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Detecting directional couplings from time series: joint distribution of distances

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Abstract– When we investigate the dynamics underlying a complex system where we can observe time series of several different components, we want to identify directional couplings between each pair of those components. In such a circumstance, we should not want to assume that the dynamics is linear or that all the components can be observed. Thus, here we propose a method for identifying directional couplings based on a joint distribution of distances. The proposed method will be easily extended to the analysis of point processes.

1. Introduction

There are various networked systems where each component of the systems may be coupled with others. For investigating such networked systems through the observations of some of these components, identifying directional couplings is the first thing we should do. By a directional coupling, we mean the influence that one component exerts upon another. Many methods have been proposed for identifying such directional couplings [1-7]. However, some common drawbacks are that (i) such methods assume the linearity of the underlying systems, (ii) such methods presume that one can observe every component in the networked systems, and (iii) such methods assume a family of models.

To overcome these drawbacks, we proposed in 2010 a method for identifying directional couplings based on time series using recurrence plots [5]. Recently, we proposed two other methods for identifying directional couplings [8]. In addition, we proposed the simultaneous use of these three methods [8]. When evaluating individual performances of the three methods, we found that one of the methods using the joint distribution of distances outperforms the other two methods. Thus, in this presentation, we focus on the joint distribution of distances for a pair of states of components to identify directional couplings.

2. Backgrounds

2.1. Takens' theorem

Suppose that we are interested in a dynamical system $f: M \to M$ on a *m*-dimensional manifold M described by $x_{i+1} = f(x_i)$ for $i \geq 0$ given an initial condition $x_0 \in M$. Let us assume that we can only observe a scalar value $s_x(i) = g(x_i)$ through an observation function $g: M \to R$. This poses the problem of how to recover the information of x_i by using the limited information of $\{s_x(i) \mid i \geq 0\}$.

To resolve this problem, the key idea is to use delay coordinates, which were proposed by Takens [9]. Given $\{s_x(i) \mid i \geq 0\}$, delay coordinates corresponding to x_i can be defined by $\vec{s}_x(i) = H_x(x_i) = (s_x(i), s_x(i+1), ..., s_x(i+d-1))$. Takens [9] showed that if $d > 2m$, it is a generic property that the relation between x_i and $\vec{s}_x(i)$ is one-to-one. Therefore, in such a case, the following diagram commutes:

$$\begin{array}{ccc} x_i & \xrightarrow{f} & x_{i+1} \\ \downarrow H_x & & \downarrow H_x \\ \vec{s}_x(i) & \xrightarrow{\tilde{f}} & \vec{s}_x(i+1) \end{array}$$

By using the delay coordinates, we can reconstruct a dynamical system that is equivalent to the original dynamics f. Thus, even if we can neither observe x_i nor know the original dynamics f, we can learn the underlying dynamics and even predict the future of s_x by $\tilde{f}$.

2.2. Stark's theorem

The above theorem by Takens has been extended by Stark to the forced system [10]. Mathematically, a setting for a forced system can be written as follows: We have two dynamical systems $f: M \to M$ and $g: M \times N \to N$, where M and N are manifolds of m and n dimensions, respectively. Therefore, the first system f is an autonomous system, while the second system g is forced by the input of the first system. In what follows, we use the following notations for describing states for these systems: $x_{i+1} = f(x_i), y_{i+1} = g(x_i, y_i)$ for $i \geq 0$.

Suppose that we do not have access to the first system and we can observe a scalar value $s_y(i) = h_y(y_i)$ depending only on the state for the second system. Similarly, we construct delay coordinates by $\vec{s}_y(i) = H_y(x_i, y_i) = (s_y(i), s_y(i+1), ..., s_y(i+d-1))$. Then it is a generic property that if $d > 2(m+n)$, the joint set of (x_i, y_i) and $\vec{s}_y(i)$ are one-to-one on the attractor [10]. This theorem means that in the forced system, we can reconstruct the information of not only the forced system but also the driving force.

2.3. A method using recurrence plots for inferring directional couplings

The first person who used the above Stark's theorem in an application is Timothy D. Sauer [11]. He assumed that one can observe several forced systems and proposed how to reconstruct the common driving force.

The second application of Stark's theorem is by two of us for inferring directional couplings [5]. We used an implication of Stark's theorem for denying the existence of a directional coupling. Let us compare two sets of delay coordinates $\vec{s}_x(i)$ and $\vec{s}_y(i)$. When the system of f drives the system of g as defined as above and d is sufficiently large, $\vec{s}_x(i)$ is one-to-one with x_i, while $\vec{s}_y(i)$ is one-to-one with (x_i, y_i). Hence, if two sets of delay coordinates $\vec{s}_y(i)$ and $\vec{s}_y(j)$ are close to each other, (x_i, y_i) and (x_j, y_j) are close to each other. This relation means that x_i and x_j are close to each other, implying that $\vec{s}_x(i)$ and $\vec{s}_x(j)$ are close to each other.

To infer the directional coupling, Ref. [5] used the contraposition of the above relation: Namely, if $\vec{s}_y(i)$ and $\vec{s}_y(j)$ are close to each other while $\vec{s}_x(i)$ and $\vec{s}_x(i)$ are not close to each other, then we can deny a directional coupling from a system described by x to a system described by y. In Ref. [5], this statement is tested by using recurrence plots [12].

3. The proposed method

3.1. Joint distribution of distances for identifying directional couplings

Similarly to Ref. [5], we used Stark's embedding theorem for inferring directional couplings [8]. This time, we interpret the theorem more straightforward.

When we use Stark's embedding theorem, if the system described by x drives the system described by y, and $\vec{s}_y(i)$ and $\vec{s}_y(j)$ are close to each other, in other words, $\vec{s}_y(i)$ and $\vec{s}_y(j)$ are neighbors, then $\vec{s}_x(i)$ and $\vec{s}_x(j)$ are also neighbors. Therefore, if we evaluate the similarity between $\vec{s}_y(i)$ and $\vec{s}_y(j)$, and the similarity between $\vec{s}_x(i)$ and $\vec{s}_x(j)$ by the corresponding distances, these distances can be plotted in a two-dimensional space as shown in Fig. 1 and can occupy the triangle region as shown by red in Fig. 1. This is the idea we would like to use here for inferring a directional coupling from a system to another.

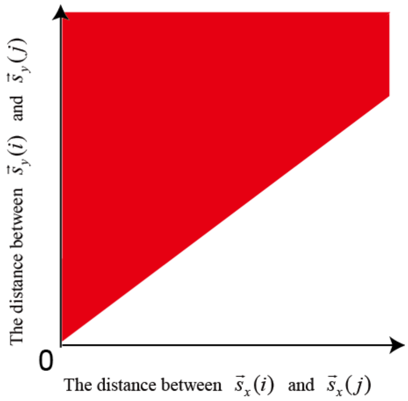

Fig. 1. The distance between $\vec{s}_x(i)$ and $\vec{s}_x(j)$ and the distance between $\vec{s}_y(i)$ and $\vec{s}_y(j)$.

3.2. Implementation

In Ref. [8], we proposed the following implementation for inferring directional couplings using the above idea: First, we calculate the Euclidean distances between every pair of embedded state vectors for the system described by x and those for the system described by y. Second, we convert the distributions of distances so that the distances have the uniform distributions between 0 and 1 (Thus, we may not be able to call them as distances strictly anymore). Third, we divide the axis of the distances for x uniformly into $2B$ bins. For each bin b, we find the minimum distance $D_y(b)$ for y. Lastly, we evaluate the distribution of $\Delta = \{D_y(B+b) - D_y(b) \mid b = 1,2,..., B\}$. Therefore, this distribution is expected to be biased towards the positive side if there is a directional coupling from x to y and the joint distribution of distances looks like one in Fig. 1. Thus, we construct a one-sided t-test based on the null hypothesis that the mean of Δ is zero. If this distribution can reject the null hypothesis, then we declare that there is a directional coupling from the system described by x to the system described by y.

4. Examples

In Ref. [8], we compared the above method of joint distribution of distances with other methods. But, here we only show the results of the above method because the space of this proceedings is limited. For detailed comparisons of the methods, see Ref. [8]. The p-values shown below are the p-values obtained by the above t-test discussed in Section 3.2.

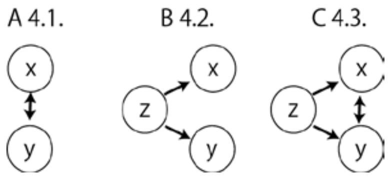

Fig. 2. Coupling configurations considered in Sections 4.1 (A), 4.2 (B) and 4.3 (C).

4.1. Mutually coupled logistic maps

First, we tested the above method using mutually coupled logistic maps (Fig. 2A). We varied the coupling strengths between 0 and 0.2, and generated a time series of length 1000 for each pair of coupling strengths.

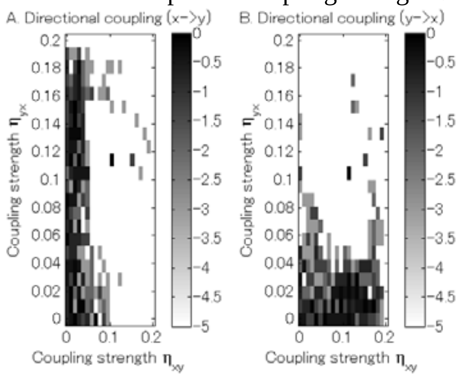

Fig. 3. Results for tests for inferring directional couplings given time series generated from mutually coupled logistic maps. In each panel, the gray scale shows the logarithm of the p-value with base 10.

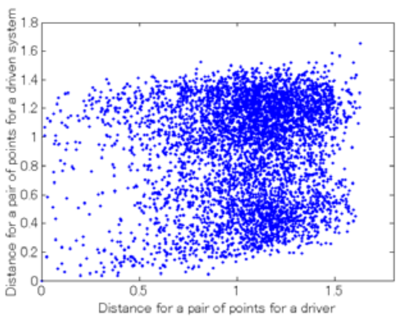

Fig. 4. Example for joint distribution of distances. Here we used a logistic map (driver) unidirectionally driving another (driven system). This example corresponds to $\eta_{yx} = 0$ and $\eta_{xy} = 0.15$ of Fig. 3.

The results are summarized in Fig. 3. The proposed method could properly infer the directional couplings if the coupling strength is modest or stronger, and the opposite coupling strength is not too strong. In addition, we show an example of joint distribution of distances in Fig. 4.

4.2. Logistic maps driven by another

Second, we tested the proposed method using logistic maps driven by another logistic map (See Fig. 2B). Here we assume that the two logistic maps we can observe are not coupled. The other simulation conditions are similar to the first example.

The results are shown in Fig. 5. We can see that the proposed method was not influenced by the common hidden driver. Thus, it seems that the proposed method does not induce false positive results, under which we declare, due to the influence of the common hidden driver, that there exists directional couplings between observed systems. This point means that the proposed method has a nice property that most of the existing methods do not have.

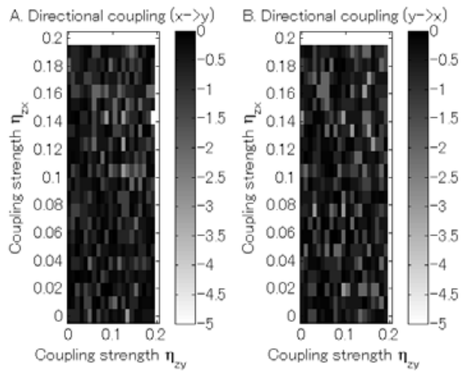

Fig. 5. Results of tests for inferring directional couplings given time series generated from two logistic maps driven by another logistic map. Here, the two logistic maps we can observe do not have direct couplings. See the caption of Fig. 3 for the interpretations.

4.3. Mutually coupled logistic maps driven by another

Third, we attempt to infer directional couplings under the existence of a common hidden driver (see Fig. 2C). Here, we also use logistic maps. The other simulation conditions are similar to the previous cases.

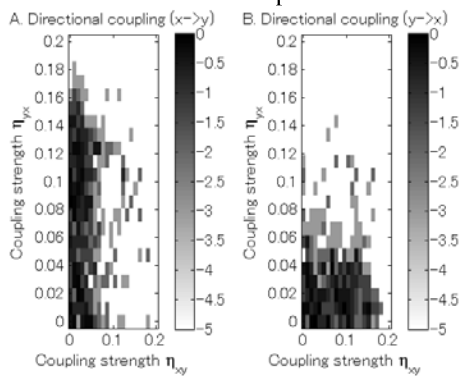

Fig. 6. Results for tests for inferring directional couplings given time series generated from mutually coupled logistic maps driven by another. See the caption of Fig. 3 for the interpretations.

The results are shown in Fig. 6. These results look similar to Fig. 3. Namely, the proposed method can work well and infer directional couplings even if there is a common hidden driver.

5. Discussions

We have proposed a method for inferring a directional coupling based on time series. Our method uses Stark's embedding theorem and infers a directional coupling using the joint distribution of distances. The method seems to work properly even if there exists a common hidden driver. Therefore, we do not have to be able to observe the entire components of a network system when we investigate its topology.

In Ref. [8], we also demonstrated that the proposed method works well even if the length of time series is 250, showing examples based on real data. The strength of the proposed method is that we use distances. Therefore, in Ref. [8], we applied our method for irregularly sampled data. Being able to obtain distances is a common condition for nonlinear time series analysis of exotic data such as a time series of network [13] and marked point processes [14-17]. Therefore, we believe that by combining with other methods, the proposed method will help us to study multivariate time series data.

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