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Stability Analysis of Periodic Orbits in Dynamic Binary Neural Networks

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Abstract—This paper studies various periodic orbits and their stability in dynamic binary neural networks with signum activation function. The network can generate various binary periodic orbits and the dynamics is integrated into a digital return map defined on a set of points. Performing basic numerical experiments, it is shown that the network can generate various periodic orbits and ternary $\{-1, 0, +1\}$ connection parameters can reinforce stability of the periodic orbits.

1. Introduction

The dynamic binary neural network (DBNN) is constructed by applying a delayed feedback to a feed forward binary neural network with signum activation [1]-[5]. Depending on connection parameters, the DBNN can generate a variety of binary periodic orbits (BPOs). The DBNN is included in digital dynamical systems such as cellular automata [8] and digital spiking neurons [10]. These digital dynamical systems are applicable to logical/sequential circuits, image processing systems, and UWB communication systems. Analysis of DBNN is important form both fundamental and application viewpoints. However, analysis of DBNN is hard because the DBNN has a large number of parameters and can generate a large variety of periodic/transient phenomena.

This paper analyzes dynamics of a simple class of DBNN. First, we introduce that the dynamics of the DBNN is integrated into a digital return map (Dmap) from a set of lattice points to itself. The Dmap can be regarded as a digital version of analog return map represented by the logistic map [11]. We then analyze three types of 6-dimensional DBNNs: the connection parameters w_{ij} are integer, they are binary $w_{ij} \in \{-1, +1\}$, and they are ternary $w_{ij} \in \{-1, 0, +1\}$, Such 6-dimensional DBNNs are applicable to control signal of basic dc/ac and ac/dc power converters [3] [5].

Performing numerical experiments it is shown that (1) the binary connection parameters are able to realize the almost same stability of BPO as the integer connection parameters, (2) the ternary connection parameters are able to reinforce stability of the BPO, and (3) the binary and ternary connection parameters can suppress spurious memories.

2. Dynamic Binary Neural Networks

The DBNN is constructed by applying a delayed feedback to a feed-forward network with the signum activation function. The dynamics is described by

$$x_i^{t+1} = \mathrm{sgn}\left(\sum_{j=1}^{N} w_{ij} x_j^t - T_i\right) \quad (1)$$

$$\mathrm{sgn}(x) = \begin{cases} +1, \text{for } x \geq 0 \\ -1, \text{for } x < 0 \end{cases} \quad i = 1 \sim N$$

where $\boldsymbol{x}^t$ is a binary state vector at discrete time t and $x_i^t \in \{-1, +1\} \equiv \boldsymbol{B}$ is the i-th element. The connection parameters w_{ij} and the threshold parameters are integer.

In order to visualize the dynamics DBNN, we introduce the Dmap. The domain of the DBNN is a set of

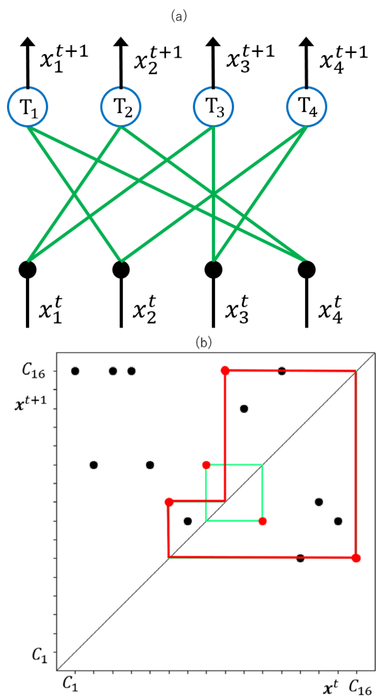

Figure 1: (a) DBNN. $w_{ij} = 0$ means no connection. The threshold parameters T_i are shown in the circles. (b) Dmap. DBNN and Dmap. Red orbit denotes TBPO. Green orbit denotes a BPO. $\gamma = 10/16$

binary vectors $\boldsymbol{B}^N$ that is equivalent to a set of points $L_D = (C_1, \cdots, C_{2^N})$, $C_i \equiv i/2^N$. Hence the dynamics of the DBNN can be integreted into the digital return map (Dmap) from L_D to itself:

$$\boldsymbol{x}^{t+1} = F_D(\boldsymbol{x}^t),\ \boldsymbol{x}^t \equiv (x_1^t, \cdots, x_N^t) \in \boldsymbol{B}^N \tag{2}$$

Figure 1 illustrates the Dmap for $N = 4$ where binary code is used to express $L_4 = \{C_1, \cdots, C_{16}\}$: $C_1 \equiv (-1,-1,-1,-1) \cdots C_{16} \equiv (+1,+1,+1,+1)$. Since the number of lattice points is 2^N, direct memory of all the inputs/outputs becomes hard/impossible as N increases. However, in the DBNN, the number of parameters is polynomial $N^2 + N$.

Since the number of the lattice points is finite, the steady states are BPOs defined as the following.

A point $\theta_p \in L_D$ is said to be a periodic point (PEP) with period p if $F^p(\theta_p) = \theta_p$ and $F^k(\theta_p) \neq \theta_p$ for $1 \leq k < p$ where F^p is the p-fold composition of F. Especially, a PEP with period 1 is said to be a fixed point. A sequence of the PEPs, $\{F(\theta_p), \cdots, F^p(\theta_p)\}$, is said to be a binary periodic orbit (BPO).

In Fig.1 the Dmap has one BPO with period 3 and BPO with period 2. Depending on the initial condition, the DBNN exhibits either BPO.

3. Teacher signal and stability

We consider storage of one BPO into the DBNN. The teacher signal binary periodic orbit (TBPO) with period T is described by

$$\begin{array}{l} \boldsymbol{z}^1, \boldsymbol{z}^2, \cdots, \boldsymbol{z}^T,\ \boldsymbol{z}^i = (z_1^i, \cdots, z_N^i) \in \boldsymbol{B}^N \\ \boldsymbol{z}^i = \boldsymbol{z}^j \text{ for } |i-j| = T,\ \boldsymbol{z}^i \neq \boldsymbol{z}^j \text{ for } |i-j| \neq T \end{array} \tag{3}$$

For simplicity, the period $T$is assumed to be order of N. In order to determine connection parameters w_{ij}, we use three methods A parameter condition for storage of TBPO.

$$\text{Integer connection: } w_{ij} = c_{ij} = \sum_{\tau=1}^{T} z_i^{\tau+1} z_j^{\tau} \tag{4}$$

$$\text{Binary connection: } w_{ij} = b_{ij} = \begin{cases} +1 & \text{for } c_{ij} \geq 0 \\ -1 & \text{for } c_{ij} < 0 \end{cases} \tag{5}$$

Ternary conection: $w_{ij} = d_{ij} \in \{1, 0, +1\}$
d_{ij} is given by zero-insertion algorithm in [6]. (6)

Ref. [6] gives a sufficient condition of parameters for storage of TBPO. Referring to the condition, as connection parameters w_{ij} are given, the threshold parameters T_i can be determined theoretically.

Here we define stability of TBPO. A TBPO is said to be stable if at least one initial point (except for the TBPO) falls into the TBPO. A TBPO is said to be globally stable if all initial points into the TBPO. In order to characterize the global stability, we introduce a simple feature quantity

$$\gamma = \frac{\#\ \text{Initial points falling into the TBPO}}{2^N} \tag{7}$$

where $T/2^N \leq \gamma \leq 1$. If $\gamma = 1$ then the TBPO is globally stable. In Fig. 1, 10 initial points fall into the TBPO with period 3 and $\gamma = 10/16$.

4. Numerical experiment

This paper considers two examples of TBPOs with period 6. For N = 6, Table 1 shows the first TBPO corresponding to a control signal of a basic AC/DC converter. Applying the three kinds of connection parameters in Eqs. (4) to (6), the first TBPO can be stored into the DBNN. Tables 3, 4, and 5 show integer, binary, and ternary connection parameters, respectively. Figures. 2, 3, and 4 show corresponding three Dmaps. The integer, binary, and ternary connection parameters give $\gamma = 17/64$, $\gamma = 12/64$, and $\gamma = 1$, respectively.

Table 2 shows the second TBPO corresponding to a control signal of a basic AC/DC converter. Applying the three kinds of connection parameters in Eqs. (4) to (6), the second TBPO can be stored into the DBNN. Tables 6, 7, and 8 show integer, binary, and ternary connection parameters, respectively. Figures. 5, 6, and 7 show corresponding three Dmaps. The integer, binary, and ternary connection parameters give $\gamma = 42/64$, $\gamma = 42/64$, and $\gamma = 1$, respectively.

In these results, we can see the following.

- The binary connection can realize the almost same global stability as the integer connection.
- The ternary connection with zero elements can realize global stability. The global stability is impossible in integer and binary connections.
- The binary and ternary connections can suppress serious memories in the case of integer connection.

Table 1: TBPO example 1

z^1	(+1,	−1,	−1,	−1,	−1,	+1)
z^2	(+1,	+1,	−1,	−1,	−1,	−1)
z^3	(−1,	+1,	+1,	−1,	−1,	−1)
z^4	(−1,	−1,	+1,	+1,	−1,	−1)
z^5	(−1,	−1,	−1,	+1,	+1,	−1)
z^6	(−1,	−1,	−1,	−1,	+1,	+1)

Table 2: TBPO example 2

z^1	(+1,	−1,	−1,	−1,	+1,	+1)
z^2	(+1,	+1,	−1,	−1,	−1,	+1)
z^3	(+1,	+1,	+1,	−1,	−1,	−1)
z^4	(−1,	+1,	+1,	+1,	−1,	−1)
z^5	(−1,	−1,	+1,	+1,	+1,	−1)
z^6	(−1,	−1,	−1,	+1,	+1,	+1)

Table 3: Integer connection parameters for example 1

i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	T_i
1	+2	−2	−2	−2	+2	+6	4
2	+6	+2	−2	−2	−2	+2	4
3	+2	+6	+2	−2	−2	−2	4
4	−2	+2	+6	+2	−2	−2	4
5	−2	−2	+2	+6	+2	−2	4
6	−2	−2	−2	+2	+6	+2	4

Table 4: Binary connection parameters for example 1

i	d_{i1}	d_{i2}	d_{i3}	d_{i4}	d_{i5}	d_{i6}	T_i
1	+1	−1	−1	−1	+1	+1	2
2	+1	+1	−1	−1	−1	+1	2
3	+1	+1	+1	−1	−1	−1	2
4	−1	+1	+1	+1	−1	−1	2
5	−1	−1	+1	+1	+1	−1	2
6	−1	−1	−1	+1	+1	+1	2

Table 5: Ternary connection parameters for example 1

i	d_{i1}	d_{i2}	d_{i3}	d_{i4}	d_{i5}	d_{i6}	T_i
1	+1	−1	−1	−1	+1	+1	2
2	+1	+1	−1	0	−1	+1	1
3	+1	+1	+1	−1	−1	−1	2
4	−1	+1	+1	0	−1	−1	2
5	−1	−1	0	+1	0	−1	2
6	−1	−1	−1	+1	+1	+1	2

Table 6: Integer connection parameters for example 2

i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	T_i
1	+2	−2	−6	−2	+2	+6	0
2	+6	+2	−2	−6	−2	+2	0
3	+2	+6	+2	−2	−6	−2	0
4	−2	+2	+6	+2	−2	−6	0
5	−6	−2	+2	+6	+2	−2	0
6	−2	−6	−2	+2	+6	+2	0

Table 7: Binary connection parameters for example 2

i	d_{i1}	d_{i2}	d_{i3}	d_{i4}	d_{i5}	d_{i6}	T_i
1	+1	−1	−1	−1	+1	+1	0
2	+1	+1	−1	−1	−1	+1	0
3	+1	+1	+1	−1	−1	−1	0
4	−1	+1	+1	+1	−1	−1	0
5	−1	−1	+1	+1	+1	−1	0
6	−1	−1	−1	+1	+1	+1	0

Table 8: Ternary connection parameters for example 2

i	d_{i1}	d_{i2}	d_{i3}	d_{i4}	d_{i5}	d_{i6}	T_i
1	0	0	−1	0	+1	+1	0
2	+1	+1	−1	−1	−1	+1	0
3	+1	+1	+1	−1	−1	−1	0
4	−1	+1	+1	+1	−1	−1	0
5	−1	−1	+1	+1	0	−1	0
6	−1	−1	−1	+1	+1	+1	0

5. Conclusions

A class of 6-dimensional DBNNs has been studied in this paper. In order to visualize the DBNN dynamics, the Dmap is introduced. In order to consider the stability of TBPO, global stability is defined and a simple feature quantity γ is introduced.

In basic numerical experiments. we have shown that the ternary connection can realize global stability and the binary/ternary connection can suppress spurious memories.

Future problems include analysis of 2DBNN with sparse connections, analysis of deep DBNNs, and engineering applications.

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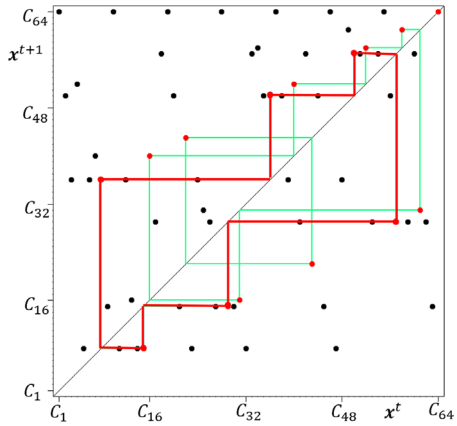

Figure 2: Dmap for Table 3. Red orbit: TBPO. Green orbit: Spurious BPO. $\gamma = 17/64$

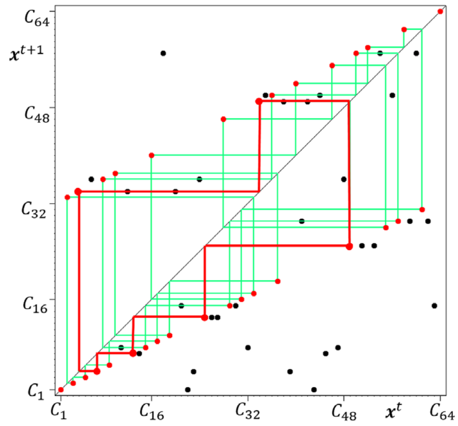

Figure 5: Dmap for Table 6. Red orbit: TBPO. Green orbit: Spurious BPO. $\gamma = 42/64$

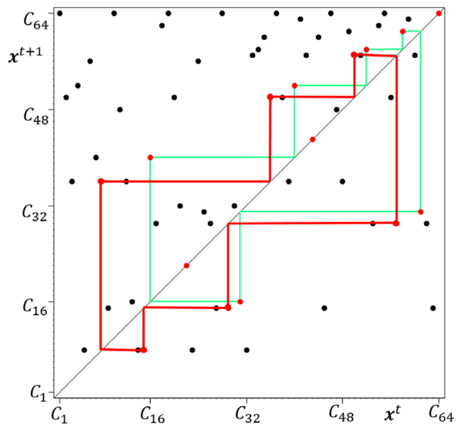

Figure 3: Dmap for Table 4. Red orbit: TBPO. Green orbit: Spurious BPO. $\gamma = 12/64$

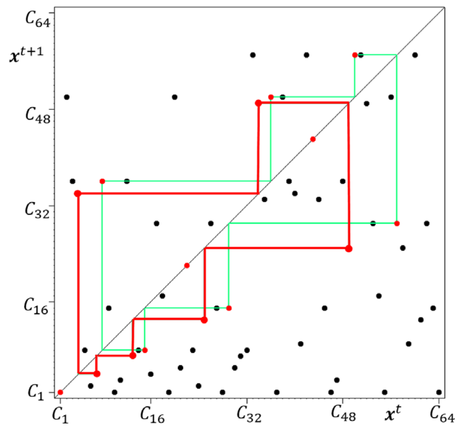

Figure 6: Dmap for Table 7. Red orbit: TBPO. Green orbit: Spurious BPO. $\gamma = 42/64$

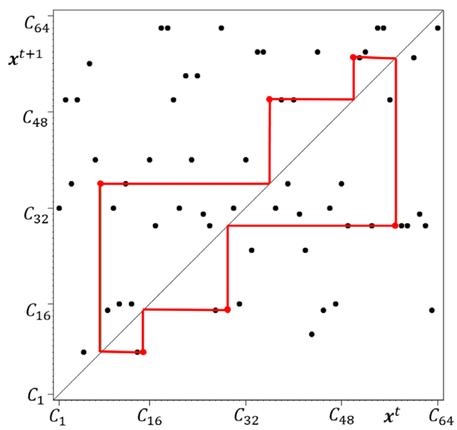

Figure 4: Dmap for Table 5. Red orbit: TBPO. $\gamma = 1$

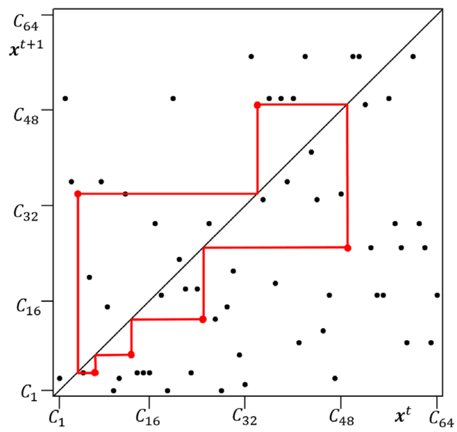

Figure 7: Dmap for Table 8. Red orbit: TBPO. $\gamma = 1$