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Super-Stabilization of Periodic Spike-Trains in the Digital Spiking Neuron

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Abstract—This paper considers super-stabilization of periodic spike-train from the digital spiking neuron consisting of two shift registers connected by a wiring. Depending on the wiring, the neuron can generate various periodic spike-trains. In order to super-stabilize a desired periodic spike-train, we present a simple deterministic rewiring method. The super-stabilized periodic spike-trains are applicable to robust and reliable encoders in multiplexing communication systems.

1. Introduction

The digital spiking neuron (DSN) is a kind of degital dynamical system inspired by integrate-and-fire neuron models [1]-[3]. The DSN is constructed by two shift registers connected by a wiring. Depending on the wiring pattern, the DSN can generate various periodic spike-trains (PSTs) and transient super-trains to the PSTs. The DSNs are applicable to spike-based communication systems and spike-based learning systems [1]-[7].

This paper considers super-stabilization of a desired PST in the DSN The super-stability means that almost all initial points fall directly (instantaneously) into the PST [3]. The super-stabilized PST is applicable to a robust and reliable encoder in multiplexing communication systems. In order to super-stabilize a PST, we introduce a super-stabilizing wiring method (SSWM [3]). The SSWM is a simple deterministic method that re-wires connection between two shift registers of the DSN and can super-stabilize a desired PST.

The dynamics of the DSN is described by a digital spike map (Dmap, [4] [5]). The Dmap is defined on a set of points and can be regarded as a digital version of analog one-dimensional map such as the logistic map [6]. Since the domain of the Dmap is a set of finite number of points, the Dmap cannot generate chaos but a variety of periodic/transient phenomena. The DSN and Dmap are well suited for computer-aided precise analysis and FPGA-based hardware implementation [1].

The Dmap is related to several digital dynamical systems such as logical/sequential circuits [8], cellular automata [9] [10], and dynamic binary neural networks [11] [12]. Such systems are applicable to information compression, signal processing, and control of switching power converters. Stability analysis of the Dmap and DSN can contribute not only to basic study of nonlinear dynamics but also to engineering applications.

2. Digital Spiking Neuron

Fig. 1 (a) illustrates the DSN consisting of two shift registers connected by a wiring. The left and right shift registers are referred to as P-cells and X-cells. The P-cells consist of M elements and operates as a pacemaker with period M. Let $\boldsymbol{P}(\tau) \equiv (P_1(\tau), \cdots, P_M(\tau))$ denote the P-cells and let $P_i(\tau) \in \{0, 1\}$ be the i-th element. The dynamics is described by

$$P_i(\tau) = \begin{cases} 1 & \text{if } \tau = i + nM \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $i \in \{1, \cdots, M\}$ and n denotes integers.

The X-cells consist of N elements and are state variables corresponding to an action potential of an analog neuron. Let $\boldsymbol{X}(\tau) \equiv (X_1(\tau), \cdots, X_N(\tau))$ denote the X-cells and let $X_j(\tau) \in \{0, 1\}$ be the j-th element. An element of the P-cells is connected to either element of the X-cells in one-way. The connection is defined by the wiring vector

$$\boldsymbol{a} = (a_1, \cdots, a_M),\ a_i = j \text{ if } P_i \text{ is connected to } X_j$$

For example, the wiring vector for Fig. 1 (a) is

$$\boldsymbol{a} = (4, 6, 10, 6, 6, 12, 7, 12)$$

Using the wiring vector, we define the base signal with period M:

$$\begin{aligned} &\boldsymbol{B}(\tau) \equiv (B_1(\tau), \cdots, B_N(\tau)),\ \boldsymbol{B}(\tau + M) = \boldsymbol{B}(\tau) \\ &B_i(\tau) = \begin{cases} 1 & \text{if } a_\tau = i \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (2)$$

where $\tau \in \{1, 2, \cdots, M\}$ and $i \in \{1, 2, \cdots, N\}$. An example of $B(\tau)$ is illustrated in Fig. 1 (b). In the X-cells, an initial condition is assumed to be

$$\begin{cases} X_k(1) = 1 & \text{for some } k \\ X_j(1) = 0 & \text{for } j \neq k \end{cases} \quad (3)$$

where $k \in \{1, 2, \cdots, N\}$ and $j \in \{1, 2, \cdots, N\}$. Only one element can be 1. The dynamics is described by

$$X_{j+1}(\tau + 1) = \begin{cases} 1 & \text{if } X_j(\tau) = 1 \text{ for } j = 1 \sim N \\ 1 & \text{if } X_N(\tau) = 1 \text{ and } B_{j+1}(\tau) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

If the top element of X-cells is active then the DSN outputs a spike $Y(\tau) = 1$ and generates a spike-train as shown in Fig. 1 (b)

$$Y(\tau+1) = \begin{cases} 1 & \text{if } X_N(\tau) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

For simplicity, we assume

$$N = 2M - 1,\ 1 < a_i - 1 \le M \quad (6)$$

where $i \in \{1, 2, \cdots, M\}$. In this case, a spike appears once per one clock period and the n-th spike appears in the n-th interval $\{(n-1)M, \cdots, nM\}$ [2] [3]. Let θ_n denote the n-th spike-phase such that

$$Y(\tau+1) = \begin{cases} 1 & \text{for } \tau = (n-1)M + \theta_n \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where $\theta_n \in \{1, 2, \cdots, M\}$ and n denotes integers. A spike-train is represented by a sequence of spike phases $\{\theta_n\}$. Since the n-th spike determines the $(n+1)$-th spike, we can define the degital spike map (Dmap) of the spike-phase:

$$\theta_{n+1} = F(\theta_n),\ \theta_n \in \{1, 2, \cdots M\} \quad (8)$$

An example of the Dmap is shown in Fig. 1 (b). The derivation process and theoretical formula of Dmaps can be found in [1]-[3]. That is, the dynamics of the DSN is integrated into the Dmap. As stated earlier, the Dmap can be regarded as a digital version of analog return maps represented by the logistic map [6]. Although the analog map can generate chaos, the Dmap cannot generate chaos because M is a finite number. However, the Dmap can generate a variety of periodic/transient phenomena as suggested in [4] [5].

3. Super-stabilizaing wiring method

In order to consider stabilization of PST, we give several definitions for the Dmap.

Definition 1: A point $p \in L_M$ is said to be a periodic point with period k if $p = f^k(p)$ and $f(p)$ to $f^k(p)$ are all different where f^k is the k-fold composition of f. A sequence of the periodic points $\{p, f(p), \cdots, f^{k-1}(p)\}$ is said to be a periodic orbit (PEO) with period k.

A PEO with period k ($f(p) = f^k(p)$) corresponds to a PST with period Mk ($Y(\tau + Mk) = Y(\tau)$). For example, the PEO with period 4 in Fig. 1 (a) c to the PST with period 4 in Fig. 1 (b). Since a PEO in the Dmap is equivalent to a PST in the DSN, we consider stabilization for the PEO in the Dmap instead of the PST for simplicity. For convenience to give definition of stability, we assume that the a period of PEO is at most $M/2$.

Definition 2: A point $q \in L_M$ is said to be an eventually periodic point (EPP) with step k if the q is not a periodic point but falls into some periodic point p after k steps: $f^k(q) = p$. An EPP with step 1 is referred to as a direct eventually periodic point (DEPP): $f(q) = p$. An EPP corresponds to an initial spike-position of a transient spike-train to the PST.

Definition 3: A PEO is said to be stable if al least one EPP falls into the PEO. A PEO is said to be super-stable if all the EPPs are DEPP falling into the PEO. For example, in Fig. 2, the Dmap has PEO with period 4 and the other 16 − 4 blue points are DEPPs falling into the PEO hence the PEO is super-stable.

Here we introduce the super-stabilizing wiring method (SSWM [3]) for a PEO. As a precondition for the SSWM, we assume that the PEO is given by some algorithm to satisfy some desired characteristics. For example, Ref. [3] has presented a simple evolutionary algorithm that gives a PEO of low autocorrelation.

For simplicity, we explain the SSWM for an example: a PEO with period 3 for $M = 8$ (PST with period 24) in Fig. 1 (a).

$$\boldsymbol{\alpha_p} \equiv \{6, 6, 12\},\ \boldsymbol{\alpha} = (4, 6, 10, 6, 12, 7, 12) \quad (9)$$

If other 5 elements are given by the following, we obtain the DSN in Fig. 2 (a).

$$\begin{aligned} \boldsymbol{a} &= (6-1, 6, 6-1, 6, 6+1, 12, 12+1, 12+2) \\ &= (5, 6, 5, 6, 7, 12, 13, 14) \end{aligned} \quad (10)$$

The difference between $\boldsymbol{\alpha}$ and $\boldsymbol{a}$ is rewiring of five blue branches in Fig. 2 (a). In the DSN, the PEO with period 3 is super-stabilized as shown in Fig. 2 (c) where 5 blue points are DEPPs falling directly into the PEO.

In general, if a PEO is given, the PEO can be super-stabilized as the following. First, let the PST with period pM be given by p elements in a wiring vector $\boldsymbol{\alpha}$

$$\begin{aligned} \boldsymbol{\alpha_p} &\equiv \{\alpha_{p_1}, \cdots, \alpha_{p_p}\} \subset \{\alpha_1, \cdots, \alpha_{p_1}, \cdots, \alpha_{p_p}, \cdots, \alpha_M\} \\ \boldsymbol{\alpha} &= (\alpha_1, \cdots, \alpha_{p_1}, \cdots, \alpha_{p_p}, \cdots, \alpha_M) \end{aligned} \quad (11)$$

The following wiring can super-stabilize the PEO.

$$\boldsymbol{a} = (a_1, \cdots, a_M)$$
$$a_i = \begin{cases} \alpha_i & \text{if } \alpha_i \in \boldsymbol{\alpha}_p \\ \alpha_j + (i-j) & \text{if } \alpha_i \notin \boldsymbol{\alpha}_p,\ \alpha_j \in \boldsymbol{\alpha}_p \end{cases}$$

where j is selected arbitrary from $\{p_1, \cdots, p_p\}$.

4. Conclusions

Super-stabilization of periodic spike-trains in the DSN is studied in this paper. In order to visualize the dynamics of DSN, the Dmap is introduced. In order to super-stabilize a desired periodic spike-train, the SSWM is presented.

Future problems include development of the SSWM into various digital dynamical systems, design of basic hardware that can generate super-stable spike-trains, and engineering application of the SSWM.

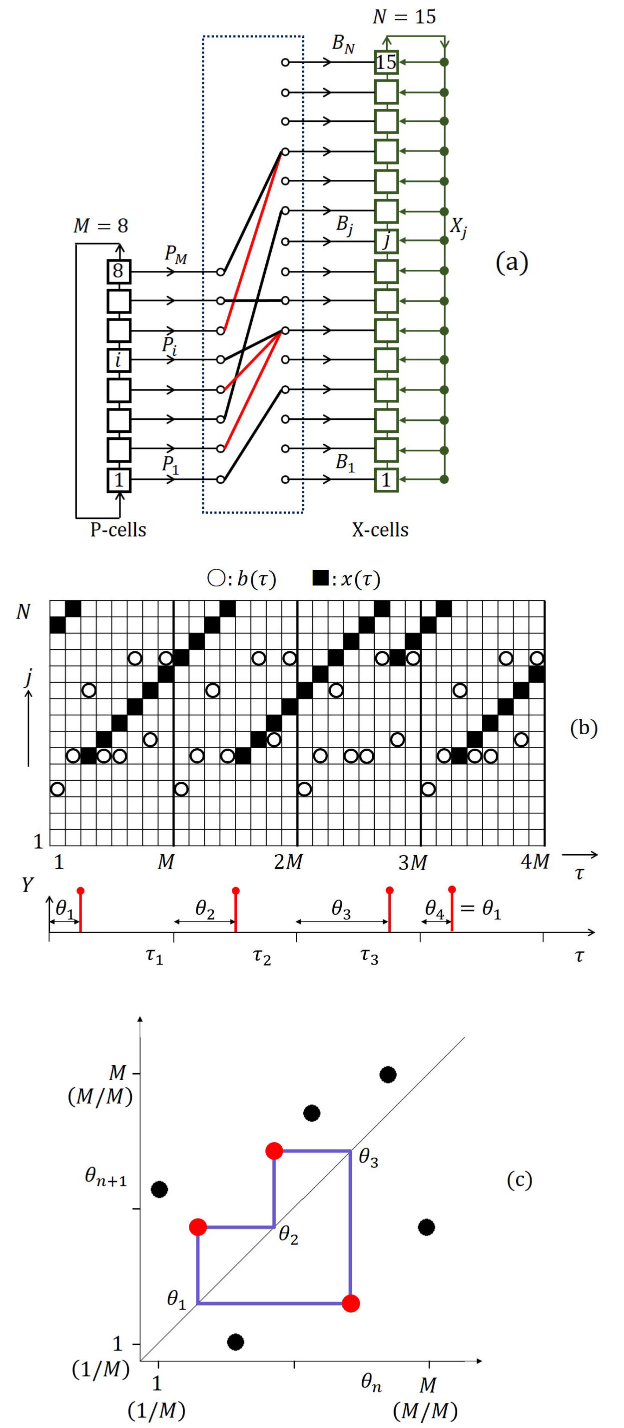

Figure 1: Digital spiking neuron (DSN). (a) Configuration. (b) Time-domain waveform of the state variable $X(\tau)$ and base signal $B(\tau)$. (c) Digital spike map. Red points construct a PEO and black points are EPPs to the PEO.

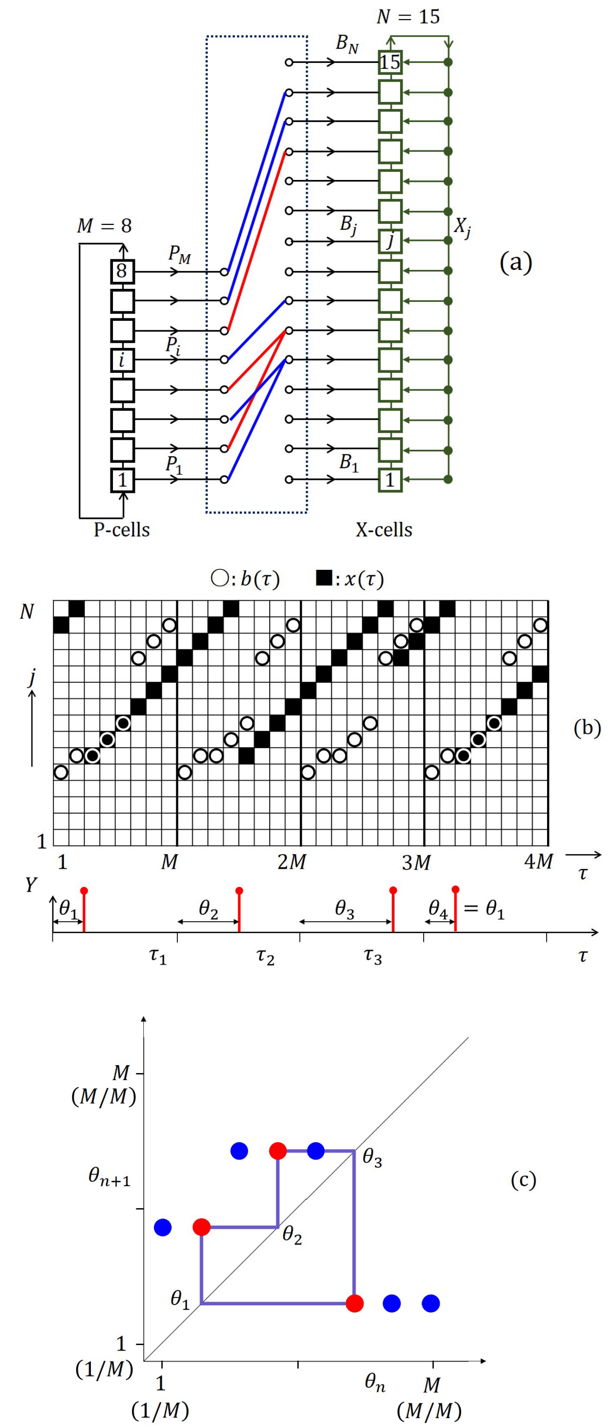

Figure 2: DSN after the SSWM. (a) Configuration. (b) Time-domain waveform of state variable and base signal. (c) Digital spike map. Red points construct a PEO with period 3 and blue points are DEPPs to the PEO.

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