# Transient Simulation of the Plane Wave Coupling to Non-Linearly Loaded Transmission Line Networks

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***Abstract*—The coupling of plane waves with arbitrary time functions, incident directions and polarizations to a transmission line network of single-wire lines above a perfectly-conducting ground plane is simulated in the time domain. The transient simulation allows taking into account non-linear loads like diodes. Based on classical transmission line theory, the network is converted into an equivalent circuit with distributed and lumped voltage sources to consider the excitation by the external field. The circuit is simulated using a circuit simulator written in MATLAB that is based on modified nodal analysis. The model is validated against an existing frequency-domain solution for linear loads. Exemplary results are shown for a three-wire network with different non-linear loads in parallel to a terminal resistor.**

***Index Terms*—transmission line, plane wave coupling, field-to-wire coupling, equivalent circuit model, diode, transmission line network, Agrawal model, modified nodal analysis**

## I. INTRODUCTION

The analysis of the field-to-wire coupling has always been of great interest for electromagnetic compatibility engineers and many methods exist for individual transmission lines [1]. In practice, lines may be connected together to form transmission line networks. The simulation of the field coupling to such networks is more complex, but has been described in [2], [3] for linear, resistive loads in the frequency domain.

Transmission line networks might also be terminated with semiconductor devices like diodes or varistors e. g. for overvoltage protection. If such non-linear loads have to be considered, a transient simulation in time domain is necessary [4]. The transmission line system can be converted into an equivalent circuit that is simulatable in a SPICE-like circuit simulator. The incident field is then considered by distributed sources along the lines. Many ideas on this topic exist [5], [6], but mainly focus on the transformation of the distributed sources into equivalent sources at the end of the lines.

Because it is difficult to include the external field excitation into existing SPICE simulators [7], the idea of this work is to integrate an own simple circuit simulator based on the modified nodal analysis (MNA) into MATLAB, where it is simpler to calculate transient fields at different spatial locations.

## II. SIMULATION MODEL

### A. Transmission Line Model

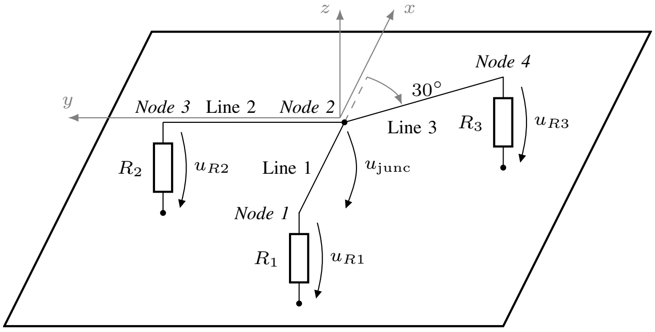

Fig. 1. Exemplary examined network of three single-wire transmission lines above the $x$-$y$-plane. The lines are connected in a junction point that is open-circuited against the ground plane. The origin of the coordinate system is located on the ground plane below this junction.

The transmission line networks of interest consist of several single-wire lines above a perfectly-conducting ground plane that acts as a return conductor. An example is shown in Fig. 1. The analysis of the field-to-wire coupling is based on classical transmission line theory [1] and has therefore some restrictions (no common mode, thin-wire approximation, TEM mode only, electrically small cross-section dimensions, see [2] for details).

Far field sources can be approximated as a plane wave. The definition of the normalized wave vector $\hat{\mathbf{k}}$ and the electric field direction $\hat{\mathbf{e}}$ in a spherical coordinate system using the polar angle $\vartheta$, the angle of azimuth $\varphi$ and the angle of polarization $\alpha$ is shown in [3, Fig. 2]. For the conversion to Cartesian coordinates see [3, Eqs. (4) and (5)].

The time-dependent electrical field with an amplitude $E_0$ at any position $\mathbf{r}$ can be calculated using [8, Eq. (3)], where $f$ is an arbitrary time function and $c$ denotes the velocity of light.

$$\mathbf{E}(\mathbf{r}) = E_0 \cdot (\hat{e}_x\hat{\mathbf{x}} + \hat{e}_y\hat{\mathbf{y}} + \hat{e}_z\hat{\mathbf{z}}) \cdot f\left(t - \frac{\hat{\mathbf{k}} \cdot \mathbf{r}}{c} + t_\beta\right) \quad (1)$$

### B. Equivalent Circuit Model

The basic equivalent circuit model is described in [10, Fig. 11.3]. The specific application to the exemplary three-wire network is shown in Fig. 2. Each line $i$ is split into cells with a length $l_{\mathrm{step},i}$.

The characteristic impedances, partial capacitances and partial inductances for each line can be calculated from the

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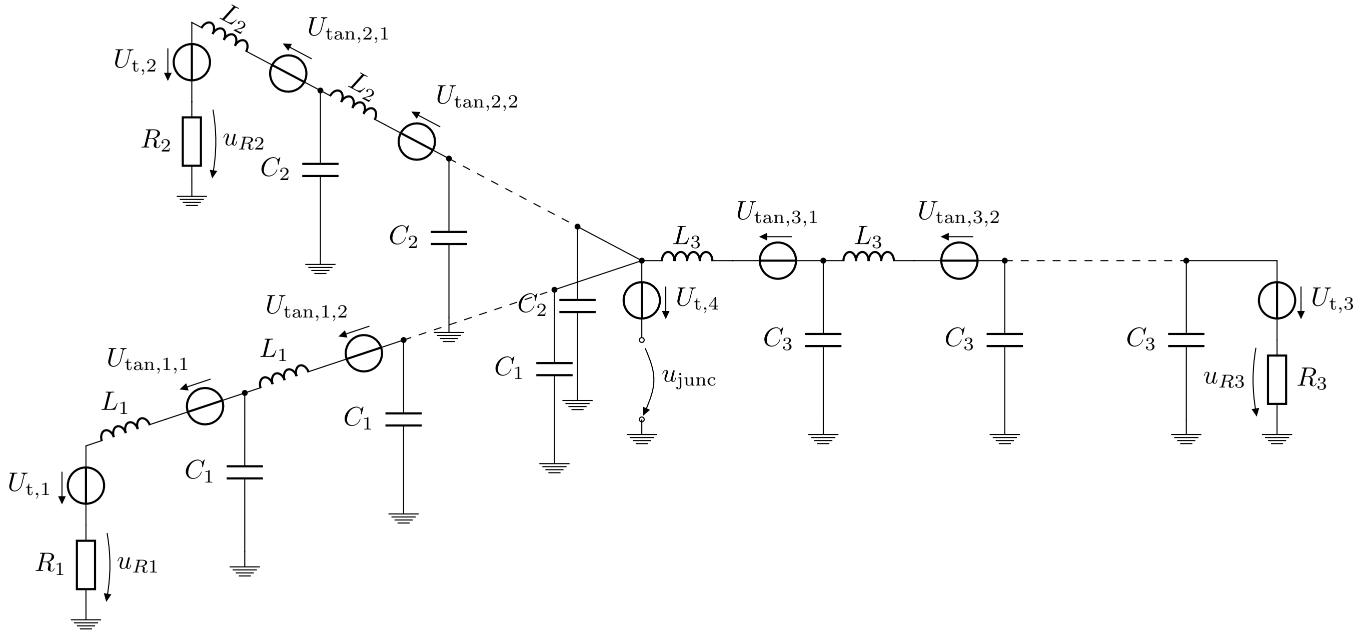

Fig. 2. Equivalent circuit model of the exemplary examined network of three single-wire transmission lines above a perfectly-conducting ground plane. The incident plane wave is considered by distributed sources along the lines and by lumped sources at the terminals using the Agrawal formulation [9] of the transmission line equations.

cross-section dimensions, where $r_0, i$ is the radius and $h_i$ is the average height of the $i$th line above ground.

$$Z_{\mathrm{c},i} = \frac{1}{2\pi}\sqrt{\mu/\varepsilon}\cdot \operatorname{arcosh}\left(h_i/r_{0,i}\right) \tag{2}$$

$$C_i = 2\pi\varepsilon \cdot \frac{l_{\mathrm{step},i}}{\operatorname{arcosh}\left(h_i/r_{0,i}\right)} \tag{3}$$

$$L_i = \frac{\mu}{2\pi}\cdot l_{\mathrm{step},i}\cdot \operatorname{arcosh}\left(h_i/r_{0,i}\right) \tag{4}$$

The distributed sources in each cell along the line are calculated according to the Agrawal formulation [9].

$$U_{\mathrm{tan},i,n} = l_{\mathrm{step},i}\cdot\left[E_{\mathrm{tan,inc},i}(\mathbf{r}_{i,n}) + E_{\mathrm{tan,ref},i}(\mathbf{r}_{i,n})\right] \tag{5}$$

The tangential field component is given by the dot product with a unit tangential vector along each line, see [3, Eq. (12)]. The electric field strength $E_{\mathrm{tan,inc},i}$ of the incident wave is calculated at $\mathbf{r}_{i,n}$ for each cell $n$ of the $i$th single-wire line. The electric field strength $E_{\mathrm{tan,ref},i}$ of the reflected wave is obtained at the equivalent position below the ground plane (with a negative $z$ coordinate of $\mathbf{r}_{i,n}$) according to image theory.

The lumped voltage source for the transverse voltage at each terminal $m$ is given by

$$U_{\mathrm{t},m} = h_i \cdot E_z(\mathbf{r}_m)\,, \tag{6}$$

where $\mathbf{r}_m$ is located on the ground plane below the corresponding terminal $m$ of the transmission line network. The lines, junction nodes, circuit elements and nodes for the MNA are automatically numbered and calculated by algorithm in developed in MATLAB that has also been used in [8].

### C. Diode Model

The diode is described by Shockley's equation [8, Eq. (5)]

$$I_{\mathrm{D}} = I_{\mathrm{S}}\cdot\left(\mathrm{e}^{\frac{U_{\mathrm{D}}}{U_{\mathrm{T}}}} - 1\right)\,, \tag{7}$$

where $I_{\mathrm{D}}$ is the forward current through the diode and $U_{\mathrm{D}}$ is the forward voltage drop across the diode. The reverse-blocking current $I_{\mathrm{S}}$ is assumed to be $1\,\mu\mathrm{A}$. The temperature voltage $U_{\mathrm{T}}$ is fixed to $25\,\mathrm{mV}$. In the MNA, the diode is linearized around an operating point that is iteratively adapted.

## III. EXEMPLARY RESULT AND DISCUSSION

### A. Parameters of the Transmission Line Network and the Incident Plane Wave

The exemplary three-wire network feature line lengths of

$$l_1 = 40\,\mathrm{cm} \qquad l_2 = 30\,\mathrm{cm} \qquad l_3 = 50\,\mathrm{cm}\,. \tag{8}$$

Exact coordinates are given in [3, Eq. (24) to (25)]. The lines share equal heights of $z_{1,i} = z_{2,i} = h_i = 1\,\mathrm{cm}$, but have different wire radii

$$r_{0,1} = 0.6\,\mathrm{mm} \qquad r_{0,2} = 0.5\,\mathrm{mm} \qquad r_{0,3} = 0.4\,\mathrm{mm}\,. \tag{9}$$

The characteristic impedances and chosen load resistances are similar as in [3].

$$Z_{\mathrm{c},1} = 210.2\,\Omega \qquad Z_{\mathrm{c},2} = 221.1\,\Omega \qquad Z_{\mathrm{c},3} = 234.5\,\Omega \tag{10}$$

$$R_1 = 210.2\,\Omega \qquad R_2 = 442.2\,\Omega \qquad R_3 = 117.3\,\Omega \tag{11}$$

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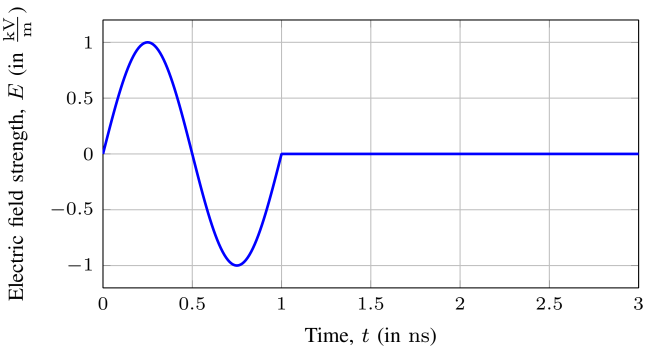


Fig. 3. Single sine pulse as the time function of the incident plane wave

The partial capacitances and inductances in Fig. 2 are calculated for each line according to (3) and (4).

$$C_1 = 47.02\,\mathrm{fF} \qquad C_2 = 44.80\,\mathrm{fF} \qquad C_3 = 42.33\,\mathrm{fF} \qquad (12)$$

$$L_1 = 2.077\,\mathrm{nH} \qquad L_2 = 2.191\,\mathrm{nH} \qquad L_3 = 2.328\,\mathrm{nH} \qquad (13)$$

The incident plane wave has a polar angle of $\vartheta_1 = 150°$ and polarization angle of $\alpha_1 = 180°$. Three different directions are simulated with azimuth angles of

$$\varphi_1 = 0 \qquad \varphi_2 = 270° \qquad \varphi_3 = 150° \qquad (14)$$

so that each line is at first hit at its load resistor. The following normalized wave vectors and electric field vectors are obtained.

$$\hat{\mathbf{k}}_1 = \begin{pmatrix} 1/2 \\ 0 \\ -\sqrt{3}/2 \end{pmatrix} \quad \hat{\mathbf{k}}_2 = \begin{pmatrix} 0 \\ -1/2 \\ -\sqrt{3}/2 \end{pmatrix} \quad \hat{\mathbf{k}}_3 = \begin{pmatrix} -\sqrt{3}/4 \\ 1/4 \\ -\sqrt{3}/2 \end{pmatrix} \quad (15)$$

$$\hat{\mathbf{e}}_1 = \begin{pmatrix} \sqrt{3}/2 \\ 0 \\ 1/2 \end{pmatrix} \quad \hat{\mathbf{e}}_2 = \begin{pmatrix} 0 \\ -\sqrt{3}/2 \\ 1/2 \end{pmatrix} \quad \hat{\mathbf{e}}_3 = \begin{pmatrix} -3/4 \\ \sqrt{3}/4 \\ 1/2 \end{pmatrix} \quad (16)$$

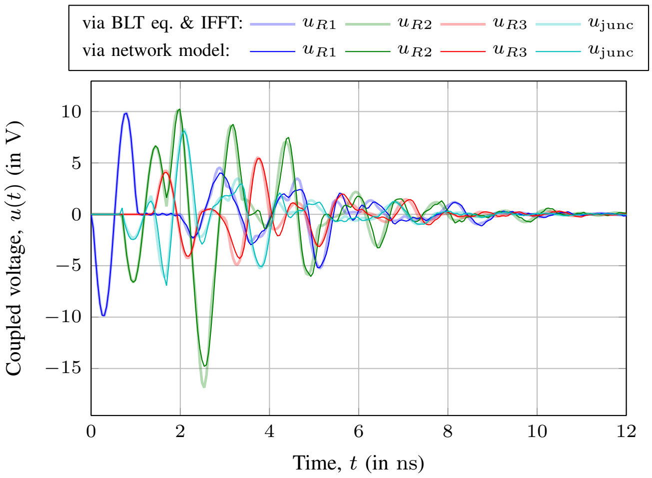
The time function $f$ of the plane wave corresponds to one single sine pulse with a peak amplitude of $1\,\frac{\mathrm{kV}}{\mathrm{m}}$ and a pulse duration of $1\,\mathrm{ns}$, see Fig. 3. This rather high field amplitude has been selected to generate coupled voltages high enough to break through additional diodes at the line terminals.

The transient simulation uses 402 time steps of $49.75\,\mathrm{ps}$ duration each. There are 135 cells for line 1 with a length of $l_{\mathrm{step},1} = 2.963\,\mathrm{mm}$, 101 cells for line 2 with $l_{\mathrm{step},2} = 2.970\,\mathrm{mm}$, and 168 cells for line 3 with $l_{\mathrm{step},3} = 2.976\,\mathrm{mm}$.

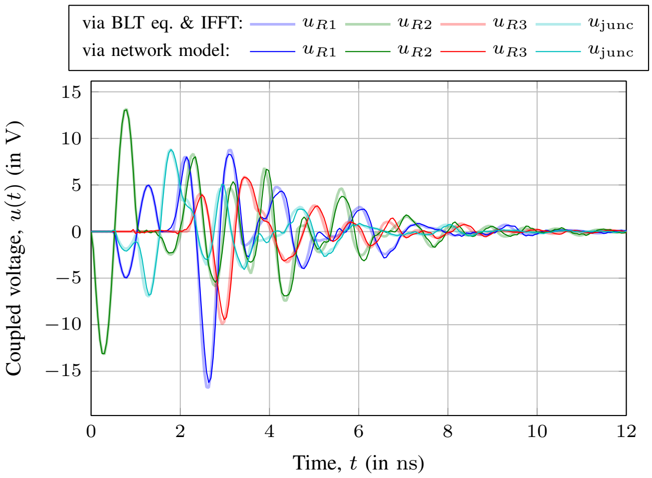
To obtain a causal results, the following time shifts $t_\beta$ have been considered for the electric field in (1).

$$t_{\beta,1} = -696\,\mathrm{ps} \qquad t_{\beta,2} = -529\,\mathrm{ps} \qquad t_{\beta,3} = -863\,\mathrm{ps} \qquad (17)$$

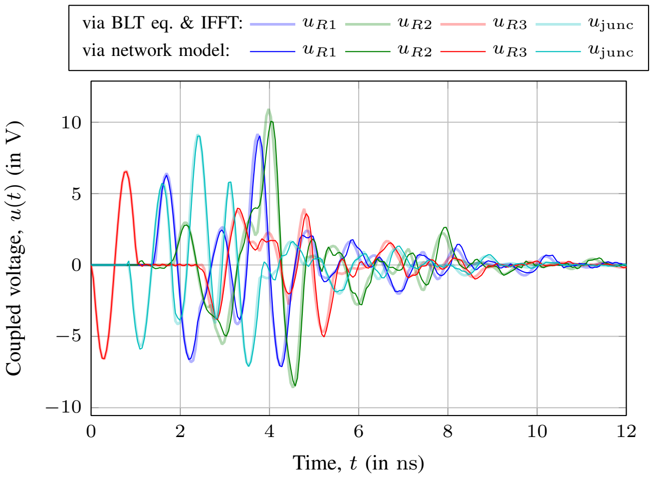
### *B. Validation with Resistive Loads against a Frequency-Domain Solution*



(a) Excitation in the direction of line 1



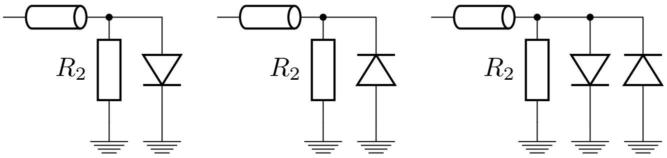
(b) Excitation in the direction of line 2



(c) Excitation in the direction of line 3

Fig. 4. Terminal voltages at the nodes of the linearly-loaded three-wire transmission line network for different directions of the incident plane wave

A comparison of the coupled voltages at the terminals and the junction point of the network for the different incident directions is shown in Fig. 4. For validation, a frequency-domain solution based on the BLT equations [3] has been calculated and converted into time domain using an inverse fast Fourier transform (IFFT). The transit times of $1.33\,\mathrm{ns}$, $1\,\mathrm{ns}$ and $1.66\,\mathrm{ns}$ along the lines as well as the reflection of the voltage waves at the mismatched loads $R_2$ and $R_3$ as well as



(a) Forward diode (b) Reverse diode (c) Anti-parallel diodes

Fig. 5. Different configurations for non-linear loads in parallel to the linear load resistor $R_2$ at the beginning of line 2

the junction point can be clearly observed.

### *C. Simulation with Non-Linear Loading*

In order to analyze the influence of non-linear loads, the configurations shown in Fig. 5 have been simulated. Corresponding results are shown in Fig. 6. It can be observed that the voltage across the load $R_2$ is limited by the diode(s), but might increase at the other loads $R_1$ and $R_3$ as well as at the junction point. Due to the increased mismatch at the diode, the coupled voltages also oscillate for longer time.

## IV. SUMMARY

The plane wave coupling to a linearly and non-linearly loaded three-wire transmission line network has been simulated in the time domain. For linear loads, the results coincide with an existing solution based on the BLT equations. Further work might include the transient analysis of inhomogeneous near fields or of stochastic fields as they occur e. g. in reverberation chambers.

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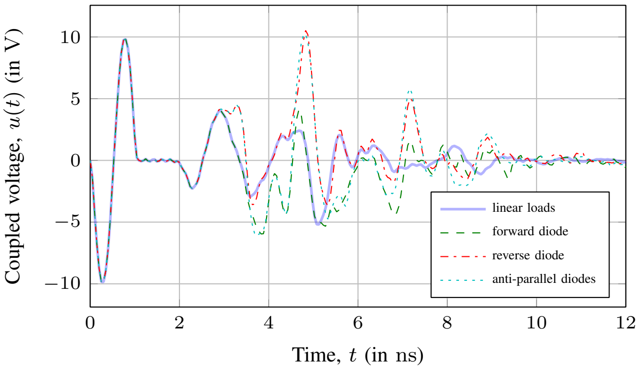
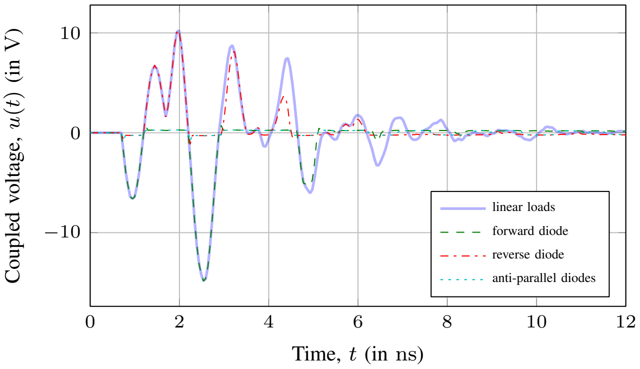
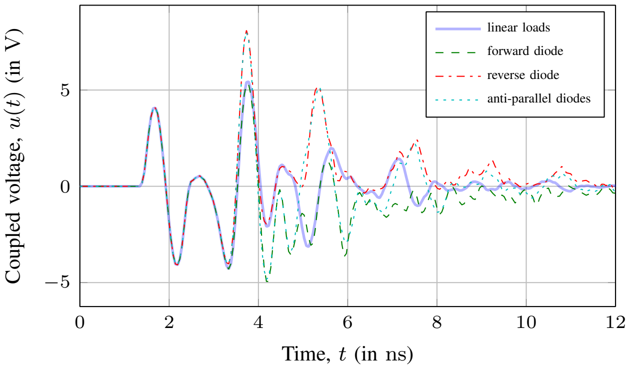


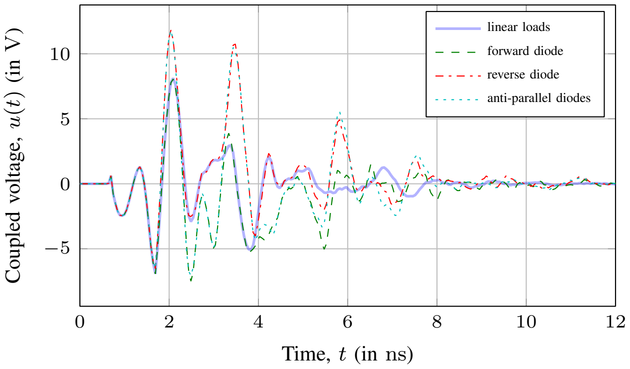
(a) Coupled voltage $u_{R1}$ at the beginning of line 1



(b) Coupled voltage $u_{R2}$ at the beginning of line 2



(c) Coupled voltage $u_{R3}$ at the end of line 3



(d) Coupled voltage $u_{\mathrm{junc}}$ at the star point

Fig. 6. Terminal voltages at the nodes of the three-wire transmission line network for different non-linear loads in parallel to the resistor $R_2$. The incident plane wave hits the network from the direction of line 1.

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