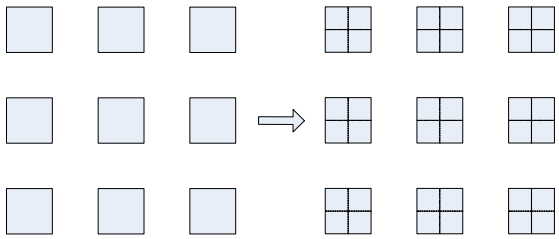


Figure 1. Periodic structures applied with MTSED method when k is 4.

In order to improve the computational accuracy, a MTSED method is proposed in this paper. According to the main idea of MTSED method, the coarse mesh on each cell is to be torn into k parts, as shown in Fig.1. With the increase of mesh density, the accuracy is improved. Theoretically, when the coarse mesh is refined to be about $\lambda/10$, the MTSED method will appear to be the conventional MoM and the accuracy can be ensured. Of course, the CPU time will be unaffordable in the case of large-scale arrays.

In MTSED method, the overall electric current $\mathbf{J}(\mathbf{r})$ in equation (2) is modified as:

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{kN} \beta_n \mathbf{f}_n^T(\mathbf{r}) \tag{6}$$

Here $\mathbf{f}_n^T(\mathbf{r})$ represent the basis functions on the torn parts of each cell. Thus, (3) can be rewritten as:

$$[\mathbf{Z}^T]\cdot[\beta^T] = [\mathbf{V}^T] \tag{7}$$

Here, $[\beta^T]$ with a size of kN denotes the tearing expansion coefficient vector, the elements of $[\mathbf{Z}^T]$ and $[\mathbf{V}^T]$ are corresponding to that of $[\mathbf{Z}]$ and $[\mathbf{V}]$ respectively, except that the integration is over the torn parts of each cell. According to MTSED method, the memory consumption is expanded k^2 times for matrix storage. The CPU time for matrix filling is the same as classic SED which is the most time consuming procedure and the CPU time for matrix solving is increased because of the expansion of matrix scale. As a result, the increasing of CPU time is slight. The MTSED method is a compromise consideration between MoM full wave analysis and classic SED method.

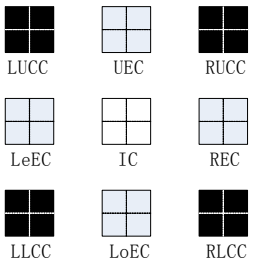


Figure 2. Periodic structures with MTSED method when k is 4.

b. MTSSED basis function method

The SSED method is a simplified solution while the electric current distribution on each element is computed by conventional MoM without considering the coupling from neighboring cells. With the coarse mesh of SSED method torn into k parts, the unknowns are increased from N to kN and the CPU time and memory consumption are also increased for accurate array analysis. Not only the current magnitude and phase are amended as it works in classic SSED, but also the current distribution on the whole array is modified. And the precision can be improved consequently as we presented in [14].

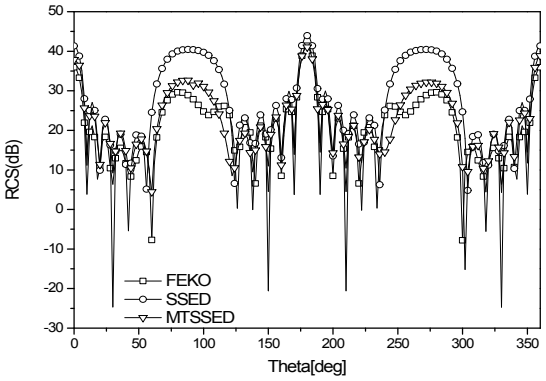
c. MTASED basis function method

The mutual coupling of neighbouring elements is considered approximately in ASED method for basis function calculation. The whole elements are catalogued into 9 kinds of basis functions as illustrated in Fig.2. The relatively accurate electric current distributions on the interior cell (IC), the left edge cell (LeEC), the right edge cell (REC), the upper edge cell (UEC), the lower edge cell (LoEC), the left upper corner cell (LUCC), the right upper corner cell (RUCC), the left lower corner cell (LLCC), and the right lower corner cell (RLCC) are obtained respectively with conventional MoM. Then the coarse mesh on each cell is torn into k parts according to MTASED and they are substituted as basis functions in equation (6). With the coupling from neighbouring cells taken into account, the accuracy is further improved.

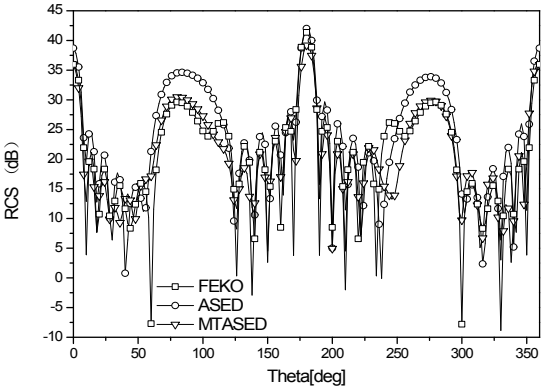
III. Numerical Results and Discussions

Metal cube array

Firstly, we give the bistatic radar cross section (RCS) of a strong-coupled 6×6 metal cube array. And the element dimension is $0.5\lambda\times0.5\lambda\times0.5\lambda$ and the gap between neighbouring element is 0.5λ in normal incidence case. The number of torn meshes (k) set to be 40. As a result, the coarse mesh that is set to be $0.4\lambda\times0.4\lambda\times6$ in classic SED method has been refined to be about $0.13\lambda\times0.13\lambda$ in MTSED method and the memory consumption is expanded to 1600 times for matrix storage. From Fig.3 (a) we can observe that the RCS curve obtained by MTSSED method is more accurate than that of SSED method compared to FEKO with 0.1λ mesh size. There is about 5.39dB error at 0° and 11.99dB error at 90° with SSED method which is mainly caused by the ignorance of mutual coupling from neighbouring elements in the simplified sub-entire domain basis function computation. With the application of MTSSED method, the accuracy has been improved significantly for the current distribution on each cell is modified with relatively dense mesh. The error at 0° and 90° has reduced to 1.91dB and 3.13dB, respectively. The average error for all angles has decreased from 2.68dB to 0.89dB. Fig.3 (b) shows the RCSs calculated by ASED method, MTASED method and FEKO with the same mesh. It is obvious that the RCS calculated by MTASED method is more consistent with FEKO full wave analysis. The error at 0° has reduced from 2.77dB to 0.02dB and the error at 90° has reduced from 4.65dB to 1.23dB. Consequently, the MTASED method is more accurate than MTSSED method for the more accurate basis function.



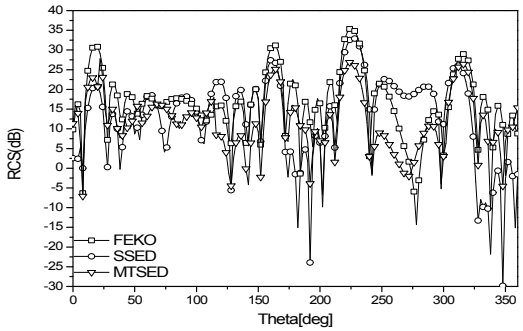
(a)



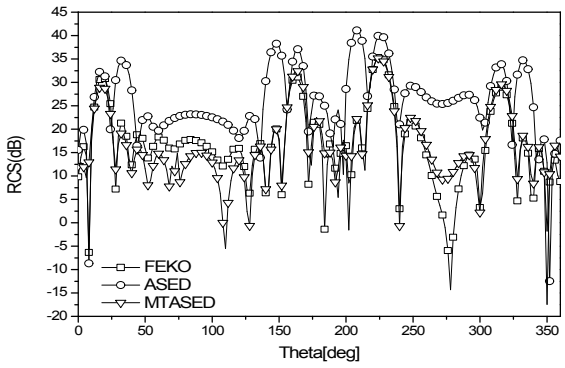
(b)

Figure.3. RCSs of the 6×6 cube array in normal incidence case. (a) MTSSED method. (b) MTASED method.

When the angle of incident wave is oblique, the increase of multiple interactions between array elements will be enhanced. Therefore, the accuracy of SED method will usually be declined. Fig.4 shows the bistatic RCSs of a 6×6 cube array consists of associated elements calculated by FEKO with the same mesh, SED method and MTSED method in oblique incidence case (θ=45°). The torn mesh on each single cell is about $0.15\lambda\times0.15\lambda$ (k=42). From Fig.4 (a) we can observe that the error at 0° has decreased from 8.81dB to 1.54dB and the error at 90° has reduced from 5.06dB to 3.99dB with MTSSED method and there is significant improvement of accuracy at 74° with the error decreased from 11.45dB to 0.66 dB. When MTASED method is employed, as shown in Fig.4 (b), the error at 0° has decreased from 5.73dB to 2.34dB and the error at 90° has reduced from 5.74dB to 2.34dB and there is significant improvement of accuracy at 36° with the error decreased from 15.22dB to 1.86dB. Compared to SED method, the computational accuracy has been improved obviously with MTSED method in oblique incidence case. The accuracy is expected to be further improved with denser torn meshes or with basis function computed with more layers of dummy elements.



(a)



(b)

Figure.4. RCSs of the 6×6 cube array in normal incidence case (θ=45°). (a) MTSSED method. (b) MTASED method.

To test the efficiency of the proposed method, we record the computation time when computing the RCSs of different arrays with SED method and MTSED method, as shown in Table I. Our codes are implemented by a personal computer with Intel(R) Core(TM) 2 Duo CPU T6400-2.00GHz and 2GB RAM. We can conclude that there is about 2 times and 5 times computation time increasing compared to classic SED method with the application of MTSSED and MTASED method, respectively. Although the efficiency of MTASED method is lower than that of MTSSED method with the same torn meshes, the MTASED method can usually achieve a better accuracy. To improve the efficiency of our proposed method, parallel programming such as message passing interface (MPI) [15], shared address space (OpenMP) [16] and meta process model (MpC) [17] can be incorporated.

TABLE I
COMPARISON OF COMPUTATION TIME OF 6×6 CUBE ARRAY

Theta	Time[s]			
	SSED	MTSSED	ASED	MTASED
0°	224.57	343.85	205.75	1071.64
45°	223.56	589.35	211.68	972.51

IV. CONCLUSION

In this paper, the MTSED method is proposed by tearing the coarse mesh of SED basis function on each unit cell into several parts. The numerical results show that the modelling precision for strong-coupled arrays can be improved more than 13 dB maximally with less than four times computational time increasing compared to classic SED method which demonstrate that the proposed MTSED method is an accurate method for electromagnetic scattering analysis of strong-coupled finite periodic structures.

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