2016 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2016, Yugawara, Japan, November 27th-30th, 2016

Approaching Rate Distortion Bound with Reinforcement Message Passing

Takumi Fujita†, Koji Okino‡ and Tatsuto Murayama†

†Graduate School of Science and Engineering ,University of Toyama
‡Information Technology Center, University of Toyama
†Graduate School of Science and Engineering ,University of Toyama
3190 Gofuku, Toyama, 930-8555, Japan
Email: m1571118@ems.u-toyama.ac.jp, okino@itc.u-toyama.ac.jp, murayama@eng.u-toyama.ac.jp

Abstract—This work reviews the methods for lossy coding of Bernoulli(1/2) source with message passing algorithms/heuristics being imposed some reinforcement operations for an optimal convergence. In particular, we consider a kind of feedback technique and presents the optimal interval for the accurate decoding of the original binary sequence. By increasing the intensity of the reinforcement control, we always observe a drastic breakdown of the system performance at a certain level of feedback.

1. Introduction

Since the start point of information theory by Shannon, lossy encoding of binary information gathers attention to the wide range of mathematicians, engineers, and practitioners. However, at least when it comes to the practical point of view, it had been quite difficult to make a good pair of encoder and decoder for the Bernoulli(1/2) source, i.e., a class of the purely random sequences, until some breakthrough was made with what we call the message passing methods today [1, 2, 3]. In these techniques, we need to impose some reinforcement conditions for the equation system to converge. In this paper, we focus on the role of feedback operations in one of the earliest methods among such algorithms/heuristics to think over the physics behind the convergence.

2. System model

Let $\boldsymbol{J}$ be an M-bit source sequence, $\boldsymbol{\xi}$ an N-bit codeword, and $\hat{\boldsymbol{J}}$ an M-bit reproduction sequence, respectively. Being given a distortion D and a randomly-constructed Boolean matrix A of dimensionality $M \times N$, we look for the N-bit codeword sequence $\boldsymbol{\xi}$, which satisfies $\boldsymbol{J} = A\boldsymbol{\xi}$ (mod 2), where the fidelity criterion $D = E[d(\boldsymbol{J}, \hat{\boldsymbol{J}})]$ holds. We suppose that binary alphabets are drawn randomly from a non-biased source and that the Hamming distortion measure is selected for the fidelity criterion, where the matrix is characterized by K ones per row and C per column, i.e., K and C define a particular generator matrix.

3. Algorithm/Heuristics

We proposed a feedback systems model for generating the proper codewords without what we call the dimension curse [1]. The newly introduced variables $m_{\mu i}(t)$, $\hat{m}_{\mu i}(t) \in [-1, +1]$ emulate the density evolution with

$$\hat{m}_{\mu i}(t+1) = \tanh(\beta J_\mu) \prod_{i' \in \mathcal{L}(\mu) \setminus i}^{m} m_{\mu i'}(t) \tag{1}$$

and

$$m_{\mu i}(t) = \tanh\left(\sum_{\mu' \in \mathcal{M}(i) \setminus \mu} \tanh^{-1} \hat{m}_{\mu' i}(t) + \tanh^{-1} \gamma m_i(t) \right). \tag{2}$$

The above pair of equations (1) and (2) give an iterative procedure for codeword generation with

$$m_i(t) = \tanh\left(\sum_{\mu \in \mathcal{M}(i)} \tanh^{-1} \hat{m}_{\mu i}(t) + \tanh^{-1} \gamma m_i(t) \right). \tag{3}$$

Here, we write the set of codeword indexes i that participate in the source index μ by $\mathcal{L}(\mu) = \{i \mid a_{\mu i} = 1\}$ with $A = (a_{\mu i})$. Similarly, we also denote another set $\mathcal{M}(i)$ such that it defines the set of source indexes linked to the codeword index i. Practical encoding procedure for this compression model would be as follows. First, given the source sequence $\boldsymbol{J}$, we just translate the Boolean alphabets $\{0, 1\}$ into that of Ising ones $\{+1, -1\}$. Then, for a certain good pair of control parameters, β and γ, the equations (1) and (2) with (3) are recursively calculated until they converge to a fix point. Finally, according to the equation (3), we calculate the codeword sequence $\boldsymbol{\xi}$ by the Boolean translation.

4. Feedback Induced Order

Numerical experiments show that the algorithm with optimal parameter selection can achieve the bound for sparse construction of the codes [1]. Here, the optimal selection implies a good pair of β and γ with respect to the system performance measure, i.e., the resulting distortion D. It has been already reported and widely known that the optimal value of β should be determined by a set of saddle

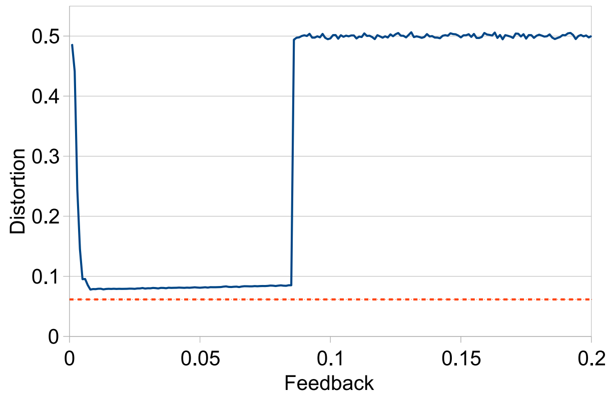

Figure 1: Empirical Performance: The feedback parameter $\gamma = 0.008$ gives the smallest distortion D around 0.08 for $K = 2$ and $C = 3$. Numerically we observe feedback induced order within the interval of $[0.008, 0.085]$. However, beyond the upper bound $\gamma = 0.085$, the system starts to converge to a non-optimal state.

point equations in the corresponding replica analysis [4]. However, when it comes to the best value of γ, we still don't have any theoretical background for the parameter selection problem. Therefore, in this paper, we examine the systematic response induced by the existence of such feedback γ. More precisely, by imposing the optimal value $\beta = 2.35$ for the case, we numerically measures the distortion D for an interval of γ. Figure 1 shows our results for the investigation. Notice here that we find an interval for nearly achieving the Shannon limit, beyond which the system suddenly loses control of decoding the original information.

5. Conclusion

In this paper, we focus on the system level effect induced by the feedback term. According to our preliminary study, it is likely that we typically observe an abrupt change of system performance at a critical value for the feedback. Future research includes the verification of this assumption using much larger scale simulations.

Acknowledgments

The authors would like to thank members in the laboratory for information theory and coding for their fruitful suggestions and comments.

References

[1] T. Murayama, "Thouless-Anderson-Palmer approach for lossy compression" *Physical Review E*, vol.69, no.3, pp.035105, 2004.

[2] M. J. Wainwright and M. Elitza, "Lossy source encoding via message-passing and decimation over generalized codewords of LDGM codes" *2005 IEEE International Symposium on Information Theory*, pp.1493–1497, 2005.

[3] S. Ciliberti, M. Mézard and R. Zecchina, "Lossy data compression with random gates" *Physical Review Letters*, vol.95, no.3, pp.038701, 2005.

[4] T. Murayama and M. Okada, "One step RSB scheme for the rate distortion function" *Journal of Physics A: Mathematical and General*, vol.36, pp.11123–11130, 2003.