# Analytical Models for Calculating Attenuation in Coplanar Waveguides

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### 1. Introduction

Coplanar waveguides (CPW's) have received considerable attention due to several advantages offered for monolithic microwave integrated circuits (MMIC's) applications. The available commercial software tools are capable of determining the characteristic impedance and propagation constant accurately; they fail to provide realistic estimates of line loss. Holloway and Kuester (HK) [1, 2] have used the concept of stopping distance with the standard perturbation method to compute the conductor loss of a coplanar waveguide (CPW) shown in Fig.1. However, theoretical stopping-distance is valid only for the isolated strip conductor [7]. It is not sufficiently accurate for computation of conductor loss of CPW. In the present work, we studied the nature of the stopping distance and it is also extracted from expressions of the stopping- distance. Those expressions are used with HK model to obtain closed- form expressions of the stopping- distance. Those expressions are used with HK model to obtain the improved HK models. The accuracy of the improved HK models is tested against Haydl *et al.* [4, 5] and Ponchak *et al.* [6].

### 2. Closed- Form Models For Stopping Distance

The parameters of a coplanar transmission line are illustrated in Fig. 1, with ground-to-ground outer spacing 2c, ground-to-ground inner spacing 2b = d, slot width w, strip width 2a = s, substrate thickness h and conductor thickness t.



Fig. 1. Cross section of coplanar transmission line.

Holloway and Kuester [1,2] have generated a table for the normalized reciprocal stopping distance  $(t/\Delta)$  for the isolated strip conductor of thickness t with 90° and 45° conductor edges, where,  $\Delta$  is the stopping distance that avoids the edge singularity of the current density on the strip conductor while carrying out integration in the perturbation method. The stopping distance is frequency dependent. Therefore the normalized reciprocal stopping distance  $y = (t/\Delta)$  is a function of the normalized skin-depth  $(t/2\delta_s)$ .

In this paper, the experimental data demonstrates that the stopping distance  $\Delta$  is structure dependent, which may comprise of conductor thickness, slot-width, strip-width, substrate thickness and ground-to ground inner spacing. Fig.2 shows dependence of the experimentally extracted  $(t/\Delta)$  on the geometrical parameter of CPW on InP substrate. All the parameters are in  $\mu$ m. The  $(t/\Delta)$  is both structure and frequency dependent. For the case  $t/(2\delta_s) \le 0.2$ ,  $(t/\Delta)$  is structure independent while for  $t/(2\delta_s) > 0.2$  i.e. at higher frequency it is significantly structure dependent. It is large for the wide central strip-width and wide slot-gap.



**Fig.2.** Structure dependence of stopping distance for s/d =0.73, t = 0.25  $\mu$ m,  $\epsilon_r$  = 12.6

The CPW structure of finite substrate thickness and finite width ground planes is shown in Fig.1. Holloway and Kuester have taken the CPW with infinite width ground conductors,  $c \rightarrow \infty$ . Following the perturbation method; as used by Holloway and Kuester, we obtain the following expression for the conductor loss of CPW shown in Fig.1 in terms of stopping distance,

$$\alpha \approx \frac{R_{sm}b^2}{16Z_0K^2(k)(b^2 - a^2)} \cdot \left\{ \frac{1}{a} \ln \left[ \left( \frac{2a}{\Delta} - 1 \right) \left( \frac{b - a + \Delta}{b + a + \Delta} \right) \left( \frac{c + a - \Delta}{c - a - \Delta} \right) \right] + \frac{1}{b} \ln \left[ \left( \frac{2b}{\Delta} + 1 \right) \left( \frac{b - a + \Delta}{b + a - \Delta} \right) \left( \frac{c - b - \Delta}{c + b - \Delta} \right) \right] \right\}$$
(1)

where, a = s/2, b = s/2+w, stopping distance ( $\Delta$ ) is measured from edge of the central strip. The surface impedance  $R_{sm}$  of the strip conductor of thickness t is obtained from expressions given in [10]. The characteristic impedance of the CPW shown in Fig.1 is obtained from standard expression [8, 10]. Ponchak *et al.* [6] have taken 2c = (2w+9s). The elliptic integral ratio K(k')/K(k) is evaluated by using the closed-form expressions [11].

We extracted the stopping distance ( $\Delta$ ) from the extensive experimental results of the conductor loss of CPW provided by Haydl *et al.* [4, 5] for the frequency range 1 GHz to 60 GHz. We noted that the theoretical reciprocal normalized stopping distance  $(t/\Delta)$  obtained by Holloway and Kuester [1] follows the experimental results for the reciprocal normalized skin-depth  $(t/2\delta_s) \le 0.5$ . The extracted results of stopping distance have been curve-fitted in two different ways to obtain two closed-form models of stopping distance: improved HK model # 1 and improved HK model # 2.

### Improved HK model # 1

For the thin strip conductors used in MMIC technology, there is significant deviation in some frequency range. Therefore, we obtained the following curve- fitted expression for normalized stopping distance from the experimentally extracted stopping distance from one set of graphical results of Haydl *et al.* for the parameter s/d = 0.13, t = 0.25 µm, h = 500 µm,  $\varepsilon_r = 12.6$  (InP substrate) d = 30 µm, s = 4 µm, w = 13 µm over frequency range 1 GHz – 60 GHz. In general  $(t/\Delta)$  is structure dependent. However, for simplicity we have obtained the following empirical expression that is only frequency dependent through the normalized conductor thickness parameter  $t/(2\delta_s)$  over the range 0.045 < x < 2.0.

$$y = \begin{cases} 1668 \ x^{4} - 1504 \ .1x^{3} + 595 \ .85 \ x^{2} - 18 \ .823 \ x + 0 \ .6874 \ , & for \ 0.045 \ \le \ x < 0.7 \\ 2394 \ .4 \ x^{4} - 5861 \ .9 \ x^{3} + 4839 \ .1x^{2} - 1025 \ .3 \ x - 51 \ .873 \ , for \ 0.7 \ \le \ x < 2.0 \end{cases}$$
(2)

#### Improved HK model # 2

To avoid the multidimensional curve-fitting, we have taken average value of structure dependent  $(t/\Delta)$  with respect to  $(t/2\delta_s)$ . The experimentally extracted terms with very large

variation in  $(t/\Delta)$  with respect to  $(t/2\delta_s)$  are neglected. We have considered only up to two times variation in the minimum  $(t/\Delta)$  for generating a table of average of  $(t/\Delta)$ . We obtained following curve-fitted expression for  $y \equiv (t/\Delta)$  with respect to  $x \equiv (t/2\delta_s)$  over the range 0.045 < x < 2.0:

$$y = \begin{cases} 67672 \ x^5 + 55171 \ x^4 - 15875 \ x^3 + 2141 \ .3x^2 - 122 \ .24x + 2.7098 \ , & for \ 0.045 \le x < 0.32 \\ -8682 \ .4x^4 + 18587 \ x^3 - 13702 \ x^2 + 4342 \ .9x - 491 \ .11 \ , & for \ 0.32 \le x < 0.7 \\ -39906 \ x^5 + 211580 \ x^4 - 440418 \ x^3 + 451351 \ x^2 - 227213 \ x + 44983 \ , for \ 0.7 \le x < 2.0 \end{cases}$$
(3)

### 3. Comparison of Models

The accuracy of the original Holloway- Kuester (HK) model HK [2], Ponchak *et al.* model [6], improved HK model # 1 and improved HK model # 2 are compared against two sources of experimental data. The first source of 336 experimental data on the conductor loss of CPW is the graphical experimental results of Haydl *et al.* [4, 5] on GaAs ( $\varepsilon_r$ =12.9) and InP ( $\varepsilon_r$ =12.6) substrates of thickness 0.5 mm, conductor thickness t= 0.25, 0.5, 1.0  $\mu m$  and s/d = 0.13, 0.4, 0.73 in the frequency range 1 GHz – 60 GHz. The second source for the 36 experimental data points is the graphical experimental results of Ponchak *et al.* [6] for the CPW on GaAs, InP and Si substrates for characteristic impedances 35  $\Omega$ , 50  $\Omega$ , 65  $\Omega$ .

Fig.3 (a) illustrates typical comparisons of four models against the experimental results of Haydl *et al.* in the frequency range 1 GHz – 60 GHz for conductor loss of a CPW structure on InP substrate with ground-to-ground inner spacing, d=60  $\mu$ m. For a fixed strip width (s) / slot-gap (w) ratio, the conductor loss decreases with increase in the strip-width and the slot-gap. The models have higher % deviation against the experimental results for the narrow strip-width and narrow slot- gap.

Fig. 3(b) further compares four models against the experimental results of Ponchak *et al.* [6] for  $50 \Omega$  CPW lines on GaAs ( $\varepsilon_r$ =12.85) h=500µm substrate at 23 GHz for wide range of s/w. The present improved HK models have average error much less as compared to other two models. We have further compared four models for the CPW on InP and Si substrates used by Ponchak *et al.* The results on % deviation for four models are summarized in table -1. The variation in average % deviation is due to the error in reading the graphical data. However, deviations computed by us and Ponchak *et al.* are close to each other, showing acceptable level of accuracy of data extraction from the experimental graphs. Against this backdrop, 4.71% average deviation in the improved HK model # 1 and 3.7% average deviation in the improved HK model # 2 show higher accuracy of our curve-fitted expressions for experimentally extracted stopping distance.



**Fig.3.** (a) Comparison of conductor loss models against experimental results [5] for  $\varepsilon_r$ =12.6, s/d=0.73, d=60µm, t=0.5µm, h=500µm; (b) Deviation in models against experimental results [6] for t=1.58µm, f=23 GHz and Z<sub>0</sub>=50Ω for CPW lines on GaAs

Models	Haydl		Ponchak	
	Av	Max	Av	Max
нк[2]	13.69	34.32	13.79	30.98
Ponchak[6]	17.07	26.75	5.52	24.6
IHK #1	5.75	19.69	4.71	33.26
IHK #2	3.73	19.65	3.7	32.73

Table-I: % Deviation of models against experimental results of Haydl [4, 5], Ponchak [6]

[Data range:  $t = 0.25 \mu m - 1.58 \mu m$ ; Freq = 1 GHz - 60 GHz]

### 4. Conclusions

We have presented two closed-form models for experiment – based stopping distance that provide the (M)MIC-CAD oriented simple models to compute conductor loss of CPW. They are accurate and simple approximate analytic models which improve significantly conductor loss computation against the available experimental results. The present improved Holloway and Kuester (IHK) model # 1 has average accuracy of 5.62% and model # 2 has average accuracy of 3.73% against the experimental results from different sources in the frequency range 1 GHz – 60 GHz with conductor thickness 0.25 $\mu$ m - 1.58 $\mu$ m.The nature of the stopping distance has also been studied and has found to be dependent on structural parameters and frequency.

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