Design of Experiments for Analyzing the Efficiency of a Multi-Coil Wireless Power Transfer System Using Polynomial Chaos Expansion

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***Abstract*—Pitch distances of single coils in multi-coil wireless power transfer systems have a substantial impact on the efficiency. But traditional full factorial (FF) design of experiments is time-consuming due to the large number of measurements in order to describe the efficiency with varying pitch distances. In this paper, polynomial chaos expansion (PCE) is applied to construct a design of experiments and to obtain a surrogate model of the efficiency of a multi-coil system. Instead of large-scale FF, the proposed method reduces the maximum number of necessary measurements by a factor of 6 to 20, while providing accurate information on the efficiency. The agreement between the results of the PCE function and FF design is analyzed in order to validate the method.**

***Keywords*—Full factorial design, multiple coils, polynomial chaos expansion (PCE), wireless power transfer.**

I. Introduction

Traditionally, full factorial design (FF), which is accurate but time-consuming, is applied in the engineering field to investigate the performance of real-world systems with regard to the variation of key parameters. Recently, polynomial chaos expansion (PCE) [1-4] has proven to be feasible and helpful for the modeling of complex systems. For many applications for multi-coil wireless power transfer (WPT), the efficiency varies and is highly dependent on the positioning and distances of coils [5][6]. In this contribution, FF is applied to analyze the efficiency of a 4-coil WPT system with each pitch distance (d_1, d_2, and d_3) shown in Figure 1 subject to change. As an attractive alternative, PCE is introduced to design the experiment of the WPT system in order to investigate the transfer efficiency using an order of magnitude less experimental setups. A test WPT system has been manufactured and the efficiency of certain sampling nodes preselected by PCE is measured to generate a surrogate function of the efficiency. The function can describe the efficiency with regard to the wide-range variation of each pitch distance. In order to reduce the complexity of the problem, the pitch distances, d_1 and d_3, are varied while the total distance (D_{total}) is kept fixed.

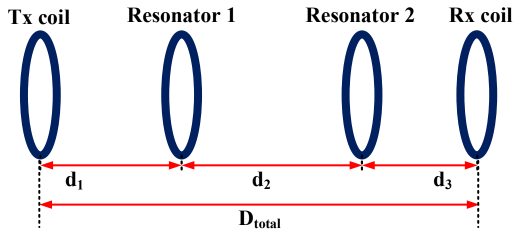


Fig. 1. Setup of a 4-coil wireless power transfer system. The total distance between coil 1 and 4 is fixed. d_1 and d_3 are independent variables and d_2 is a dependent variable.

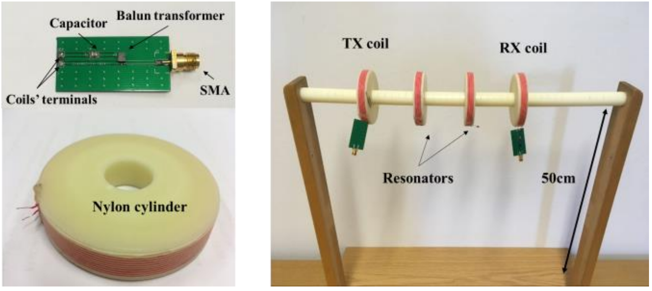
The remainder of this contribution is structured as follows: the experimental device of a 4-coils WPT system constructed for measurements of the efficiency is shown in Section II together with some results of FF. In Section III, the PCE method is introduced and the results of PCE are compared to those from the FF design. Lastly, a conclusion is given in Section IV.

II. Experimental Setup and Results of Full Factorial

A 4-coil passive structure was used for measurements in this work. The S-parameter S_{21} is typically used to describe the transmission efficiency η in WPT systems [5]. $|S_{21}|^2$ refers to the ratio of output and input power,

$$\eta = \frac{P_{out}}{P_{in}} = |S_{21}|^2, \tag{1}$$

where P_{out} is the delivered power to the matched load of the Rx coil and P_{in} is the incident power of the Tx coil. For the measurements of S-Parameters, Tx and Rx coils are connected to a 2-port VNA through a feeding circuit that includes a balun transformer. The total distance between Tx and Rx coils is fixed. Distances d_1 and d_3 are independent and uniformly distributed variables in a given range while d_2 is a dependent variable. The measurements for distances d_1 and d_3 were made, thus resulting in the full factorial design of experiments.



(a) (b)

Fig. 2. (a) The Nylon cylinder and feeding circuit of the Tx or Rx coil. The feeding circuits include Balun transformer, chip capacitor and SMA connector and the nylon cylinder has isometric spiral grooves on the side surface. The resonant coils' terminals are connected by an in-line capacitor directly. (b) The setup of the 4-coils system. The system is supported by a wooden bracket with an height of 50 cm above the ground.

A. *Setup of 4-coil WPT System*

At first, helical coils are made for the Tx coils, Rx coils and resonant coils which are shown in Figure 2 (a). The Tx coil is composed of a cylinder and a feeding circuit. The supporting cylinder is made of Nylon with isometric spiral grooves on the side surface. Soft copper wires with insulation are wound around the cylinder with a height of 9 mm. Tx and Rx coils have both 9 turns of copper wires on the cylinder with diameter of 8 cm. Hence, we can calculate that the total length of copper wires on each coil is 2.26 m. A serial chip capacitor Cs is placed in the feeding circuit connected to strip lines whose characteristic impedance is 50 Ohms. In the case of the resonant coils, the serial in-line capacitor is connected to the two terminals directly. The overview of the setup is illustrated in Figure 2 (b).

B. *Results of Full Factorial Design of Experiments*

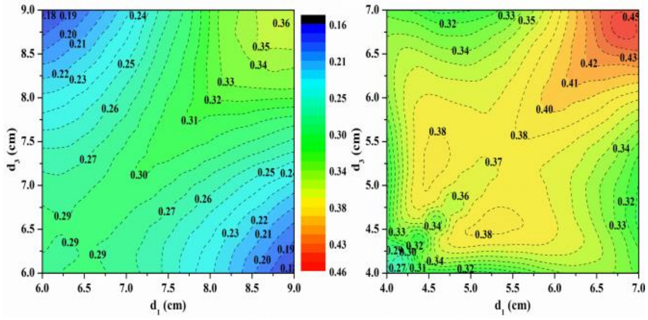
The efficiency is measured in the frequency band from 12 MHz to 16 MHz with 401 frequency points. Two groups of measurements were finished and the specific experimental set-ups are indicated as the following two cases.

1) Case 1: D_{total}=22 cm, d_1 and d_3 vary from 6 cm to 9 cm

In this case, distances d_1 and d_3 are varied using equally large step sizes Δd = 2 mm. The maximum obtainable efficiency in the frequency range from 12 to 16 MHz for varying distances d_1 and d_3 is depicted as a contour map in Fig. 3(a). Here, we can see that the peak values of the efficiency are symmetrical with respect to the diagonal because of the reciprocal nature of the 4-coil system. Additionally, peak values for η can be observed if $d_1 = d_3$. The total maximum efficiency for case 1 is η = 0.36 and corresponds to frequency f = 13.84 MHz for $d_1 = d_3$ = 8.8 cm. Here, the insertion loss of the feeding circuits is included, meaning the real efficiency will be higher.

2) Case 2: D_{total}=18 cm, d_1 and d_3 vary from 4 cm to 7 cm

In this case, the step sizes for d_1 and d_3 is the same as in case 1. The maximum of all peak values of the efficiency is 0.46 and the corresponding frequency is 13.84 MHz for d_1=d_3=6.8 cm, as shown in Figure 3(b). When d_1 and d_3 are smaller than 5.7 cm the maximum efficiency shifts away



D_{total} = 22 cm D_{total} = 18 cm
(a) (b)

Fig. 3. The maximum obtainable efficiency in the frequency range from 12 to 16 MHz as obtained through FF design of experiment. (a) Case 1 with D_{total} = 22 cm. (b) Case 2 with D_{total} = 22 cm.

from the diagonal of the design space. The results of Fig. 3 suggest that in order to obtain the maximum efficiency the total length D_{total} should be reduced while the distances of resonator coils to either Tx or Rx coil should be larger and equal to each other.

III. PCE Analysis and Comparison to FF

In this work, we apply PCE to the measured results in order to arrive at a surrogate model for the efficiency η of the multi-coil system for WPT. The main advantage of PCE method is that only a few measurements are required to obtain comparable results to those of FF design of experiments.

A. *PCE Analysis*

The PCE method for analyzing the efficiency η is as follows: Introducing $\xi = (d_1, d_3)^T$ as the vector containing the N = 2 independent input variables, we can then write the efficiency $\eta(\xi, f)$ as a function of ξ and frequency f by expanding η into a series

$$\eta(\xi, f) = \sum_{i=0}^{D} \hat{\eta}_i(f)\phi_i(\xi) \tag{2}$$

where $D = \frac{(P+N)!}{P!N!} - 1$ is the number of expansion terms that is dependent on the order of approximation P. A pre-analysis suggests that the results of PCE are accurate enough if P = 5. $\{\phi_i\}_0^D$ in equation (2) is the set of joint basis polynomials in ξ. In this work, we chose the input variables d_1 and d_3 to be uniformly distributed within a specific interval, so Legendre polynomials must be chosen for $\{\phi_i\}_0^D$. Lastly, $\{\hat{\eta}_i\}_0^D$ is the set of yet unknown PCE coefficients. Given the distribution of ξ and the order of approximation P, PCE specifies a set of samples $\{\tilde{\xi}_k\}_1^Q$ of size $Q = (P+1)^N$ in order to determine the PCE coefficients.

$$f : \{\tilde{\xi}_k\}_1^Q \mapsto \{\hat{\eta}_j\}_0^D \tag{3}$$

The function in equation (3) directly maps the set of samples onto the PCE coefficients and is based on pseudo-spectral projection which involves projecting $\eta(\xi, f)$ onto the basis polynomials [1]. Exploiting the orthogonality of the Legendre polynomials one can simply solve for and compute $\{\hat{\eta}_i\}_0^D$ by using a numerical integration technique such as Gaussian quadrature [3]. The intermediate steps of the function in equation (3) are omitted here in the interest of brevity, so the reader is referred to the extensive literature on this subject.

The specific samples of d_1 and d_3 for an order of approximation of P = 5 are given in Table I. For each case of D_{total}, d_1 and d_3 are equal to any combination of the values in the corresponding column. Then, the total results of all samples of d_1 and d_3 are used to calculate the coefficients $\{\hat{\eta}_i\}_0^D$ of the function $\eta(\xi, f)$. In a way, the PCE method proposed can be understood as a combination of small-scale design of experiments and polynomial regression.

TABLE I GQ-NODES OF D_1 OR D_3 FOR ORDER P=5

Set-up of d_1 or d_3 (D_{total} = 22cm)	Set-up of d_1 or d_3 (D_{total} = 18cm)
6.10 cm	4.10 cm
6.51 cm	4.51 cm
7.14 cm	5.14 cm
7.86 cm	5.86 cm
8.49 cm	6.49 cm
8.90 cm	6.90 cm

B. Comparison of PCE and FF Results

In order to validate the accuracy of the PCE method, the efficiency predicted by the PCE function is compared to the efficiency obtained from the FF design of experiments. The efficiency is a function of frequency as well as distances d_1 and d_3. Hence, the comparison is made in two separate steps. In the first step, the frequency is fixed and d_1 and d_3 are varied. Here, two arbitrary frequency points (13.8 MHz and 14.2 MHz) are selected arbitrarily to compare the efficiency and measured for two kinds of total distances, namely 18 cm and 22 cm. The efficiency results obtained by FF and the PCE function are illustrated in Figure 4. Apart from Figure 4 (b), all results agree sufficiently well. The disagreement in Figure 4(b) can be explained that the order of approximation P = 5 is not always optimal for all the frequency points and setups of d_1 and d_3.

In the second step, the efficiencies are compared in the frequency range from 12 to 16 MHz using fixed distances d_1 and d_3 for two different total lengths, namely D_{total} = 18 cm and D_{total} = 22 cm. The values of d_1 and d_3 are also selected arbitrarily for demonstration purpose. The results are shown in Figure 5 and it can be seen that there is good agreement between the PCE function and the FF design of experiments.

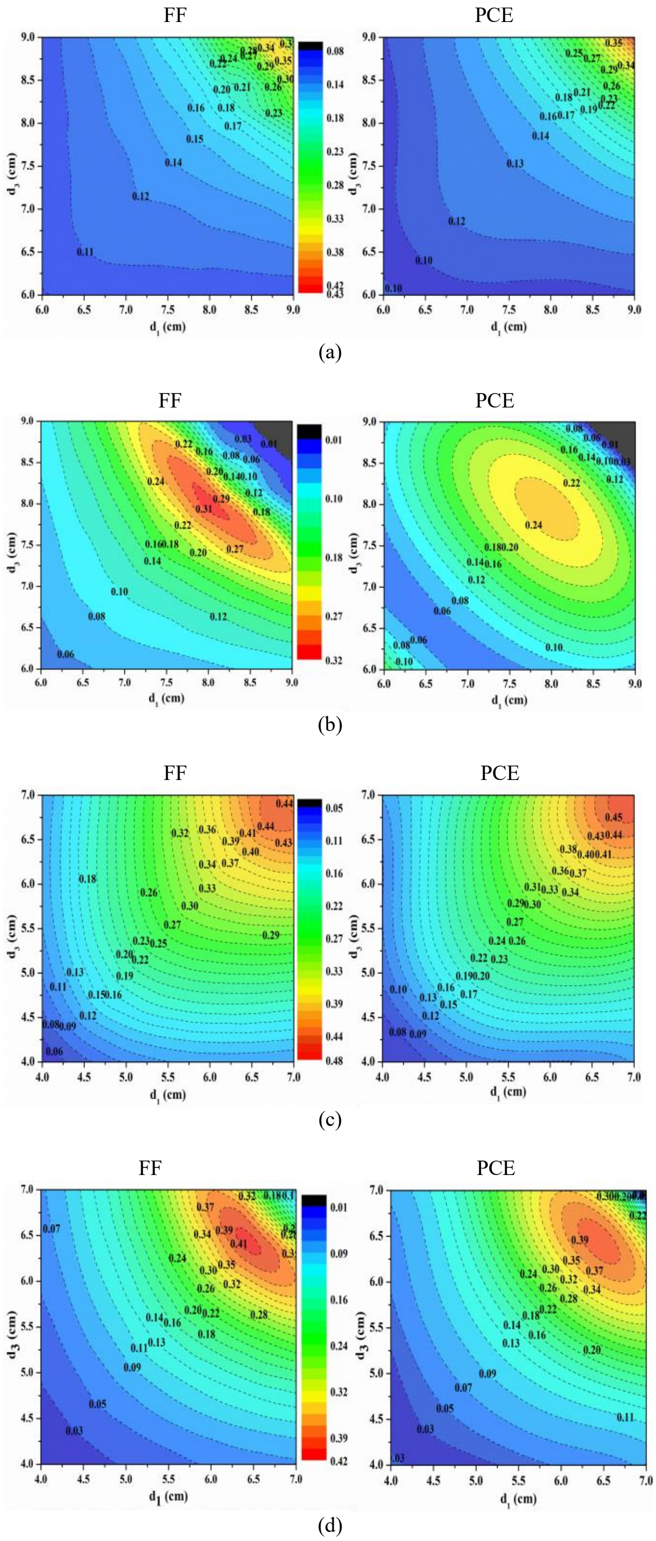


Fig. 4. Comparison of the efficiencies as obtained through full factorial design of experiments and PCE. The measurements were done under 4 different conditions: (a) f = 13.8 MHz, D_{total} = 22 cm, (b) f = 14.2 MHz, D_{total} = 22 cm, (c) f = 13.8 MHz, D_{total} = 18 cm, and (d) f = 14.2 MHz, D_{total} = 18 cm.

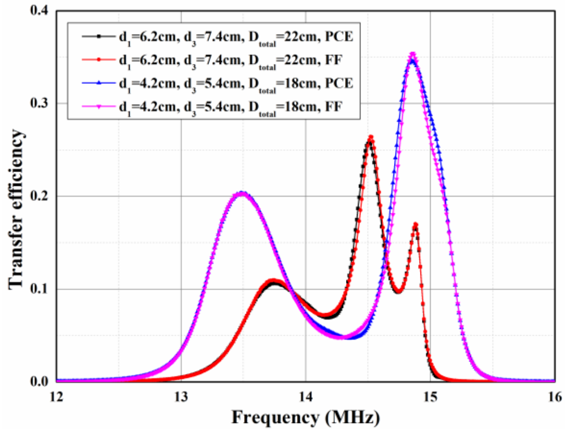


Fig. 5. Two set-ups are selected to show the results of PCE function with varying frequency *f*, in which d1 and d3 are selected arbitrarily for demonstration. It can be seen that the efficiency calculated by the PCE function is in good agreement with the results obtained through FF.

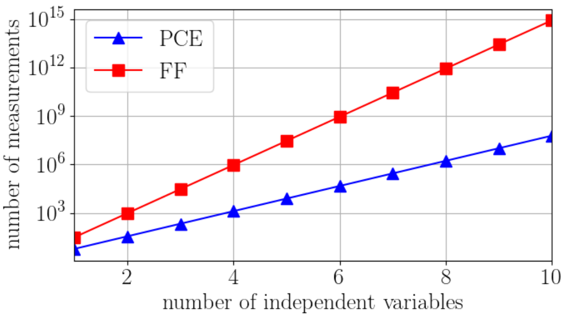


Fig. 6. The total number of necessary measurements is reduced significantly using the PCE method if the number of independent variables of the system increases.

C. Efforts of PCE and FF

The main advantage of the method proposed is the reduction of the number of measurements. When the step of d_1 and d_3 is 2 mm or 1 mm, 136 or 496 measurements are necessary for FF design, respectively. The PCE method just needs 21 measurements while providing the information of transfer efficiency with similar accuracy. The design of experiments by PCE reduces the number of measurements by a factor 6 to 20. Implicit in this analysis is the fact that the total number of necessary measurements is effectively halved due to symmetry of the experimental setup. Figure 6 depicts the computational effort of the general case where symmetries are not exploited and with similar discretization of d_1 and d_3 as before. It can be seen that when the number of independent variables increases there is a much more significant reduction of the necessary number of measurements. For $N = 4$, PCE reduces the required number by three orders of magnitude.

D. Sensitivity Analysis of Experiments

For a fixed experimental setup with $d_1 = d_3 = 7$ cm, $D_{total} = 18$ cm and f = 13.8 MHz, the efficiency was measured 50 times in a row. The range of efficiency is from 0.3 to 0.6 and it is equally divided into 70 intervals. The actual number of measured efficiencies in each interval is shown in Figure 7.

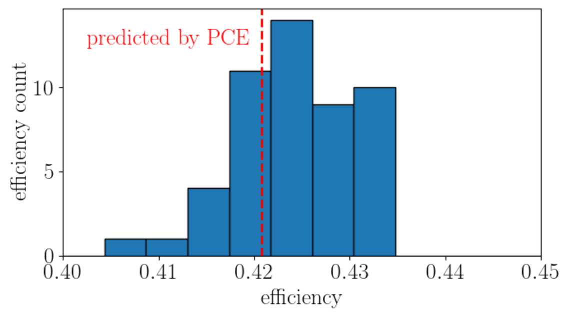


Fig. 7. Histogram of the efficiency from 50 measurements. The range of efficiency is from 0.3 to 0.6 and is divided into 70 equal intervals. The expectation of the efficiency is 0.42, which is quite close to the value of PCE (0.4208). The standard deviation is 0.006.

The mean value of all the efficiency samples is 0.42 which is quite close to the value of PCE function (0.4208). The standard deviation of efficiency is 0.006 and it indicates that the sensitivity of measurements is low enough. We may therefor conclude that the deviation of the predicted PCE result is well within the observed measurement accuracy.

IV. Conclusion

In this paper, the surrogate model of the efficiency of a multi-coil WPT system obtained using PCE is shown to be time-saving and highly accurate when comparing the results of both the PCE function and FF design of experiments. The advantage of the method proposed only increases with the number of input variables. In addition, we can conclude that the symmetric arrangement of WPT system implies higher transfer efficiency. Furthermore, the adjacent resonators should not be too close to the Tx or Rx coil in order to optimize the efficiency.

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