

# A Novel Estimation Technique using $K$ -Order Models to Evaluate the Maximum Electric Field of Multiple-Antenna Transmitters

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**Abstract**—In this paper, a novel estimation technique using  $K$ -order models is proposed for evaluating the maximum electric fields radiated from multiple-antenna transmitters. We first develop the  $K$ -order models of the electric fields radiated from multiple transmitting antennas, then utilize them for estimations. Fundamental concepts of the estimations and detailed electric field formulas will be presented and discussed. Several validations are also illustrated to verify the performance and the effectiveness of the proposed technique. For example, in evaluating the maximum electric fields radiated from a two-dipole configuration in an ordinary room, the maximum error caused by conventional estimations can be as high as 8.5%. By applying the proposed technique, it can be decreased to below 0.24% and 0.02% with the 2- and 3-order estimations, respectively.

**Index Terms**—Electric Field, Estimation, MIMO.

## I. INTRODUCTION

In the next generations of wireless communications, multiple-antenna devices will be one of core technologies. Recently, the LTE-Advanced specification indexes  $2 \times 2$  multi-input multi-output (MIMO) schemes. Furthermore, massive MIMO techniques are expected to be a key development in 5G systems. From the viewpoint of RF safety compliance tests [1], finding the maximum electric field (E-field) radiated from such devices is a challenging work, owing to the complexity of their operations. For example, at a measurement point, different sets of the relative phases of radiation sources give different values of the total E-field. Thus, it is difficult to identify the set of the relative phases that gives the maximum E-field.

In specific absorption rate (SAR) measurements [2] for multiple-antenna transmitters, conventional measurement techniques generally require measurements for all possible sets of the relative phases with a phase step [2]–[5]. These techniques are time-consuming or even impractical due to a large number of necessary measurement, particularly when the phase step is decreased and/or the number of antennas is increased.

Some advanced techniques [6]–[9] focus on particular types of signals [6], specific antenna configurations [8], using vector E-field probes [7], or only limited to spatial-averaged SAR [9]. Some of them provide estimated SAR with high errors (the difference between estimated and measured/calculated SARs), up to 5.2% in validations of two antennas in [7] or even higher in [9].

In our previous studies, we developed a fast and simple technique to estimate the SARs of multiple-antenna devices [10]. It requires a limited number of initial measurements for predefined sets of relative phases and estimates the SARs/E-fields for any other relative phase sets. It is verified through different scenarios, including different frequencies, antenna configurations, or measurement systems, providing estimated SARs with relatively small estimation errors.

In this paper, we propose a more advanced technique using  $K$ -order models of E-fields, which greatly reduces the estimation errors. In the following sections, we will explain the need for  $K$ -order estimations and then develop useful expressions for the estimations as well as validate them through various scenarios in simulations and experiments.

## II. $K$ -ORDER ELECTRIC FIELD EXPRESSIONS

In the estimation technique introduced in our previous study [10], the estimation errors are kept relatively small. However, we observed that the error tends to increase when the number of antennas is increased. For instance, it is around 3% for a two-antenna case [10], or 6% for a case of three antennas [11] in practical measurements. Additionally, when using the estimation models in [10] to evaluate the maximum E-field of a multiple-antenna configuration in air in an ordinary room, we found that the error was considerably higher than the case of SAR measurements in a human-body phantom. There are several possible reasons for this phenomenon including the contribution of reflected E-field components to the total E-field at a measurement point. The reflections can be from the inner surfaces of phantom shells in SAR measurements or from room walls in E-field measurements in the air. Generally, the more antennas in a transmitter, the more reflected E-field components contributing to the measurement point. Also, the strength of reflected signals in the air is stronger than that in a human-body phantom. Thus, to reduce estimation error, a new estimation model with considerations on the reflected components is necessary.

Now, let us consider to the total E-field radiated from two antennas at a measurement point. In consideration of the direct and reflected components of the E-fields, the total E-field of

the measurement point can be expressed as

$$E = E_1 + E_2 e^{j\beta} + \sum_{i=1}^K E_i^{(ref)} e^{j\Delta\beta_i}, \quad (1)$$

where  $\beta$  is the relative phase of the two antennas,  $j$  is the imaginary unit ( $j^2 = -1$ ),  $E_1$  and  $E_2$  represent the direct components, and  $E_i^{(ref)}$  ( $i = 1, 2, \dots$ ) represent all reflected components from the two antennas. The  $\Delta\beta_i$  is the phase difference caused by different path lengths of the reflected components. The values of  $\Delta\beta_i$  are randomly distributed and the number of reflected components is indeterminable.

Owing to the complexity of (1), it is difficult to use as an estimation model. Therefore, we need to approximate it to a simplified form with controllable parameters. We will do so from the mathematical perspective. Note that the total E-field in (1) includes the exponential functions of  $j\beta$ , which are periodic with a period of  $2\pi$ . Furthermore, in general, the  $\Delta\beta$  and  $\pm k\beta$  ( $k = 1, 2, \dots$ ) for all different points in a measurement scheme would occupy all values from  $-\pi$  to  $\pi$ . Thus, it is reasonable and equivalent to approximately express the  $\Delta\beta$  as the multiples of  $\beta$ . By doing so, the total E-field at the measurement point can be rewritten as

$$E = \sum_{k=-K}^K a_k e^{jk\beta}, \quad (2)$$

where  $K$  is the order of the mathematical model of the E-field ( $K = 1, 2, 3, \dots$ ) and  $a_k$  ( $k = -K, \dots, K$ ) are complex values, representing the  $K$ -order component of the field. The complex values  $a_k$  are independent of changes in the relative phases. If they can be determined through several initial measurements for predefined phase sets, it is possible to estimate the E-fields for the other phase sets. Here, the phase and amplitude of E-fields must be captured by vector E-field probes [13]. The estimation using such probes is called *vector estimation*.

In practice, some measurement systems can only provide information on the amplitude of the measured E-field (i.e., the  $|E|^2$ ). Thus, the E-field model in (2) can not be used. Now, we will extend the model in (2) for the square of the amplitude of the E-field. The estimation based on this model is called *scalar estimation*. By taking square operations, the square of the amplitude of the total E-field in (2) will be

$$|E|^2 = A_0 + \sum_{k=1}^{2K} [B_k \cos(k\beta) + C_k \sin(k\beta)]. \quad (3)$$

In the above equation, there are  $(4K + 1)$  real parameters  $A_0$ ,  $B_k$ , and  $C_k$  ( $k = 1, \dots, 2K$ ). To determine them,  $(4K + 1)$  initial measurements of  $|E|^2$  are required. This number can be large for high order estimations. If we reduce the order of  $\beta$  to  $K$  in the sine and cosine functions in (3), the number of these parameters reduces to  $(2K + 1)$ . By doing so, we eliminate the components with orders higher than  $K$  and accept that the expression becomes less accurately approximated. Then, the E-field model becomes:

$$|E|^2 \simeq A_0 + \sum_{k=1}^K [B_k \cos(k\beta) + C_k \sin(k\beta)]. \quad (4)$$

TABLE I  
K-ORDER MODELS OF TWO-ANTENNA TRANSMITTERS AND PREDEFINED SETS OF THE RELATIVE PHASE

K	No. of meas.	Formula for $ E ^2$	Predefined $\beta$ [degree]
1	3	$A_0 + B_1 \cos \beta + C_1 \sin \beta$	0, 90, 180
2	5	$A_0 + B_1 \cos \beta + B_2 \cos 2\beta + C_1 \sin \beta + C_2 \sin 2\beta$	0, 90, 180, 225, 270
3	7	$A_0 + B_1 \cos \beta + B_2 \cos 2\beta + B_3 \cos 3\beta + C_1 \sin \beta + C_2 \sin 2\beta + C_3 \sin 3\beta$	0, 90, 135, 180, 225, 270, 315

When  $K = 1$ , the model in (4) becomes the fundamental model for scalar estimation developed in our previous work [10]. Table I lists some examples of the E-field models and the predefined sets of the relative phases for the case of two-antenna. The E-fields/SARs for these sets will be used to calculate the estimation factors.

Now, at a measurement point, in the  $k^{\text{th}}$  measurement when the relative phase of the sources is set to  $\beta_{(k)}$ , let  $|E_k|^2$  be the measured value of the square of amplitude of the total E-field ( $k = 1, \dots, (2K + 1)$ ). The estimation factors of the point can be determined by solving the following linear equations:

$$\begin{cases} |E_1|^2 & = A_0 + B_1 \cos(\beta_{(1)}) + \dots + \\ & B_K \cos(K\beta_{(1)}) + C_1 \sin(\beta_{(1)}) \\ & + \dots + C_K \sin(K\beta_{(1)}); \\ \vdots & \vdots \\ |E_{2K+1}|^2 & = A_0 + B_1 \cos(\beta_{(2K+1)}) + \dots + \\ & B_K \cos(K\beta_{(2K+1)}) + C_1 \sin(\beta_{(2K+1)}) \\ & + \dots + C_K \sin(K\beta_{(2K+1)}), \end{cases} \quad (5)$$

When the number of antennas is greater than two, we can expand the models in similar ways. The following is an example for three-antenna devices. The E-field at a measurement point can be expressed as

$$E = \sum_{p=-K}^K \sum_{q=-K}^K a_{pq} e^{jp\beta} e^{jq\alpha}, \quad (6)$$

where  $\beta$  and  $\alpha$  are the relative phases between the second and third antennas versus the first antenna, respectively.

Similarly, for the scalar model, the formula for three-antenna transmitters can be expressed as

$$\begin{aligned} |E|^2 \simeq & A_0 + \sum_{p=0}^K \sum_{q=1}^K [B_{pq} \cos(p\beta + q\alpha) + C_{pq} \sin(p\beta + q\alpha)] \\ & + \sum_{p=1}^K \sum_{q=-K}^0 [B_{pq} \cos(p\beta + q\alpha) + C_{pq} \sin(p\beta + q\alpha)]. \end{aligned} \quad (7)$$

where  $A_0$ ,  $B_{pq}$ , and  $C_{pq}$  are also the scalar estimation factors.

For both vector and scalar estimations, the above analyses suggest that the number of necessary measurements for predefined sets of relative phases is  $(2K + 1)$  for the two-antenna case and  $(2K + 1)^2$  for the case of three-antenna transmitters. In general, the number of necessary measurements for  $N$ -antenna transmitters in the  $K$ -order model is  $(2K + 1)^{N-1}$ .

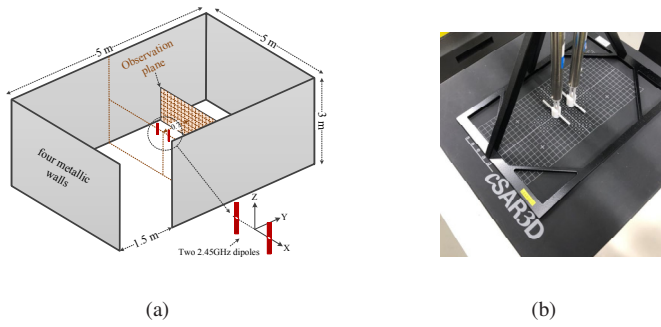


Fig. 1. Validation examples: (a) two 2450-MHz dipoles in an ordinary room, and (b) two 2140-MHz dipoles in SAR measurements.

### III. VALIDATIONS

To verify the effectiveness of the proposed  $K$ -order estimation models, we present here two typical examples. The first is a numerical validation, where we evaluate the maximum E-field of two 2450-MHz dipoles in an ordinary room. The second is an experimental validation, where we conduct the SAR measurements for a configuration of two 2140-MHz dipoles. In all cases, the dipole antennas are excited so that the amplitudes of the sources are handled at their maximums, and the relative phase  $\beta$  of the two dipoles changes from  $0^\circ$  to  $360^\circ$  with a step. Computed E-fields ( $|E|^2$ ) or measured SARs for the predefined sets of the relative phase  $\beta$  shown in Table I are used for calculating the estimation factors. After the estimation factors are determined, vector and scalar estimations will be performed for different sets of the relative phases accordingly to (2) and (4). For convenience, the estimated and the measured SAR or the computed E-fields ( $|E|^2$ ) are normalized to the maximum value of them.

The estimation error is defined as the difference between the normalized estimated and measured SAR (or the computed E-field) for each relative phase. Now, let  $\overline{\text{SAR}}_{est}$  and  $\overline{\text{SAR}}_{meas}$  be the normalized SARs at the observation plane obtained from estimation and measurement, respectively. The estimation error can be written as

$$\text{Error} = 100 \cdot (\overline{\text{SAR}}_{est} - \overline{\text{SAR}}_{meas}) \quad [\%]. \quad (8)$$

#### A. Numerical validation with two dipoles in an ordinary room

Fig. 1(a) shows the two-dipole configuration in an ordinary room. For simplicity, the room is modeled with all four walls made of a perfectly conducting material. The size of the room is  $5 \times 5 \times 3$  [m<sup>3</sup>]. Two dipoles, separated by a quarter wavelength, are placed at the center of the room, and the observation plane is 0.7 m from the antennas. In general, we can use any other planes in the validations. This example can also be viewed as an office room with metallic blind curtains, where signals are reflected many times inside the room. A smaller room would result in stronger reflections. The antennas can be considered as radiation sources from communication devices such as WiFi hotspots or similar sources.

Fig. 2 shows the estimation errors calculated at the maximum point in the observation plane in different relative phases.

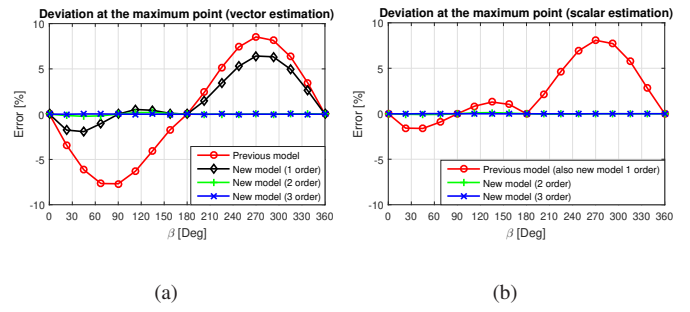


Fig. 2. Deviations between estimated and simulated  $|E|^2$  at the maximum point in the observation plane: (a) vector estimation, (b) scalar estimation.

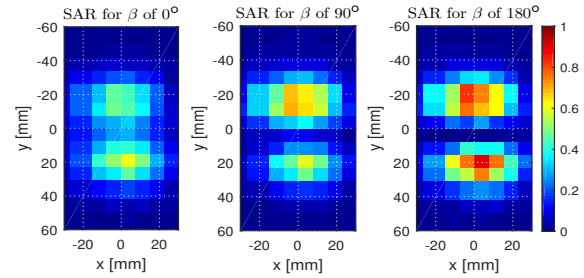


Fig. 3. Measured SAR distributions for different values of  $\beta$ .

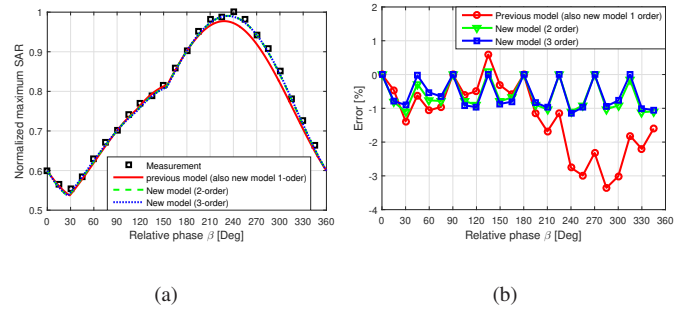


Fig. 4. SAR measurement results of two dipoles: (a) normalized maximum local SAR in the observation plane, and (b) estimation error.

As can be seen from this figure, the estimation errors in the  $K$ -order models (2- and 3-order models) are much smaller than those of the previous models. For both vector and scalar estimations, the difference of nearly 10% between estimated and simulated E-field at the relative phase of  $270^\circ$  in the previous model can be decreased to below 0.2% for 2- and 3-order estimations. This significant reduction highlights the excellent performance of the  $K$ -order models in providing precisely estimated E-fields.

#### B. Experimental validation in SAR measurement

Fig. 1(b) shows a photograph of the antenna configuration in SAR measurements. The dipoles are separated by a distance of a quarter wavelength, and placed 10 mm above a flat phantom of a SAR measurement system [14]. Note that SAR is proportional to the square of amplitude of internal E-field, thus the  $K$ -order SAR estimation model for two-antenna case is the same to (4). Since we only have the measured SAR point

TABLE II  
COMPARISON OF HIGH-ORDER ESTIMATIONS

Properties	Our work	Technique in [9]
Supports vector estimations	Yes	No
Supports point estimations	Yes	Not given
Volume scan in SAR measurement	Once	Every measurement
Estimation errors	Small	Large
Estimation factor computations	Linear	Nonlinear (curve fitting)

data from the SAR measurement system, we will present the validation of the scalar estimation of the  $K$ -order model using measured data.

Fig. 3 shows the normalized local SAR for the relative phases of  $0^\circ$ ,  $90^\circ$ , and  $180^\circ$  in the observation plane. For different relative phases, we can see different distributions and peak value of local SAR. These data will be used to calculate the estimation factors for 1-order estimation. Fig. 4(a) presents the normalized maximum local SAR in the measurement plane. From this figure, it is easy to identify the maximum local SAR and its corresponding relative phase among different phase sets. Fig. 4(b) shows the estimation errors calculated at the maximum SAR point, where a clear improvement of the high-order estimations can be seen. For example, the estimation error caused by the model in [10] (also the 1-order of the new model) is larger 3%, which is reduced to about 1% by the 2-order or 3-order of the new model.

In the numerical validations, we can see major improvements of the estimation accuracy in high-order estimations (2- or 3-order), as shown in Figs. 2(a) and 2(b), for instance. However, in the experiments, the 3-order estimation (or higher order) does not further reduce the estimation error. It is because of the uncertainty of the measurement systems, and the incorrect setting of the relative phases of the sources during measurements as analyzed in [12]. Thus, for the actual SAR evaluation of multiple-antenna devices, the 2-order model is recommended. It also does not require a significant number of measurements for estimations.

#### IV. DISCUSSION AND RELATED WORKS

We have conducted a various number of other validations, including different antenna types, frequencies, number of antennas, and phantoms. In all examined cases, the  $K$ -order estimations can greatly reduce the estimation errors. Although it requires the larger a number of initial measurements, compared to that of the conventional technique in [10], the proposed  $K$ -order estimations provide an excellent solution for evaluating the maximum E-fields of multiple-antenna transmitters.

Regarding the determination of the maximum spatial-averaged SAR of multiple-antenna transmitters in portable devices, a closely related work with a high-order estimation model has recently been reported by Li *et al.* [9]. Table II gives a comparison of our proposed  $K$ -order estimation technique and the high-order estimation in [9]. While they technique mainly focuses on estimating the maximum spatial-averaged SAR, our proposal is for point-based estimations, which generally give smaller estimation errors. By supporting

both vector and scalar E-field probes, our proposed technique can be applicable for more general purposes such as to estimate E-fields or the Poynting vector in the near field of multiple-antenna 5G terminals in millimeter wave bands. Furthermore, since the set of the relative phases corresponding to the maximum SAR can be found by estimations in a reference plane, our technique does not require volume scans in every SAR measurement. Also, the computational method in determining estimation parameters in [9] is nonlinear using least-squares curve fitting, whereas they can be determined by solving linear equations in our technique, resulting in much faster and more accurate computations.

#### V. CONCLUSION

We propose in this paper a novel technique using  $K$ -order models to evaluate the E-fields radiated from multiple-antenna transmitters. Beside presenting expressions of the  $K$ -order models, we successfully demonstrate the performance and the effectiveness of the proposed technique. Compared with previously developed estimation techniques, the proposed  $K$ -order estimation technique not only gives smaller estimation errors but can also be used for a wider range of applications and in measurement systems for multiple-antenna transmitters such as for SAR evaluation and E-field (phase and amplitude) determination in various environments.

Since the proposed  $K$ -order estimation may require a relatively large number of measurements for predefined combinations of relative phases, there is a trade-off between the number of necessary measurements and the accuracy of measurements. An appropriate choice of the estimation model and its order is needed, depending on the circumstances, to satisfy other factors such as the evaluation time and cost.

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