Computationally-efficient Algorithm for Robust Optimization of Electromagnetic Design Problems

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***Abstract*—In the design stage of electromagnetic device, the way to handle the uncertainty of design parameters has attracted wide attention. Monte Carlo Method works well when dealing with uncertainties but it consumes too much time and computing resource. This paper proposes a computationally efficient way to achieve robustness based on Stochastic Collocation Method and TEAM 22 is used as a verification example. It is demonstrated that the approach combining Stochastic Collocation Method and genetic algorithm provides high computational efficiency without losing accuracy comparing with Monte Carlo Method.**

Keywords—uncertainty, robust, Stochastic Collocation Method, Monte Carlo Method, TEAM

I. INTRODUCTION

For finding more robust and reliable designs, the uncertainties from design variables and design process have been widely considered in the design stage of electromagnetic (EM) devices. Many scholars have used a variety of methods to obtain a robust device or control method. Some examples of the state-of-the-art optimization methods used in the field comprise a possibility-based optimal design algorithm [1], a surrogate modeling technique based on a second-order equation [2], and the space-time kriging surrogate model [3].

The previous methods require a lot of time to obtain a robust solution, which significantly affect computational efficiency of EM optimization [4-6] .Therefore, some measures are taken such as evaluating the robustness by the worst-case optimization [4], reusing global surrogate model [5], and adopting an efficient serial-loop optimization strategy [6]. However, these methods more or less show limitations when they are applied to other optimization problems.

To improve the numerical efficiency of EM optimization while maintaining design robustness, a general optimization strategy based on Stochastic Collocation Method (SCM) [7,8] is proposed in this paper. The basic concepts of SCM are given in Section II. Section III shows the process of obtaining the robust optimal solution based on SCM. Section IV expounds the robust TEAM 22 problem [9] and compares the robustness and time consuming of the optimization results obtained by the different methods. Finally, the paper's conclusions are in Section V.

II. OUTLINE OF THE STOCHASTIC COLLOCATION METHOD

Over recently years, uncertainty analysis methods have become of more interest in computational electromagnetics (CEM) in order to take account of practical complexity and unpredictability within a simulation. To this end, design parameters of EM simulation are presented by random variables with properly assigned distributions.

The Stochastic Collocation method (SCM) is a popular choice for the stochastic processing of complex systems where well-established deterministic codes exist. By utilizing the SCM method, the relationship between the result of uncertainty analysis and the variables is approximated by the sum of the polynomials. One way is to use a Lagrange interpolation approach that is given by

$$y(x) = Lag(f(x)) = \sum_{i=0}^{n} f(x_i) l_i(x) \tag{1}$$

where $y(x)$ is a polynomial approximation of the true solution $f(x)$. x_i and $f(x_i)$ stand for the collocation points and the corresponding deterministic solutions of these points, respectively. $l_i(x)$ are the Lagrange interpolation polynomials structured by the collocation points.

$$l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)} \tag{2}$$

As for the SCM, the collocation points is given by the zero points of the generalized Polynomial Chaos [7]. The orthogonal polynomial basis is selected according to the probability distribution of the random variables, as shown in Table I. Specially, if the variables are multidimensional, the interpolating points are the tensor product form of interpolating points in every dimension [8]. The accuracy of the SCM is elaborated in [10].

TABLE I. THE CORRESPONDING BETWEEN THE TYPE OF GENERALIZED POLYNOMIAL CHAOS AND RANDOM VARIABLES

Random variables	**Wiener-Askey chaos**	**Support**
Gaussian	Hermite-chaos	$(-\infty, +\infty)$
Gamma	Laguerre-chaos	$[0, +\infty)$
Beta	Jacobi-chaos	$[a, b]$
Uniform	Legendre-chaos	$[a, b]$

III. SCM Based Robust Optimal Design

The traditional optimal design problem is constructed as

$$\begin{aligned} &\min f(\boldsymbol{P}_{\mathrm{d}}) \\ &s.t.\ g_i(\boldsymbol{P}_{\mathrm{d}}) \le 0, \quad i = 1,\ldots,m \end{aligned} \tag{3}$$

where $f(\boldsymbol{P}_{\mathrm{d}})$ is the objective function for design variable set $\boldsymbol{P}_{\mathrm{d}}$; $g_i(\boldsymbol{P}_{\mathrm{d}})$ are the constraint functions for i=1,…,m.

When the uncertainties are taken into account, the random design variable set $\boldsymbol{P}$ is defined as

$$\boldsymbol{P} = \overline{\boldsymbol{P}} + \xi$$

where $\overline{\boldsymbol{P}}$ is the mean value of $\boldsymbol{P}$ according to the statistical definition, while ξ is the random variable whose distribution can be assumed to be uniform in order to obtain a robust solution to the optimization problem. Thus, $\boldsymbol{P}$ can be redefined by normalization as

$$\boldsymbol{P} = \overline{\boldsymbol{P}} + \boldsymbol{\eta} \cdot x \tag{4}$$

where $\boldsymbol{\eta}$ is half of the range of the distribution and x a random number between [-1,1].

The incorporation of the robustness analysis with formulation (4) incurs high computational time. For the traditionally used Monte Carlo method (MCM), serious computational burden is imposed due to the required large sample size as well as the iterative nature of the design optimization process. In contrast, the SCM is similar to MCM in the sense that it involves only the solution of a sequence of deterministic calculations at given collocation points in the stochastic space.

By applying the SCM to robust optimal design problem, $f(\boldsymbol{P})$ can be approximated by Lagrange interpolation polynomial according to (1).

$$f(\boldsymbol{P}) \approx y(\boldsymbol{P}) = \sum_{i=1}^{n} f(\boldsymbol{P}_i) l_i(\boldsymbol{P}) \tag{5}$$

where $\boldsymbol{P}_i$ (i=1,2,…,n) is the collocation points for univariate or tensor product form of interpolating points for multivariate with the similar form as (4)

$$\boldsymbol{P}_i = \overline{\boldsymbol{P}} + \boldsymbol{\eta} \cdot x_i$$

Generally, the zero points of generalized Polynomial Chaos in Table I are chosen to be x_i. For instance, the Legendre polynomials are orthogonal with respect to the uniform distribution. Its expression is as follow.

$$L_0(x) = 1, L_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left\{\left(x^2 - 1\right)^n\right\}, \qquad n = 1, 2, \ldots \tag{6}$$

Let the zero points of the n-dimensional Legendre polynomial be $x_1, x_2, \ldots, x_n$, and the Lagrange basis polynomial $l_i(x)$ are given by (2).

After the $y(\boldsymbol{P})$ is constructed, the mean of the obtained n values is defined as the objective function value of $\boldsymbol{P}$ as MCM does. It is denoted as:

$$\hat{f}_S(\boldsymbol{P}) = \frac{1}{n}\sum_{i=1}^{n} y(\boldsymbol{P}_i) \tag{7}$$

The SCM is not subject to the number of sampling points but to the number of variables. For problems where the solution is a smooth function of the random input variables and the dimension of the stochastic space is moderate, SCM has been shown to converge much faster than MCM [11].

IV. Application to TEAM 22

A. TEAM workshop problem 22

The TEAM workshop problem 22 is an optimization case of the Superconducting Magnetics Energy Storage (SMES) that has been used as a benchmark problem in magneto statics. The goal of TEAM 22 is to find the best configuration in SMES device to maintain the stored energy while minimizing the stray field. The stray field is represented by magnetic flux density $\boldsymbol{B}_{\mathrm{stray}}$ and it is evaluated in 21 equidistant points marked on lines a and b in Fig.1.

$$\boldsymbol{B}_{\mathrm{stray}}^2 = \frac{\sum_{i=1}^{21} \boldsymbol{B}_{\mathrm{stray},i}^2}{21} \tag{8}$$

Furthermore, to keep the superconductivity characteristic, the restriction is given by the inequality:

$$\|J\| \le (-6.4\|\boldsymbol{B}_{\max}\| + 54) \ \ (\mathrm{A/mm^2}) \tag{9}$$

Since the current density of both coils is fixed in the TEAM 22, the inequality (9) can also be expressed as:

$$\|\boldsymbol{B}_{\max}\| \le 4.92\mathrm{T} \tag{10}$$

When the robustness is taken into account in the optimization of TEAM 22, it is assumed that the adjustable

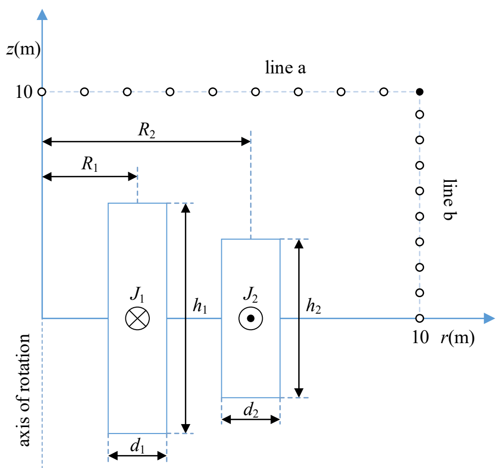

Fig.1. The configuration of the TEAM 22

parameters R_2, h_2, d_2 in outside coil suffer undesirable and unavoidable presence of the uncertainties that can cause damage in the optimization system and its results. The goals of the robustness-considered case remain the same as the classical problem. Consider the design variables $\boldsymbol{P}=\{R_2, h_2, d_2\}$, the robust TEAM 22 problem can be formulated with the objectives and the restriction above as

$$\min f(\boldsymbol{P}) = \omega_1 \frac{\boldsymbol{B}_{\text{stray}}^2}{\boldsymbol{B}_{\text{norm}}^2} + \omega_2 \frac{|E-E_0|}{E_0}$$
$$s.t.\ \ \|J\| \le (-6.4\|\boldsymbol{B}_{\max}\| + 54)(\text{A/mm}^2)$$

where $\boldsymbol{B}_{\text{norm}}$=200μT; $\boldsymbol{B}_{\max}$ is the maximum magnetic flux density; E is the energy that is actually stored in the designed device; E_0 is the target stored energy with a fixed value 180MJ; ω_1, ω_2 are the barycentric weights. Take ω_1=0.001, ω_2=1 in this case to keep the relative value of the leakage flux in the same order of magnitude as the relative error of the stored energy:

$$\min f(\boldsymbol{P}) = 0.001 \times \frac{\boldsymbol{B}_{\text{stray}}^2}{\boldsymbol{B}_{\text{norm}}^2} + \frac{|E-E_0|}{E_0} \qquad (11)$$
$$s.t.\ \ \|B_{\max}\| \le 4.92\text{T}$$

The genetic algorithm (GA) is used to optimize the TEAM22, and the robustness is achieved by using MCM and SCM respectively at the fitness function. The termination condition of GA is set to iterate 20 generations and the optimization results are compared after 20 generations. The workstation used in this paper is Dell T7610 with Intel(R) Xeon(R) E5-2687W v2 3.4GHz and 128G RAM.

B. Robustness Achieved by SCM

The cubic polynomials can lead the uncertainty analysis to convergence considering both computational efficiency and accuracy according to [12]. Thus, the number of configuration points are chosen to be 3 in SCM. The three-dimensional form of the Lagrange interpolation formula is given by:

$$\begin{aligned} f(x,y,z) &\approx y(x,y,z) \\ &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} f(x_i, y_j, z_k) l_k(z) l_j(y) l_i(x) \end{aligned} \qquad (12)$$

As for the TEAM 22, the interpolation formulation is formed as:

$$y(\boldsymbol{P}) = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} f(R_{2(i)}, h_{2(j)}, d_{2(k)}) l_k(z) l_j(y) l_i(x) \qquad (13)$$

where

$$\begin{cases} R_{2(i)} = \overline{R_2} + \eta_{R_2} \cdot x_i \\ h_{2(j)} = \overline{h_2} + \eta_{h_2} \cdot y_j \\ d_{2(k)} = \overline{d_2} + \eta_{d_2} \cdot z_k \end{cases}$$

with x, y, z random numbers between [-1,1]. The interpolating points are $\{I_1, I_2, I_3\} = \left\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\right\}$ where I can be x, y or z, and the tensor product form is $\left\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\right\} \otimes \left\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\right\}$ .Thus, equation (13) can provide the answer.

Similarly, 100 values of x, y and z are each randomly taken and substituted into (14) to obtain the corresponding objective function value .The notation $\hat{f}_S(\boldsymbol{P})$ is defined as the mean of the 100 obtained values to represent the objective function value of $\boldsymbol{P}$ given by:

$$\hat{f}_S(\boldsymbol{P}) = \frac{1}{100}\sum_{i=1}^{100} y(\boldsymbol{P}_i) \qquad (14)$$

The result of optimization is shown in Fig.2. The optimal solution is $\boldsymbol{P}$ = {3.123, 0.534, 0.344}.

C. Results and Discussion

The maximum value of objective function M in the robust interval $\boldsymbol{D} = 0.6\boldsymbol{\eta}$ is found after obtaining the optimized solution. The rate of change in function value is defined as

$$\delta = \frac{M-f}{f}$$

where f is the optimal function value obtained by different methods.

The "normal" solution that does not consider robustness is also obtained. And it is compared with the solutions obtained above, as outlined in Table II where t is the time consumed for obtaining the optimal solution. The f obtained by SCM is similar to the "normal" one after 20 generations of operation. It can be seen from δ that the performance of EM device

TABLE II. COMPARISON OF ROBUST OPTIMIZATION RESULTS OBTAINED BY DIFFIRENT METHODS

Method	R(m)	h(m)	d(m)	f	M	δ	t(s)
Normal	3.08	0.508	0.389	0.0180	0.0321	78.33%	4863.35
MCM	3.115	0.608	0.311	0.0197	0.0241	22.34%	146415.48
SCM	3.123	0.534	0.344	0.0185	0.0241	30.27%	23288.96

considering robustness is more stable. When taking the robustness into account, the time consumed increases substantially. And the efficiency of achieving robustness by using SCM is improved by nearly 84.09% comparing with the time consumed by using MCM. Thus the SCM is an effective technique in terms of accuracy and computational efficiency.

V. Conclusion

The uncertainties which widely existed in design variables and design process is taken into account in the optimization of EM device. The SCM exhibits comparable calculation accuracy to MCM with time consuming much less than MCM. When dealing with TEAM 22, both MCM and SCM obtained robust solution. The computing efficiency using SCM is improved by nearly 84.09%.

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