



## Asymmetric emission caused by chaos-assisted tunneling and synchronization in two-dimensional microcavity lasers

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It has been experimentally reported that low threshold lasing and directional emission can be simultaneously achieved in a two-dimensional microlaser with the quadrupolar cavity [1]. The quadrupolar cavity is defined in the polar coordinates  $(r, \theta)$  as

$$r(\theta) = r_0(1 + \epsilon \cos 2\theta), \quad (1)$$

where  $r_0$  is the size parameter and  $\epsilon$  is the deformation parameter fixed as  $\epsilon = 0.09$ . In this cavity, a resonant mode that localizes along a pair of stable triangular orbits is confined by total internal reflection. This strong light confinement leads to a low lasing threshold, while weak light emission occurs by chaos-assisted tunneling [2]. The tunneling induces light intensity leakage from the stable triangular orbits to chaotic orbits that eventually escape from the cavity violating the critical angle condition for total internal reflection, where the chaotic dynamics governed by unstable manifolds results in directional emission [3].

The emission patterns of two-dimensional microcavity lasers can be theoretically studied by analyzing the resonant modes of the two-dimensional Helmholtz equation [4]

$$[\nabla^2 + n^2(x, y)k^2]\psi(x, y) = 0, \quad (2)$$

where  $n(x, y)$  is the refractive index and  $k$  is the vacuum wave number. The real part of  $k$  represents the resonant wave number, while the imaginary part represents the decay rate of the resonant mode. We consider transverse-magnetic polarization, namely,  $\psi(x, y)$  represents the  $z$ -component of the electric field  $E_z$ . The refractive index inside the cavity is  $n_{in} = 3.3$ , while it is  $n_{out} = 1$  outside the cavity. Equation (2) can be numerically solved by, for example, the extended boundary element method [5].

In the experiment in Ref. [1], asymmetric directional emission patterns were observed despite the symmetry of the quadrupolar cavity. The wave functions of the resonant modes are divided into the four symmetry classes

$$\psi_{ab}(-x, y) = a \psi_{ab}(x, y) \quad (3)$$

$$\psi_{ab}(x, -y) = b \psi_{ab}(x, y), \quad (4)$$

with the parities  $a, b \in \{-, +\}$ . Therefore the emission patterns of individual resonant modes have the same symmetry as the cavity shape (i.e., symmetric with respect to both  $x$  and  $y$  axis). However, when we consider nonlinear modal interaction through a lasing medium, the locking, or synchronization of two different parity modes can occur, and it yields an asymmetric emission pattern [6].

In this presentation, we show that the experimentally observed asymmetric emission pattern can be explained by the locking of a nearly degenerate pair,  $\psi_{++}$  and  $\psi_{+-}$  (i.e., the modes with slightly different wave numbers). First, we demonstrate that although  $\psi_{++}$  and  $\psi_{+-}$  are localized along the pair of stable triangular orbits, their superposition turns out to be localized only along one of the pair. Secondly, by using the Maxwell-Bloch model that takes account the nonlinear effect of a lasing medium [7], we actually simulate the locking of the nearly degenerate pair as well as the appearance of the asymmetric emission patterns.

### References

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