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A Study on Satisfaction Method of Constraints by Approximating Constraints

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Abstract—Genetic Algorithm (GA) is actively applied to real-world problems. Most of these real world problems are constrained optimization problems which optimize objective functions while satisfying constraints. In constrained optimize problems, the constraints need to be determined whether each of them is satisfied or not, which is regarded as a two-class classification. Therefore, the performance of GA for constrained optimization problems can be improved by classifiers as the approximation of constraints. In this paper, the approximation method of objective functions and constraints using Multiple Regression Analysis was applied to five benchmark problems as an introduction to the classification of constraints and studied the improvement of the performance of search in constrained optimization problems.

1. Introduction

Genetic Algorithm (GA) is actively applied to real-world problems such as the design of front nose of N700, the design of aircraft wing of MRJ, and Nurse Scheduling Problem[2][3][4]. Most of these real world problems are constrained optimization problems which optimize objective functions while satisfying constraints. In the conventional GA researches, it has been reported that the performance of search can be improved by approximation of objective functions[5][6]. However, the approximation of fitness values of objective functions requires high approximation accuracy and it takes many individuals and actual evaluations as the information of approximation. Therefore, it is difficult to apply it, or it does not show high performance in real-world problems. On the other hand, constraints need to be determined whether each of them is satisfied or not, which is regarded as a two-class classification. And usually all of the constraints have to be satisfied. This paper studies the improvement of the performance of search in constrained optimization problems using Multiple Regression Analysis[1] as an introduction to approximation of constraints.

2. Proposed method

In GA, actual evaluation is generally required to acquire the fitness value(s) of objective function(s) and whether it violates constraint(s) for each generated offspring. Thus, offsprings which have worse fitness value(s) of objective function(s) than those of their parents or violate constraint(s) can be generated and that leads a waste of search. Therefore, in the proposed method, only offsprings which have better fitness value(s) than their parents and do not violate any constraints are generated based on Multiple Regression Analysis (MRA) and actual evaluation are done for them to improve the performance of search. In the proposed method, the crossover of same parents is repeated until the offsprings described above are generated or the number of crossover reaches the upper limit. The procedures of usual GA and the proposed method are shown in Fig.1.

3. Experiment

In this paper, the conventional method and the proposed method were applied to five benchmark problems (CF1 to CF5) which were employed for Multi-objective Optimization Test Instances for the CEC 2009 Special Session and Competition[7]. All of these benchmark problems, CF1 to CF5, are constrained optimization problems having one constraint and two objective functions. The details of these benchmark problems are shown in Table 1. The number of evaluations was 300,000 which was the condition used for the competition. In this experiment, the population size was 300 and the search was ended at 1,000th generation. In the proposed method, the upper limit of the number of crossover by same parents was 100 and the regression equations for the objective functions and constraint were calculated by newly obtained individuals at each generation.

The individuals satisfying the constraint for each benchmark problem are shown in Fig.2 to Fig.6. In all benchmark problems, the performance of the proposed method which uses MRA to approximate the objective functions and constraint was worse than that of the conventional GA. It is supposed that the worse performance of the proposed method was caused by the insufficiency accuracy of the approximation by MRA. The actual evaluated fitness values and the approximated values of the objective functions and constraint are shown in Fig.7 to Fig.11. The gradational color of each point in the figures is correspond to the generation, which changes from red (first generation) to blue (last generation). These figures show that the accuracy of the approximation was lower especially in early generations and it made the performance of the search worse. Because the more the search was conducted, individuals were more converged, it is thought that the accuracy of the approximation became better in later generations. However, remarkable inclinations of this accuracy improvement were not observed in these experiments. It cannot be denied that is the limit of a linear approximation method. In addition, as shown in Fig.7, extremely small approximated values by MRA was observed in CF1. It is supposed that was caused by the multicollinearity.

4. Conclusion

This paper studied the improvement of the performance of search in constrained optimization problems using Multiple Regression Analysis as an introduction to approximation of constraints. The proposed method which approximates objective function and constraints by MRA and generates offsprings by crossover considering them was applied to five constrained optimization problems and compared with the conventional GA. As the result, the performance of the proposed method was worse than that of the conventional GA. It was supposed that the low performance was caused by the insuficiency accuracy of the approximation by MRA especially in early generations. We will study on the approximation method for objective functions and constraints considering the convergence of individuals. We will also study on nonlinear approximation methods for the approximation.

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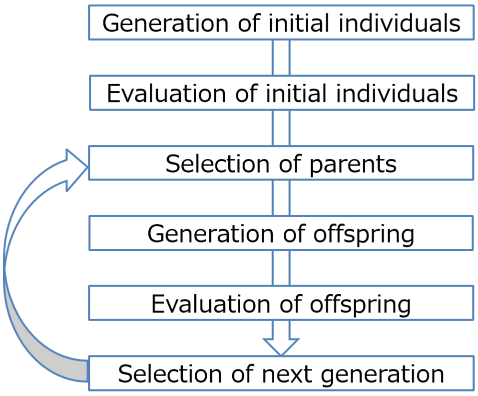
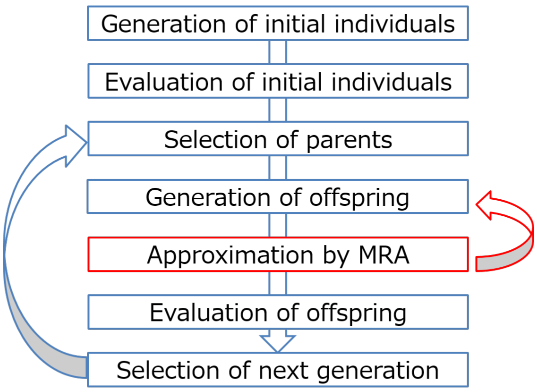
Table 1: Benchmark problems

CF1	$f_1(\boldsymbol{x}) = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ $f_2(\boldsymbol{x}) = 1 - x_1 + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ $1 - f_1 - f_2 + a\lvert\sin(N\pi(f_1 - f_2 + 1))\rvert \leq 0$	$a = 1$ $N = 10$ $n = 10$
CF2	$f_1(\boldsymbol{x}) = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2(\boldsymbol{x}) = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - \cos(6\pi x_1 + \frac{j\pi}{n}))^2$ $\frac{t}{1+e^{4\lvert t\rvert}} \leq 0$ $t = 1 - \sqrt{f_1} - f_2 + a\sin(N\pi(\sqrt{f_1} - f_2 + 1))$	$a = 1$ $N = 2$ $n = 10$
CF3	$f_1(\boldsymbol{x}) = x_1 + \frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_1} {y_j}^2 - 2\prod_{j\in j_1}\cos(\frac{20y_j\pi}{\sqrt{j}}))$ $f_2(\boldsymbol{x}) = 1 - {x_1}^2 + \frac{2}{\lvert J_2\rvert}(4\sum_{j\in J_2} {y_j}^2 - 2\prod_{j\in j_2}\cos(\frac{20y_j\pi}{\sqrt{j}}))$ $y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n})\ (j = 2, ..., n)$ $1 - {f_1}^2 - f_2 + a\lvert sin(N\pi({f_1}^2 - f_2 + 1))\rvert \leq 0$	$a = 1$ $N = 2$ $n = 10$
CF4	$f_1(\boldsymbol{x}) = x_1 + \sum_{j\in J_1} h_j(y_j)$ $f_2(\boldsymbol{x}) = 1 - x_1 + \sum_{j\in J_2} h_j(y_j)$ $y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n})\ (j = 2, ..., n)$ $h_2(t) = \begin{cases} \lvert t\rvert & (t < \frac{3}{2}(1 - \frac{\sqrt{2}}{2})) \\ 0.125 + (t-1)^2 & (otherwise) \end{cases}$ $h_J(t) = t^2\ (j = 3, 4..., n)$ $\frac{t}{1+e^{4\lvert t\rvert}} \leq 0$ $t = -0.25 + 0.5x_1 - x_2 + \sin(6\pi x_1 + \frac{2\pi}{n})$	$n = 10$
CF5	$f_1(\boldsymbol{x}) = x_1 + \sum_{j\in J_1} h_j(y_j)$ $f_2(\boldsymbol{x}) = 1 - x_1 + \sum_{j\in J_2} h_j(y_j)$ $y_j = \begin{cases} x_j - 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}) & (j \in J_1) \\ x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & (j \in J_2) \end{cases} \quad (j = 2, ..., n)$ $h_2(t) = \begin{cases} \lvert t\rvert & (t < \frac{3}{2}(1 - \frac{\sqrt{2}}{2})) \\ 0.125 + (t-1)^2 & (otherwise) \end{cases}$ $h_J(t) = 2t^2 - \cos(4\pi t) + t\ (j = 3, 4..., n)$ $-0.25 + 0.1x_1 - x_2 + 0.8x_1\sin(6\pi x_1 + \frac{2\pi}{n}) \leq 0$	$n = 10$
	$J_1 = \{j\mid j \text{ is odd and } 2 \leq j \leq n\}$ $J_2 = \{j\mid j \text{ is even and } 2 \leq j \leq n\}$	

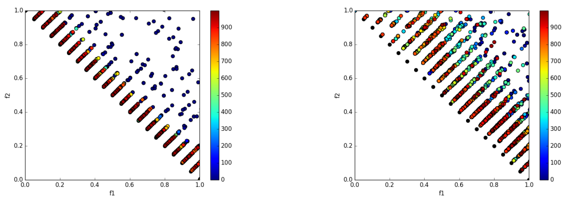


(a) Conventional GA



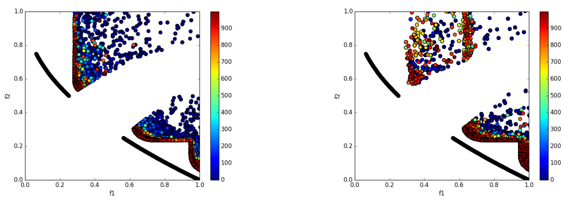
(b) Proposed method

Figure 1: Procedure of GA



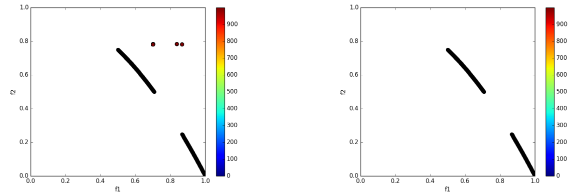
(a) Conventional GA (b) Proposed method

Figure 2: Result of CF1



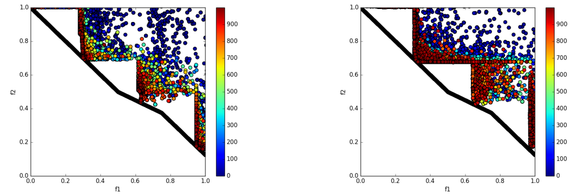
(a) Conventional GA (b) Proposed method

Figure 3: Result of CF2



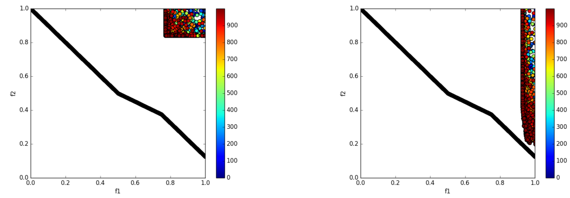
(a) Conventional GA (b) Proposed method

Figure 4: Result of CF3



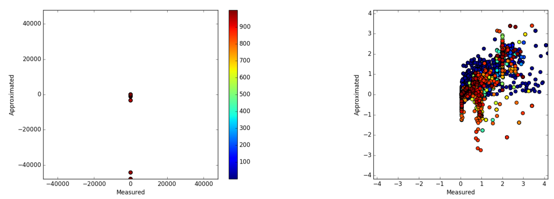
(a) Conventional GA (b) Proposed method

Figure 5: Result of CF4

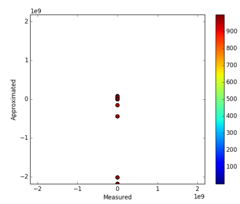


(a) Conventional GA (b) Proposed method

Figure 6: Result of CF5

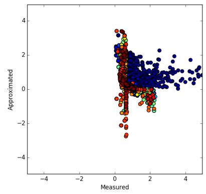


(a) Objective function : f_1 (b) Objective function : f_2

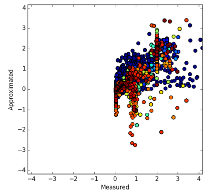


(c) Constraint

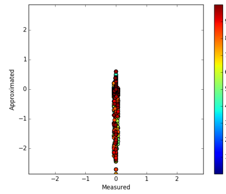
Figure 7: Accuracy of CF1



(a) Objective function : f_1

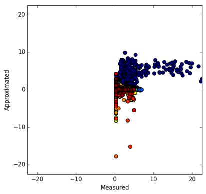


(b) Objective function : f_2

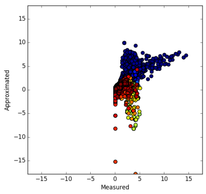


(c) Constraint

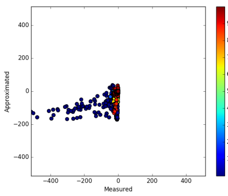
Figure 8: Accuracy of CF2



(a) Objective function : f_1

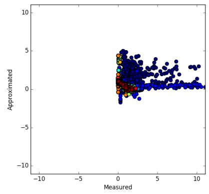


(b) Objective function : f_2

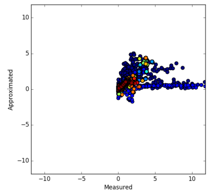


(c) Constraint

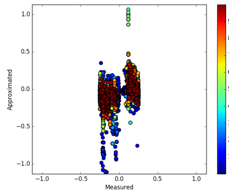
Figure 9: Accuracy of CF3



(a) Objective function : f_1

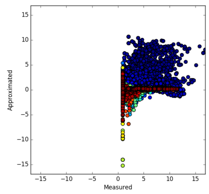


(b) Objective function : f_2

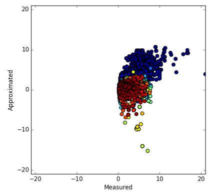


(c) Constraint

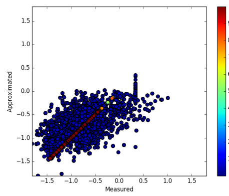
Figure 10: Accuracy of CF4



(a) Objective function : f_1



(b) Objective function : f_2



(c) Constraint

Figure 11: Accuracy of CF5