

# Minimum Q Electrically Small Spherical Magnetic Dipole Antenna – Practice

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## 1. Introduction

Since the first works on a fundamental limit  $Q_{LB}$  for the radiation quality factor  $Q$  of an electrically small antenna were published by Wheeler [1] and Chu [2], it has been a question how close a realistic antenna can get to this fundamental limit. Recently, the  $Q$  factor was derived for an electric current distribution on a spherical surface in free space radiating electric or magnetic dipole fields [3]. It is shown that if the stored energy inside the sphere is taken into account, the quality factor rises to  $Q = 1.5Q_{LB}$  and  $Q = 3.0Q_{LB}$  for an electric  $TM_{10}$  and magnetic  $TE_{10}$  spherical mode, respectively.

Until now, the result closest to the fundamental limit among practical antennas has been demonstrated by the spherical helix antenna designed by Best [4]. This antenna radiates the  $TM_{10}$  spherical mode and indeed yields the quality factor of  $Q \simeq 1.5Q_{LB}$ . In a recent paper [5], three  $TE_{10}$ -mode antenna configurations approaching the limit  $Q = 3.0Q_{LB}$  were presented. These results seem to show that a magnetic dipole antenna offers a  $Q$  factor inferior to that of its electric dipole counterpart. In fact, as suggested by Wheeler [6], the magnetic dipole antenna can theoretically be brought arbitrary close to the lower bound by filling it with magnetic material having relative permeability  $\mu \rightarrow \infty$ :

$$Q = Q_{LB} \left(1 + \frac{2}{\mu}\right). \quad (1)$$

In practice, however, application of Wheeler's suggestion is not so straightforward as it might look at first glance. Especially, if one take into account core resonances and parasitic higher-order modes, which in a realistic antenna cannot be completely eliminated.

In this paper, we discuss practical aspects of using a magnetic core to reduce the radiation  $Q$  factor of spherical magnetic dipole antennas. Taking outset in a  $TE_{10}$ -mode multiarm spherical helix antenna – one of the three spherical magnetic dipole antennas presented in [5] – we numerically investigate the influence of the core permeability on electric characteristics of the antenna and show how its  $Q$  factor approaches the lower bound. Numerical simulations are performed using a code based on the surface integral equation technique and higher-order method of moments [7].

## 2. Magnetic Sphere Resonances and $Q$ factor

As follows from (1), the  $Q$  factor rapidly decays as  $\mu$  increases, so there is no need to go for very high permeability. It is sufficient to use a core with relative permeability about 100 to bring the  $Q$  factor within 2% of the lower bound. However, we should remember that Wheeler's expression (1) is derived assuming  $ka \ll 1$ , where  $k$  is the free space wavenumber and  $a$  is the radius of minimum sphere enclosing an antenna. It means that the first magnetic sphere resonance ( $TE_{10}$ -mode resonance) is never reached for finite  $\mu$ . In practice, an antenna is considered electrically small if its  $ka < 0.5$  [8], and therefore, the  $TE_{10}$  resonance occurs for a finite value of the magnetic sphere permeability. This has two important consequences. First, though the magnetic core reduces the  $Q$  factor of a finite size magnetic antenna, the expression (1) might be inaccurate. Second, as noted in [3], core resonances should be avoided, since they boost the stored energy and thus spoil  $Q$ . Moreover, the later point is relevant not only for the  $TE_{10}$ -mode resonance, but for all resonances including  $TM_{10}$  as well as higher-order magnetic and electric resonances. As we noted above, it is next to impossible to get rid of parasitic modes in practical antennas, which means that if a parasitic mode in an antenna in free space is not

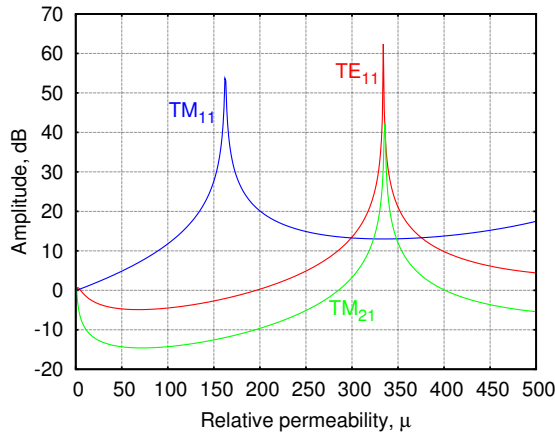


Figure 1: Amplitudes of the first three modes excited by a plane wave in a magnetic sphere of  $ka = 0.245$ .

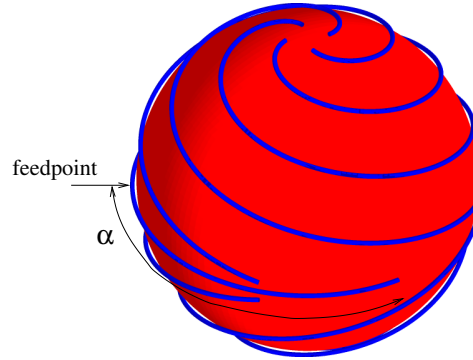


Figure 2: A  $TE_{10}$  4-arm spherical helix antenna.

sufficiently suppressed, it give rise to a core resonance with the corresponding destructive effects for the  $Q$  factor.

For preliminary theoretical analysis it is instructive to consider the analytic Mie series solution for a plane wave scattering by a magnetic sphere [9]. Although the solution is limited to the  $m = 1$  spherical modes, we can still use it as a guideline keeping in mind that sphere resonances are degenerate in  $m$ . For example, Fig. 1 shows amplitudes of the first three modes excited in a magnetic sphere of  $ka = 0.245$  as a function of the core permeability  $\mu$ . It is seen that within the plotted range of  $\mu$  there is an optimum  $\mu \simeq 70$ , where the amplitude of the  $TE_{11}$  mode inside the core is minimal. Due to the proximity of the  $TM_{11}$  resonance, things become more complicated, and it is likely that the optimum shifts toward lower values of  $\mu$ . Actual amplitudes of  $TE_{10}$  and  $TM_{1m}$  modes in a spherical magnetic core excited by antenna currents are somewhat different than those given in Fig. 1. Nevertheless, the considerations given above allow us to identify the range of interest  $10 < \mu < 100$ , where the optimum permeability is expected to be found. Next, numerical simulations of a particular antenna configuration are performed to locate an exact optimum of the core magnetic permeability.

### 3. $TE_{10}$ Antenna with Magnetic Core

A  $TE_{10}$  multiarm spherical helix antenna [5] is sketched in Fig. 2. In our investigations we have chosen a 4-arm configuration, i.e. the top and bottom hemispheres of the antenna are each composed of four wire arms. The antenna is excited by a curved dipole located in the equatorial plane and driven in the center by a delta-gap voltage generator. As demonstrated in [10, 11, 5], this kind of excitation provides an easy and convenient way for matching a magnetic dipole antenna to a desired feed line impedance. At a given frequency  $f_0$ , the antenna is tuned to the resonance by changing the number of turns  $N_{\text{turns}}$  in each arm. To make the antenna resonant at  $f_0 = 300$  MHz in free space the number of turns is set to  $N_{\text{turns}} = 2.25$ . Other geometrical parameters of the antenna are as follows:

- radius of the spherical helix  $r_h = 40$  mm;
- radius of the magnetic core  $r_c = 39$  mm ( $ka = 0.245$ );
- length of the excitation dipole  $\alpha = 143^\circ$ ;
- wire radius  $r_w = 0.5$  mm.

The radii of the helix and the core are chosen so that direct contact between them is avoided. This done to ensure the stability of the numerical solution. Thus, the radius of a minimum sphere the antenna can fit in is  $a = 40.5$  mm, which corresponds to  $ka \simeq 0.254$ .

Numerical simulations are performed for the range of core permeabilities identified in the previous section. Obviously, the magnetic core inserted in the antenna perturbs the energy balance, and hence the number of turns is adjusted for each value of  $\mu$  to recover the balance (Fig. 3). As permeability increases, magnetic energy in the core reduces, and consequently, a fewer number of turns is required to provide a matching amount of electric energy. The antenna input resistance at the resonance is also affected by the magnetic core (Fig. 3). This can easily be compensated by changing the length of the excitation dipole, as demonstrated in [5].

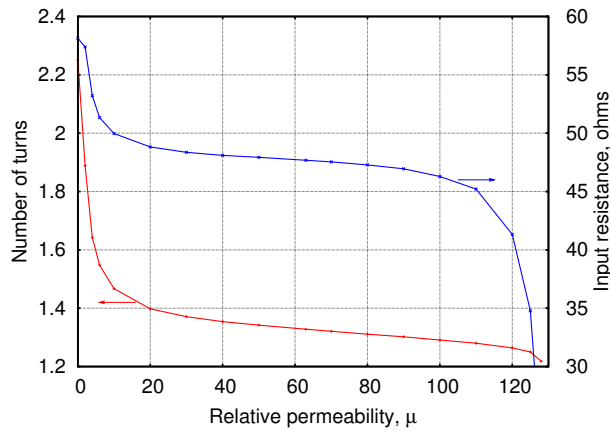


Figure 3: Numerical results for the 4-arm spherical helix antenna with a magnetic core. Number of turns is adjusted for each value of the core magnetic permeability to keep the antenna resonance frequency unchanged (300 MHz). Corresponding changes in the input resistance are also shown.

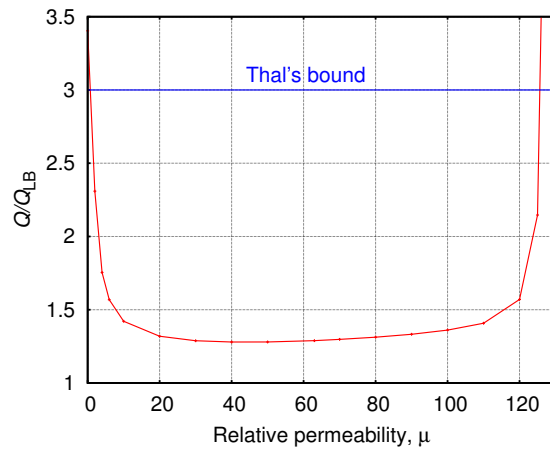


Figure 4: Numerical results for the 4-arm spherical helix antenna with a magnetic core. Computed  $Q/Q_{LB}$  ratio as a function of the core magnetic permeability. The  $Q/Q_{LB} = 3.0$  bound [3] for an ideal  $TE_{10}$ -mode antenna without a magnetic core is shown for comparison.

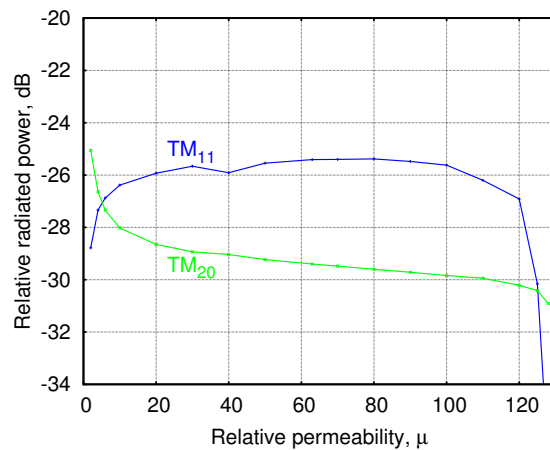


Figure 5: Numerical results for the 4-arm spherical helix antenna with a magnetic core. Radiated power of the parasitic  $TM_{11}$  and  $TM_{20}$  modes relative to the total radiated power.

The radiation  $Q$  factor is computed using [12, eq.(96)]. In free space the antenna yields  $Q \simeq 220$ , which is 3.4 times above the lower bound. The ratio  $Q/Q_{LB}$  rapidly drops as the magnetic permeability of the core increases (Fig. 4). However, after having reached a minimum  $Q/Q_{LB} \simeq 1.28$  at  $\mu \simeq 40$ , the ratio monotonically grows and finally rises steeply for  $\mu > 120$ . As anticipated, the  $TM_{11}$  resonance pulls the  $Q/Q_{LB}$  curve up despite the fact that the magnetic energy of the main  $TE_{10}$  mode is still low in the core. Spherical wave expansion of the antenna far-fields shows that despite the core resonance, the relative radiated power of the  $TM_{11}$  mode, as well as the next strongest mode  $TM_{20}$ , is very low (Fig. 5), and thus has a negligible effect on the  $Q_{LB}$  (this issue is discussed in detail in [5]).

## 4. Conclusions

Practical aspects of applying a magnetic core to approach the lower bound for the radiation  $Q$  factor of an electrically small magnetic dipole antenna are considered. It is shown that although a magnetic core does reduce the  $Q$  factor, its effect is not as strong as predicted by Wheeler's expression (1). This is due to the fact that a finite size magnetic core supports multiple internal resonances, which spoil the  $Q$  factor also away from exact resonance frequencies; and in a worst case they can even significantly increase  $Q$ . The resonances in question are not only those of the  $TE_{10}$  mode, but also resonances of all other modes that are not sufficiently suppressed in the antenna. Numerical results for a 4-arm spherical helix antenna filled with magnetic material demonstrate the destroying effect of the parasitic  $TM_{11}$  mode on the antenna  $Q$  factor.

Theoretical considerations as well as numerical results show that in a given range of magnetic permeabilities away from core resonances there is an optimum  $\mu$  for which the  $Q$  factor is lowest. In the given example for the antenna of the size  $ka \simeq 0.254$ , the ratio  $Q/Q_{LB} \simeq 1.28$  is achieved.

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