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# Network Coding and Dynamical Systems

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**Abstract**—This work considers a kind of empirical tradeoff in network coding for dynamical systems, offering some insights for real world applications. In particular, we optimize its computational cost and the transaction delay in the network that is caused by the use of our recursive method. Future application would be also mentioned from a point of view of information network sciences.

## 1. Introduction

The rise of the notion "Internet of Things" has been gathering attention to the wide range of network engineers and computer scientists. At the same time, it is becoming crucial to consider how to network many devices with the limited computational resources. One of the conventional approaches would be the naive application of what is called network coding in the area of information theory. This classical method provides an efficient procedure to broadcast digital contents to many subscribers at the largest data rate possible. However, it is also known that practical encoding for typical large networks is not that easy. In this paper, we propose a recursive construction of network coding based on a graph partitioning scheme, and try to numerically optimize their sizes to enhance the overall data rate under the same computational resources.

## 2. System model

Network coding is a kind of "store and forward" type routing technology which enables us to deal with multiple inputs and outputs efficiently. The basic concept was first proposed by Ahlswede, Cai, Li, and Yeung at the outset of the 21st century [1]. After that, Li, Yeung, and Cai offered the method of linear network coding as a practical framework to realize the new routing technology [2]. Together with the linear information flow (LIF) algorithm given by Jaggi, Sanders, Chou, Effros, Egner, Jain, and Tolhuizen, it is possible for us to implement the basic idea of network coding into real world networks [3]. In this paper, we numerically examine a class of ideal networks with the following assumptions.

- We consider the 36-layered butterfly-type network as a system model.
- We use the LIF-algorithm to encode all network transactions.
- We apply simple "wait and throw" principle for the packet collision.

Notice that we choose the butterfly network as a building block for our network decomposition approach, since using this element enables us to find the trivial solution to the packet collision problem in the layered model. The details of the decomposition will be reported elsewhere.

## 3. Numerical Results

Figure 1 shows the empirical performances of the overall coding time and the system level delays given a depth of the layered network. As is expected, an increase of the element network size increases the overall coding time and decreases the systematic delay steps.

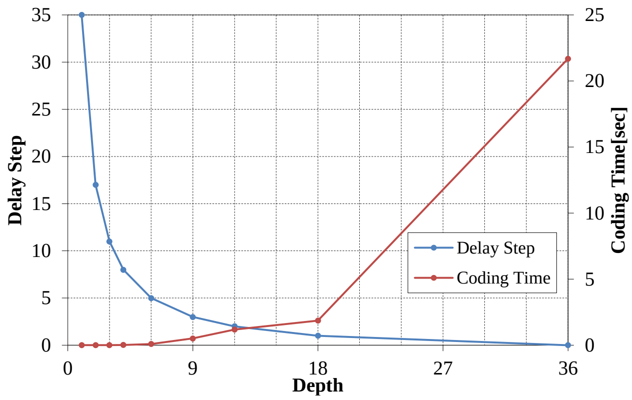


Figure 1: Empirical Performance: Red line shows the overall time required to complete the network coding at a certain depth of the unit network, while blue line represents the system level delays for typical transactions.

Figure 2 represents the empirical tradeoff between the two measures. In general, we prefer shorter coding time $x_t$ and fewer delay steps $x_d$. Write

$$\beta(x_t, x_d) = x_t + \alpha x_d \quad (0 \leq \alpha < \infty).$$

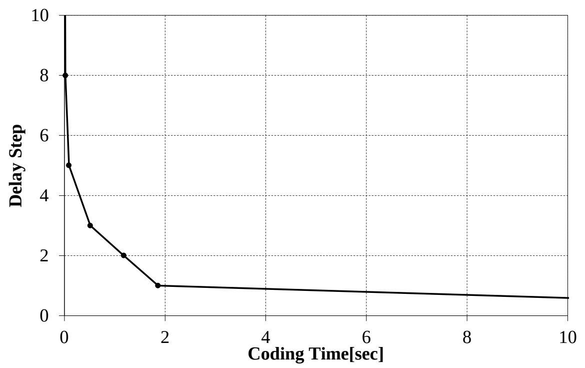


Figure 2: Empirical Tradeoff: Solid line shows Numerical tradeoff between coding time and delay steps. The transaction delay decreases as the coding time increases meaning that we use larger butterfly units for the network decomposition.

Here the parameter $\alpha$ denotes our preference for the two measures. By choosing $\alpha = 0$ we have $\beta(x_t, x_d) = x_t$, which corresponds to the user who prefers shorter coding time needing to reconfigure the dynamical network every minute. On the other hand, if we consider $\alpha \to \infty$ then $\beta(x_t, x_d)$ equals $\alpha x_d$ and this implies the user who does not like any delays in the transactions. That is, by imposing the value of $\alpha$ for the potential user preference, we could decide the optimal depth level to enhance the system performance.

## 4. Conclusion

Since there would be so many mobile devices connecting to the internet already, we need to regard networks as dynamical systems rather than static ones. In this paper, we reconsider computational cost of its overhead processes required to realize the typical network coding from a point of view of the standard pareto optimization. Natural directions for future research include generalizing the network architecture and the theoretical sophistication.

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## References

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