Reproduction of nonlinear cochlea response by asynchronous bifurcation processor
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Abstract—Biological cochleae have highly nonlinear responses to sound stimuli, e.g., response curves (so-called tuning curves) are highly nonlinear with respect to stimulation frequencies. In this paper, the nonlinear dynamics of a Hopf-bifurcation-type cochlea model based on a concept of an asynchronous bifurcation processor is investigated. It is shown that the model can reproduce nonlinear tuning curves of not only a mammalian cochlea (cat) but also a reptilian cochlea (turtle).

1. Introduction

Biological cochleae have highly nonlinear responses to sound stimuli such as nonlinear frequency tuning curve, multi-tone suppression, first and second pitch shifts, adaptation, parallel spike density encoding, and so on [1]. Among such nonlinear responses, the nonlinear frequency tuning curve is focused in this paper. Fig. 1 shows an example of the frequency tuning curve of a reptilian cochlea (turtle). In this figure, a frequency at a minimum peak of the curve is called a characteristics frequency and is corresponding to a specific position of a basilar membrane in the cochlea, i.e., the basilar membrane works as a kind of nonlinear mechanical Fourier transformer. There are many mathematical and circuit models the cochlea [2]-[10]. One of simple-but-powerful mathematical cochlea models is the Hopf-cochlea [6]-[10]. On the other hand, in this paper, the following four discrete states \( \{X, Y, P, Q\} \) are used in a cochlea model.

\[ X \in \mathbb{Z}_N = \{0, \cdots, N-1\}, \quad Y \in \mathbb{Z}_N, \]
\[ P \in \mathbb{Z}_M = \{0, \cdots, M-1\}, \quad Q \in \mathbb{Z}_M, \]

where \( N \) and \( M \) are positive integers, which determine the resolution of the state space \( \{(X, Y, P, Q) | X \in \mathbb{Z}_N, Y \in \mathbb{Z}_N, P \in \mathbb{Z}_M, Q \in \mathbb{Z}_M\} \).

Based on the normal form of the Hopf-bifurcation, cochlea models have been presented and investigated [8]-[10].

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The discrete states $X$ and $Y$ are used to reproduce nonlinear oscillatory behaviors of the basilar membrane, and the discrete states $P$ and $Q$ are used to control velocities of the discrete states $X$ and $Y$, respectively. Then the following functions $g_x : Z_N \times Z_N \rightarrow R$ and $g_y : Z_N \times Z_N \rightarrow R$ are introduced.

\[ g_x(X, Y) = \delta l(X - N/2) - \omega l(Y - N/2) - \frac{l}{2}(X - N/2)(Y - N/2)^2, \]

\[ g_y(X, Y) = \omega l(X - N/2) - \delta l(Y - N/2) - \frac{l}{2}(Y - N/2)(X - N/2)^2, \]

where $l = m/N$, and $\delta \in R$, $\omega \in R$, and $m \in R$ are parameters. Note that the functions $g_x$ and $g_y$ are used to design a vector field and are not implemented in hardware. Letting $Z_N = \{- (M - 1), \ldots, 0, \ldots, M - 1\}$, the following functions $F_x : Z_N \times Z_N \rightarrow Z_N$ and $F_y : Z_N \times Z_N \rightarrow Z_N$ are introduced.

\[ f_x(X, Y) = \begin{cases} 
M - 1 & \text{if } \text{Int}(\frac{X}{N}, \frac{Y}{N}) \geq M - 1, \\
-(M - 1) & \text{if } \text{Int}(\frac{X}{N}, \frac{Y}{N}) \geq -(M - 1), \\
\text{otherwise}, & 
\end{cases} \]

\[ f_y(X, Y) = \begin{cases} 
M - 1 & \text{if } \text{Int}(\frac{X}{N}, \frac{Y}{N}) \geq M - 1, \\
-(M - 1) & \text{if } \text{Int}(\frac{X}{N}, \frac{Y}{N}) \geq -(M - 1), \\
\text{otherwise}, & 
\end{cases} \]

where $T_x \in R^+$ and $T_y \in R^+$ are periods of the following internal clocks $C_x(t)$ and $C_y(t)$, respectively.

\[ C_x(t) = \begin{cases} 
1 & \text{if } t = 0, T_x, 2T_x, \ldots, \\
0 & \text{otherwise}, 
\end{cases} \]

\[ C_y(t) = \begin{cases} 
1 & \text{if } t = 0, T_y, 2T_y, \ldots, \\
0 & \text{otherwise}. 
\end{cases} \]

Note that the ratio $T_x/T_y$ of the periods can be either rational (i.e., the clocks $C_x$ and $C_y$ are phase-locked or synchronized) or irrational (i.e., the internal clocks $C_x$ and $C_y$ are asynchronous). Since the clock generators are uncoupled, the period ratio $T_x/T_y$ is generically irrational and thus the internal clocks $C_x(t)$ and $C_y(t)$ are generically asynchronous. Recall that the asynchrony of the internal clocks $C_x(t)$ and $C_y(t)$ are used to realize a smooth vector field of our model. The internal clocks $C_x$ and $C_y$ trigger the following transitions of the discrete states $P$ and $Q$, respectively.

If $C_x(t) = 1$, then

\[ P(t) := \begin{cases} 
P(t) + 1 & \text{if } P(t) < |F_x|, \\
0 & \text{if } P(t) \geq |F_x|. 
\end{cases} \]

If $C_y(t) = 1$, then

\[ Q(t) := \begin{cases} 
Q(t) + 1 & \text{if } Q(t) < |F_y|, \\
0 & \text{if } Q(t) \geq |F_y|. 
\end{cases} \]

Here the symbol "\(t_+\)" denotes "\(\lim_{t \to t^+} \varepsilon\)" and the symbol ":=" denotes an "instantaneous state transition" hereafter. The internal clocks $C_x$ and $C_y$ trigger the following transitions of the discrete states $X$ and $Y$, respectively.

If $C_x(t) = 1$ and $P(t) \geq |F_x|$, then

\[ X(t) := \begin{cases} 
X(t) + 1 & \text{if } X(t) \neq N - 1, \\
0 & \text{if } X(t) \geq 0, \\
\text{otherwise}. 
\end{cases} \]

If $C_y(t) = 1$ and $Q(t) \geq |F_y|$, then

\[ Y(t) := \begin{cases} 
Y(t) + 1 & \text{if } Y(t) \neq N - 1, \\
0 & \text{if } Y(t) \geq 0, \\
\text{otherwise}. 
\end{cases} \]

Let $s(t) = A \sin(2\pi ft)$ be a stimulation signal. Then the following stimulation input $S(t)$ is introduced.

\[ S(t) = \begin{cases} 
1 & \text{if } t = \tau_p(1), \tau_p(2), \cdots, \\
-1 & \text{if } t = \tau_n(1), \tau_n(2), \cdots, \\
0 & \text{otherwise}, 
\end{cases} \]

where the instantaneous density of the pulse positions $\{\tau_p(1), \tau_p(2), \cdots\}$ is proportional to $s(t)$ for $s(t) > 0$ and is zero for $s(t) \leq 0$; and the instantaneous density of the pulse positions $\{\tau_n(1), \tau_n(2), \cdots\}$ is proportional to $-s(t)$ for $s(t) < 0$ and is zero for $s(t) \geq 0$. Note that such a density modulation can be easily realized by using a standard density modulator. The stimulation input $S(t)$ triggers the following transitions of the discrete state $P$ and $Q$.

If $S(t) = 1$ and $F_x \geq 0$, then

\[ P(t) := \begin{cases} 
P(t) + 1 & \text{if } P(t) < |F_x|, \\
0 & \text{if } P(t) \geq |F_x|. 
\end{cases} \]

If $S(t) = -1$ and $F_x \geq 0$, then

\[ P(t) := \begin{cases} 
P(t) - 1 & \text{if } P(t) > 0, \\
|F_x| & \text{if } P(t) = 0, 
\end{cases} \]

If $S(t) = 1$ and $F_x < 0$, then

\[ P(t) := \begin{cases} 
P(t) - 1 & \text{if } P(t) > 0, \\
|F_x| & \text{if } P(t) = 0, 
\end{cases} \]
Figure 2: Typical time chart.

If \( S(t) = -1 \) and \( F_X < 0 \), then
\[
P(t_+) := \begin{cases} 
P(t) + 1 & \text{if } P(t) < |F_X|, \\
0 & \text{if } P(t) \geq |F_X|. 
\end{cases}
\]

Then, the stimulation input \( S(t) \) triggers the following transitions of the discrete state \( X \) and \( Y \).

If \( S(t) = 1 \) and \( F_X \geq 0 \), then
\[
X(t_+) := \begin{cases} 
X(t) + 1 & \text{if } X(t) \neq N - 1 \\
X(t) & \text{otherwise}, 
\end{cases}
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If \( S(t) = -1 \) and \( F_X \geq 0 \), then
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X(t_+) := \begin{cases} 
X(t) + 1 & \text{if } X(t) \neq N - 1 \\
X(t) & \text{otherwise}. 
\end{cases}
\]

Fig. 2 shows a typical time chart of the model and Fig. 3 shows typical time waveforms of the model. In order to characterize the time waveforms, the following RMS of the discrete state \( X \) is introduced.

\[
RMS(X) = \lim_{T \to \infty} \sqrt{\frac{1}{T} \int_{-T}^{T} (X(t) - N/2)^2 dt}.
\]

Figure 3: Typical time waveforms. The parameter values are \((N, M, T_X, T_Y, \delta, \omega, m) = (256, 256, 10^{-7}, 10.1^{-7}, -10, 2\pi \times 10^3, 64)\). The stimulation frequency is \( f = 1k[Hz] \).

In the case of Fig. 3, the RMS is about 67. Also, in order to characterize the stimulation input \( S(t) \), the following magnitude \( p(S(t)) \) is introduced.

\[
p(S) = \lim_{\tau \to \infty} \frac{\text{Number of spikes } |S(t)| = 1 \text{ for } t \in [0, \tau]}{\tau}
\]

Using the magnitude \( p \), the following sound power level \( SPL_S \) of \( S(t) \) is introduced.

\[
SPL(S) = (p(S) - p_0) \times 10^{-5}(dB),
\]

where \( p_0 = 1.0 \times 10^6 \) is a reference magnitude corresponding to 0 (dB) of the \( SPL \). Note that the sound power level \( SPL \) in this paper corresponds to a sound pressure level often used in the literatures on the physiology of cochlear. Using the \( SPL \) of the stimulation input \( S(t) \) and the RMS of the discrete state \( X \), the following minimum sound power level \( SPL_{min} \) is introduced.

Minimum sound power level \( SPL_{min} \)

The minimum sound power level \( SPL_{min} \) is the minimum sound power level \( SPL \) of the stimulation input \( S(t) \) such that the RMS of the discrete state \( X \) is greater than or equal to a given threshold value \( RMS_{th} \).

Then, using the minimum sound power level \( SPL_{min} \) and the stimulation frequency \( f \), the following frequency tuning curve is introduced.
Figure 4: Frequency tuning curves of our model, where parameter values are the same as those in Fig. 3. The dashed line shows audible range which have 30-700[Hz] of a turtle [13]. It can be seen that the curves mimic some of the physiologically measured frequency tuning curves of the turtle in Fig. 1.

The characteristics curve of the minimum sound power level $SPL_{min}$ with respect to the input frequency $f$ is said to be a frequency tuning curve.

Fig. 4 shows frequency tuning curves of our model, where a dashed line shows audible range 30-700[Hz] of a turtle [13]. Comparing the frequency tuning curves in Fig. 4 (our model) and that in Fig. 1 (turtle), it can be seen that our model can mimic the physiologically measured frequency tuning curves of the turtle.

3. Conclusions

The nonlinear dynamics of a Hopf-bifurcation-type cochlea model based on a concept of an asynchronous bifurcation processor was investigated. It was shown that the model can reproduce nonlinear tuning curves of not only a mammalian cochlea (cat) but also a reptilian cochlea (turtle). Future problems are including (a) analysis of the occurrence mechanism of the bifurcation by using discrete-continuous hybrid Poincare map, (b) comparison of a frequency tuning curve of an FPGA-implemented circuit of our model with biological data, (c) reproductions of more highly nonlinear responses to sound stimuli such as multitone suppression, first and second pitch shifts, adaptation, parallel spike density encoding, and so on. This work was partially supported by JSPS KAKENHI Grant Number 15K00352.

References


