A Fast Time Domain PMCHW Equations for Predicting Transient Responses of Dielectric Objects

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***Abstract*—The time domain discontinuous Galerkin Poggio-Miller-Chang-Harrington-Wu integral equation method based on an improved marching-on-in-degree scheme is proposed for accelerating the simulation of the transient scattering responses of arbitrary shaped homogeneous dielectric objects. A combination of three weighted Laguerre polynomials is set as new temporal basis functions to derive the final equations. Therefore, the entire solution process can be accelerated. Some numerical examples are given to validate its efficiency and accuracy.**

Keywords—Improved marching-on-in-degree (MOD) scheme; Poggio-Miller-Chang-Harrington-Wu (PMCHW) integral equation method; time domain (TD); weighted Laguerre polynomials (LP)

I. Introduction

In recent years, transient electromagnetic responses is important for the EMC design of many structures. Among all the simulation methods, time domain surface integral equation (TDSIE) methods have been widely used to get the transient electric and magnetic responses in time domain [1], [2]. The Time domain Poggio-Miller-Chang-Harrington-Wu (PMCHW) integral equation method is widely adopted for simulating dielectric targets [3]. Because it is only required to mesh the surface of targets and the total number of unknowns can be reduced. Time domain Poggio-Miller-Chang-Harrington-Wu (PMCHW) integral equation method has two main solutions for processing the time term, i.e., marching-on-in-time (MOT) scheme and marching-on-in-degree (MOD) scheme [4], [5], respectively. Compared with TD-PMCHW-MOT method, TD-PMCHW-MOD method can achieve very high stability even for the very late time. However, as we all know, TD-PMCHW-MOD suffers from its high computational cost. Although the memory consumption is tolerable, the entire solution process is time-consuming.

In this paper, a TD-PMCHW with improved MOD method is presented. RWG basis functions are chosen as the spatial basis functions as usual [7]. Further, a combination of three weighted Laguerre polynomials is chosen as a new temporal basis function set [6], [8]. Thus, the CPU time required for this algorithm can be reduced and the computational efficiency can be improved, compared to TD-PMCHW with original MOD scheme. Some numerical results are presented to validate its efficiency and accuracy.

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II. Formulation

A. TD-PMCHW

We consider a dielectric object of permittivity ε_2 and permeability μ_2 in an unbounded space of permittivity ε_1 and permeability μ_1. According to the equivalence principle, there are induced electric and magnetic currents, which are denoted by $\vec{J}$ an $\vec{M}$, respectively. We have the following electric field integral equation:

$$-L_2(\vec{J}) + K_2(\vec{M}) - L_1(\vec{J}) + K_1(\vec{M}) = \vec{E}^i(\vec{r},t)\Big|_{\tan} \tag{1}$$

$$-L_2(\vec{M}) - K_2(\vec{J}) - L_1(\vec{M}) - K_1(\vec{J}) = \vec{H}^i(\vec{r},t)\Big|_{\tan} \tag{2}$$

In order to avoid the time integral in (1) and (2), we introduce a set of new source vectors, which defined as

$$\vec{J}(\vec{r},t) = \partial\vec{e}(\vec{r},t)/\partial t \tag{3}$$

$$\vec{M}(\vec{r},t) = \partial\vec{h}(\vec{r},t)/\partial t. \tag{4}$$

Therefore, we have

$$-L_2(\vec{J}) - L_1(\vec{J}) = \sum_{v=1}^{2}\left\{\frac{\mu_v}{4\pi}\int_S \frac{1}{R}\partial_t^2\vec{e}(\vec{r},\tau_v)dS' - \frac{1}{4\pi\varepsilon_v}\nabla\int_S\frac{1}{R}\nabla'\cdot\vec{e}(\vec{r},\tau_v)dS'\right\} \tag{5}$$

$$K_2(\vec{M}) + K_1(\vec{M}) = \sum_{v=1}^{2}\frac{1}{4\pi}\nabla\times\int_S\frac{1}{R}\partial_t\vec{h}(\vec{r},\tau_v)dS' \tag{6}$$

$$-K_2(\vec{J}) - K_1(\vec{J}) = -\sum_{v=1}^{2}\frac{1}{4\pi}\nabla\times\int_S\frac{1}{R}\partial_t\vec{e}(\vec{r},\tau_v)dS' \tag{7}$$

$$-L_2(\vec{M}) - L_1(\vec{M}) = \sum_{v=1}^{2}\left\{\frac{\varepsilon_v}{4\pi}\int_S \frac{1}{R}\partial_t^2\vec{h}(\vec{r},\tau_v)dS' - \frac{1}{4\pi\mu_v}\nabla\int_S\frac{1}{R}\nabla'\cdot\vec{h}(\vec{r},t_v)dS'\right\} \tag{8}$$

Firstly, we expand $\vec{e}$ and $\vec{h}$ with RWG basis function $\vec{f}_n(\vec{r})$ [7], as described as

$$\vec{e}(\vec{r},t)=\sum_{n=1}^{N}e_n(t)\vec{f}_n(\vec{r}) \tag{9}$$

$$\vec{h}(\vec{r},t)=\sum_{n=1}^{N}h_n(t)\vec{f}_n(\vec{r}) \tag{10}$$

Next, by using $\vec{f}_m(\vec{r})$ as the testing functions, we perform the Galerkin testing procedure to (1) and (2), and we can get

$$\begin{aligned}&\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left\{\left[\mu_v a_{mn}\partial_t^2 e_n(\tau_{mn,v})+\frac{1}{\varepsilon_v}b_{mn}e_n(\tau_{mn,v})\right]\right.\\&\left.+\left[\frac{1}{c_v}I_{mn,k1}\partial_t^2 h_n(\tau_{mn,v})+I_{mn,k2}\partial_t h_n(\tau_{mn,v})\right]\right\}=V_m^E(t)\end{aligned} \tag{11}$$

$$\begin{aligned}&\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left\{-\left[\frac{1}{c_v}I_{mn,k1}\cdot\partial_t^2 e_n(\tau_{mn,v})+I_{mn,k2}\cdot\partial_t e_n(\tau_{mn,v})\right]\right.\\&\left.+\left[\varepsilon_v a_{mn}\cdot\partial_t^2 h_n(\tau_{mn,v})+\frac{1}{\mu_v}b_{mn}^{pq}\cdot h_n(\tau_{mn,v})\right]\right\}=V_m^H(t)\end{aligned} \tag{12}$$

where

$$a_{mn}=\frac{1}{4\pi}\int_S\vec{f}_m(\vec{r})\cdot\int_{S'}\frac{1}{R}\vec{f}_n(\vec{r}')dS'dS \tag{13}$$

$$b_{mn}=\frac{1}{4\pi}\int_S\nabla\cdot\vec{f}_m(\vec{r})\int_{S'}\frac{1}{R}\nabla'\cdot\vec{f}_n(\vec{r}')dS'dS \tag{14}$$

$$I_{mn,k1}=\frac{1}{4\pi}\int_S\vec{f}_m(\vec{r})\cdot\int_S\vec{f}_n(\vec{r}')\times\frac{\hat{R}_k}{R_k}dS' \tag{15}$$

$$I_{mn,k2}=\frac{1}{4\pi}\int_S\vec{f}_m(\vec{r})\cdot\int_S\vec{f}_n(\vec{r}')\times\frac{\hat{R}_k}{R_k^2}dS' \tag{16}$$

N_D is the total number of spatial basis functions.

Different from the traditional MOD method, we adopt a combination of three weighted Laguerre polynomials with successive degrees [7], [9]-[11], as given by

$$e_n(t)=\sum_{j=0}^{\infty}e_{n,j}\left(\phi_j(st)-2\phi_{j+1}(st)+\phi_{j+2}(st)\right) \tag{17}$$

$$h_n(t)=\sum_{j=0}^{\infty}h_{n,j}\left(\phi_j(st)-2\phi_{j+1}(st)+\phi_{j+2}(st)\right) \tag{18}$$

where $e_{n,j}$ are the unknown coefficients. s is the scaling factor. $\phi_j(st)=e^{-st/2}L_j(st)$, and L_j is the Laguerre polynomial of degree j. We assume that $e_{n,j}=0$ and $h_{n,j}=0$ for $j<0$, the above equation can be written as

$$\begin{aligned}&\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left\{\mu_v a_{mn}\sum_{j=0}^{\infty}\frac{s^2}{4}\phi_j(s\tau_{mn,v})\left[e_{n,j}+2e_{n,j-1}+e_{n,j-2}\right]\right.\\&+\frac{1}{\varepsilon_v}b_{mn}\sum_{j=0}^{\infty}\phi_j(s\tau_{mn,v})\left(e_{n,j}-2e_{n,j-1}+e_{n,j-2}\right)\\&+\frac{1}{c_v}I_{mn,k1}\sum_{j=0}^{\infty}\frac{s^2}{4}\phi_j(s\tau_{mn,v})\left[h_{n,j}+2h_{n,j-1}+h_{n,j-2}\right]\\&\left.+I_{mn,k2}\sum_{j=0}^{\infty}\frac{s}{2}\phi_j(s\tau_{mn,v})\left(h_{n,j}-h_{n,j-2}\right)\right\}=V_m^E(t)\end{aligned} \tag{19}$$

$$\begin{aligned}&\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left\{-\frac{1}{c_v}I_{mn,k1}\cdot\sum_{j=0}^{\infty}\frac{s^2}{4}\phi_j(s\tau_{mn,v})\left[e_{n,j}+2e_{n,j-1}+e_{n,j-2}\right]\right.\\&-I_{mn,k2}\cdot\sum_{j=0}^{\infty}\frac{s}{2}\phi_j(s\tau_{mn,v})\left(e_{n,j}-e_{n,j-2}\right)\\&+\varepsilon_v a_{mn}\cdot\sum_{j=0}^{\infty}\frac{s^2}{4}\phi_j(s\tau_{mn,v})\left[h_{n,j}+2h_{n,j-1}+h_{n,j-2}\right]\\&\left.+\frac{1}{\mu_v}b_{mn}^{pq}\cdot\sum_{j=0}^{\infty}\phi_j(s\tau_{mn,v})\left(h_{n,j}-2h_{n,j-1}+h_{n,j-2}\right)\right\}=V_m^H(t)\end{aligned} \tag{20}$$

After that, we use $\phi_j(st)$ as the temporal testing functions and perform the standard Galerkin testing procedure to the above equations. In order to obtain the unknown coefficients $e_{n,i}$ and $h_{n,i}$ of degree i, we reorganize these equations. The incident EMP and all known electric and magnetic current coefficients of degree j ($j=0,1,2,\cdots$, and $(i-1)$) are put in the right-hand side, with a set of compact TD-PMCHW equations with the new MOD scheme derived as

$$\begin{aligned}&\sum_{n=1}^{N_D}Z_{11}^{mn}e_{n,i}+\sum_{n=1}^{N_D}Z_{12}^{mn}h_{n,i}=V_{m,i}^E-P_{m,i}^E-Q_{m,i}^E\\&\sum_{n=1}^{N_D}Z_{21}^{mn}e_{n,i}+\sum_{n=1}^{N_D}Z_{22}^{mn}h_{n,i}=V_{m,i}^H-P_{m,i}^M-Q_{m,i}^M\end{aligned} \tag{21}$$

where

$$V_{m,i}^E=\int_0^\infty\phi_i(st)\left[\int_S\vec{f}_m(\vec{r})\cdot\vec{E}^i(\vec{r},t)dS\right]d(st) \tag{22}$$

$$V_{m,i}^H=\int_0^\infty\phi_i(st)\left[\int_S\vec{f}_m(\vec{r})\cdot\vec{H}^i(\vec{r},t)dS\right]d(st) \tag{23}$$

$$Z_{11}^{mn}=\sum_{v=1}^{2}\left[\frac{\mu_v s^2 a_{mn}}{4}\cdot\exp\left(-s\frac{R}{2c_v}\right)+\frac{b_{mn}}{\varepsilon_v}\cdot\exp\left(-s\frac{R}{2c_v}\right)\right] \tag{24}$$

$$Z_{12}^{mn}=\sum_{v=1}^{2}\left[\frac{s^2}{4c_v}\cdot I_{mn,k1}\cdot\exp\left(-s\frac{R}{2c_v}\right)+\frac{s}{2}\cdot I_{mn,k2}\cdot\exp\left(-s\frac{R}{2c_v}\right)\right] \tag{25}$$

$$Z_{21}^{mn}=-\sum_{v=1}^{2}\left[\frac{s^2}{4c_v}\cdot I_{mn,k1}\cdot\exp\left(-s\frac{R}{2c_v}\right)+\frac{s}{2}\cdot I_{mn,k2}\cdot\exp\left(-s\frac{R}{2c_v}\right)\right] \tag{26}$$

$$Z_{22}^{mn}=\sum_{v=1}^{2}\left[\frac{\varepsilon_v s^2 a_{mn}}{4}\cdot\exp\left(-s\frac{R}{2c_v}\right)+\frac{b_{mn}}{\mu_v}\cdot\exp\left(-s\frac{R}{2c_v}\right)\right] \tag{27}$$

$$\begin{aligned}P_{m,i}^E=\sum_{v=1}^{2}\sum_{n=1}^{N_D}&\left[\mu_v a_{mn}\frac{s^2}{4}I_{ii}(s\tau_{mn,v})\left[2e_{n,i-1}+e_{n,i-2}\right]\right.\\&+\mu_v a_{mn}\sum_{j=0}^{i-1}\frac{s^2}{4}I_{ij}(s\tau_{mn,v})\left[e_{n,j}+2e_{n,j-1}+e_{n,j-2}\right]\\&+\frac{1}{\varepsilon_v}b_{mn}I_{ii}(s\tau_{mn,v})\left(-2e_{n,i-1}+e_{n,i-2}\right)\\&\left.+\frac{1}{\varepsilon_v}b_{mn}\sum_{j=0}^{i-1}I_{ij}(s\tau_{mn,v})\left(e_{n,j}-2e_{n,j-1}+e_{n,j-2}\right)\right]\end{aligned} \tag{28}$$

$$Q_{m,i}^E=\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left[\frac{1}{c_v}I_{mn,k1}\frac{s^2}{4}I_{ii}(s\tau_{mn,v})\left[2h_{n,i-1}+h_{n,i-2}\right]\right.$$

$$+\frac{1}{c_v} I_{mn,k1} \sum_{j=0}^{i-1} \frac{s^2}{4} I_{ij}(s\tau_{mn,v})\left[h_{n,j}+2h_{n,j-1}+h_{n,j-2}\right]$$
$$+I_{mn,k2}\frac{s}{2} I_{ii}\left(s\tau_{mn,v}\right)\left(-h_{n,i-2}\right)$$
$$\left.+I_{mn,k2}\sum_{j=0}^{i-1}\frac{s}{2} I_{ij}\left(s\tau_{mn,v}\right)\left(h_{n,j}-h_{n,j-2}\right)\right\} \quad (29)$$

$$P_{m,i}^{M}=\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left[-\frac{1}{c_v} I_{mn,k1}\cdot\frac{s^2}{4} I_{ii}(s\tau_{mn,v})\left[2e_{n,i-1}+e_{n,i-2}\right]\right.$$
$$-\frac{1}{c_v} I_{mn,k1}\cdot\sum_{j=0}^{i-1}\frac{s^2}{4} I_{ij}(s\tau_{mn,v})\left[e_{n,j}+2e_{n,j-1}+e_{n,j-2}\right]$$
$$-I_{mn,k2}\cdot\frac{s}{2} I_{ii}\left(s\tau_{mn,v}\right)\left(-e_{n,i-2}\right)$$
$$\left.-I_{mn,k2}\cdot\sum_{j=0}^{i-1}\frac{s}{2} I_{ij}\left(s\tau_{mn,v}\right)\left(e_{n,j}-e_{n,j-2}\right)\right] \quad (30)$$

$$Q_{m,i}^{M}=\sum_{v=1}^{2}\sum_{n=1}^{N_D}\left[+\varepsilon_v a_{mn}\cdot\frac{s^2}{4} I_{ii}(s\tau_{mn,v})\left[2h_{n,i-1}+h_{n,i-2}\right]\right.$$
$$+\varepsilon_v a_{mn}\cdot\sum_{j=0}^{i-1}\frac{s^2}{4} I_{ij}(s\tau_{mn,v})\left[h_{n,j}+2h_{n,j-1}+h_{n,j-2}\right]$$
$$+\frac{1}{\mu_v} b_{mn}^{pq}\cdot\sum_{j=0}^{\infty} I_{ij}\left(s\tau_{mn,v}\right)\left(-2h_{n,j-1}+h_{n,j-2}\right)$$
$$\left.+\frac{1}{\mu_v} b_{mn}^{pq}\cdot\sum_{j=0}^{i-1} I_{ij}\left(s\tau_{mn,v}\right)\left(h_{n,j}-2h_{n,j-1}+h_{n,j-2}\right)\right] \quad (31)$$

$$I_{ij}\left(s\frac{R}{c_v}\right)=\int_0^{\infty}\phi_i(st)\cdot\phi_j\left(st-s\frac{R}{c_v}\right)d(st) \quad (32)$$

B. Computational Complexity

To measure their performance, we assume the number of spatial unknowns and the maximum temporal degree are N_D and N_0, respectively. The comparison of time consumption is listed in Table I.

TABLE I. COMPARISON OF COMPUTATIONAL COMPLEXITY

Method	*TD-PMCHW-original MOD*	*TD-PMCHW-improved MOD*
Computational Complexity	$O\left(N_D^2 N_0^2\right)$	$O\left(N_D^2 N_0\right)$

III. Numerical Results And Discussion

In our numerical examples, a temporal modulated Gaussian pulse is used as the incident wave, and described by

$$\vec{E}^i(\vec{r},t)=\hat{u}E_0\left(4/\pi/T\right)e^{-\gamma^2}\cos\left(2\pi f_0 t\right) \quad (10)$$

$$\gamma=4\left(c_0 t-c_0 t_0-\vec{r}\cdot\hat{k}\right)/T \quad (11)$$

where E_0 = 1 *V*. *T* =*8 lm*, and $c_0 t_0$ = *12 lm*.

The example is a dielectric cube ($0.5m\times0.5m\times0.5m$) of relative permittivity $\varepsilon_r=2.5$. The wave vector and polarization of the excitation wave are set to be $+\hat{z}$ and $-\hat{x}$, respectively. f_0=150 *MHz*. The average mesh sizes are 0.2m, 0.15m, and 0.1m, respectively. Correspondingly, the numbers of unknowns are 234, 360, and 522. TD-PMCHW with the original and the improved MOD scheme are applied to simulate the transient scattering responses.

Fig. 1 shows the transient electric current responses and transient magnetic current responses of the dielectric cube at different points. The results agree well with each other. The RCS of the dielectric cube with different mesh sizes obtained by TD-PMCHW with improved MOD scheme are compared with those obtained by original MOD, as shown in Fig.2, which shows that TD-PMCHW with improved MOD can get very accurate results. The required CPU time is listed in Fig. 3. Further, by comparisons, we can see that TD-PMCHW with improved MOD is much faster than TD-PMCHW with the original MOD.

At last, a simple dielectric rocket model is analyzed with relative permittivity ε_r =2.5. The wave vector and the polarization vector are $\hat{k}=-\hat{y}$ and $\hat{u}=+\hat{z}$, respectively. f_0 = 200 *MHz*. The rocket is discretized with the mesh size of 0.06 m, as shown in Fig.4. The total number of unknowns is 2034.

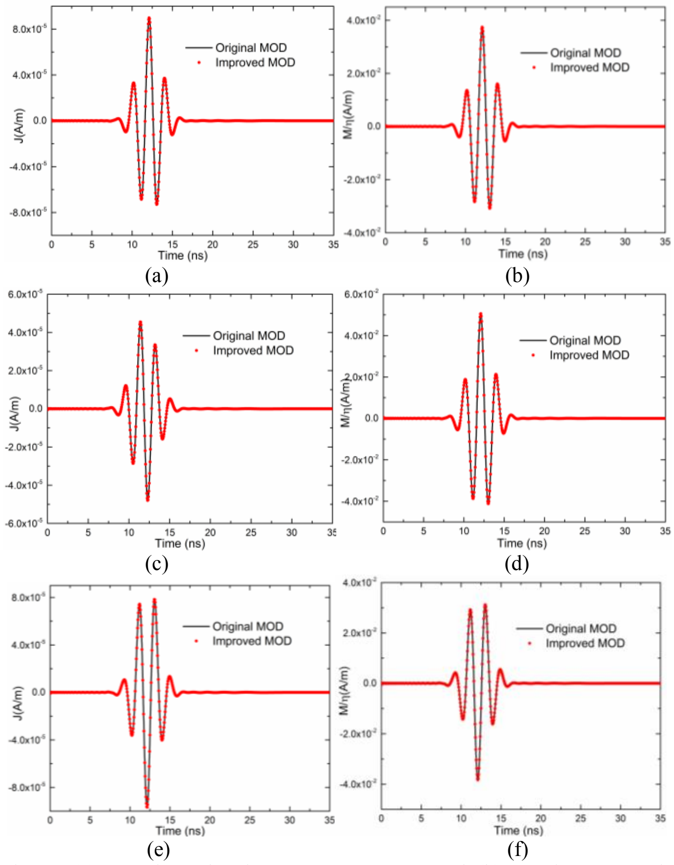


Fig. 1. (a) Transient electric current responses and (b) transient magnetic current responses of the dielectric cube at point $(0.169, 0.343, 0)m$. (c) Transient electric current responses and (d) transient magnetic current responses of the dielectric cube at point $(0.061, 0.367, 0)m$. (e) Transient electric current responses and (f) transient magnetic current responses of the dielectric cube at point $(0.278, 0.225, 0)m$.

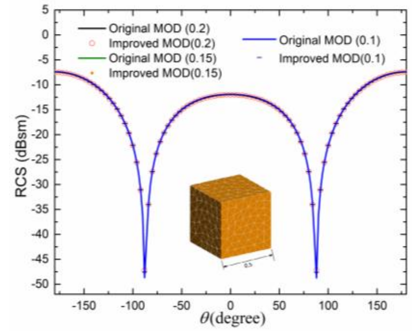


Fig. 2. RCS of the cube with different mesh sizes at f = 150 MHz, when $\varphi = 0^{\circ}$ and θ ranges from -180° to 180°.

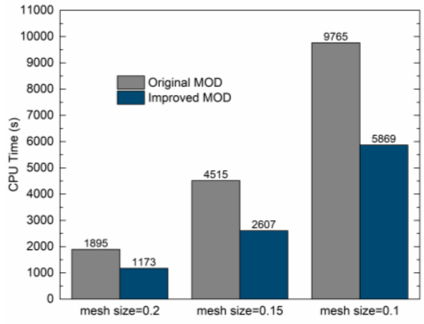


Fig. 3. CPU Time (s).

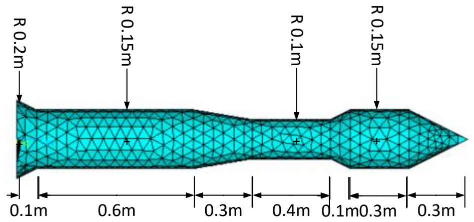
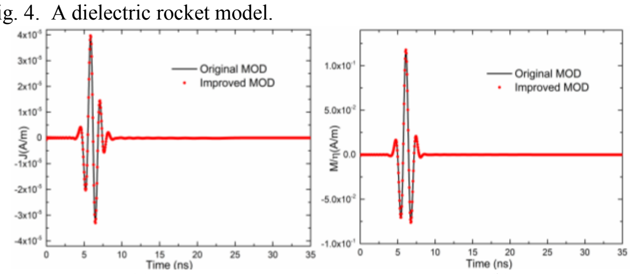


Fig. 4. A dielectric rocket model.



(a) (b)

Fig. 5 (a) Transient electric current responses and (b) transient magnetic current responses of the dielectric rocket at point $(-0.136, 0.059, 1.729)m$.

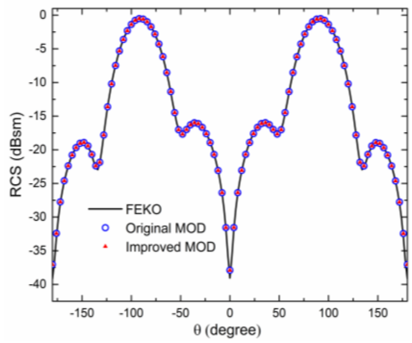


Fig. 6. RCS of the dielectric rocket at f = 200 MHz, when $\varphi = 0^{\circ}$.

TABLE II. COMPARISON OF CPU TIME FOR DIELECTRIC ROCKET

Dielectric rocket with 2034 unknowns	*TD-PMCHW-original MOD*	*TD-PMCHW-improved MOD*
CPU time (min)	1625	927

Fig. 5 shows the transient electric current response and transient magnetic current response of the dielectric rocket obtained by TD-PMCHW with improved MOD method, which shows excellent agreements with those obtained by the original MOD method. In addition, we compare the computed RCS with that obtained by software FEKO at 200 MHz. They are all in good agreements. Moreover, the comparison of the time consumption between TD-PMCHW with our method and traditional MOD is listed in Table II. It is clear that the TD-PMCHW is accelerated with an improved MOD scheme.

IV. CONCLUSION

The TD-PMCHW method with an improved MOD scheme is presented to simulate time-domain responses of dielectric structures. A single weighted Laguerre polynomial is replaced by a combination of three associated Laguerre polynomials with successive degrees. This method not only inherits the benefits of all the original TD-PMCHW-MOD methods, but also simplifies the iterative process. Thus, the computational process can be sped up significantly. The presented numerical results have demonstrated both stability and accuracy of our developed algorithm.

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