Full-Wave Coupling Analysis of Flanged Coaxial Line Array

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1 Introduction

A flanged coaxial line array finds practical applications in reflector antenna feeds, interstitial microwave applicators, and electromagnetic interference and compatibility [1-4]. A Green's function approach has been used to perform a theoretical investigation into coupling through a flanged coaxial line array [1]. A simple solution for TEM mode coupling through flanged coaxial lines has been also investigated by using Hankel transform in [4]. In this paper, we shall rigorously solve the boundary-value problem dealing with open-ended coaxial lines. We present the scattered field in terms of the discrete and continuous modes based on the eigenfunction expansion and Hankel transform. The boundary conditions are enforced to obtain a theoretical expression for mutual coupling between coaxial line modes. Computation is performed to check the validity of our formulation.

2 Field Analysis and Computation

Fig. 1 illustrates a coaxial line array consisting of a finite N number of open-ended coaxial lines $(j = 1, 2, \cdots, N)$ on an infinite flange. An incident TEM mode propagates along the j^{th} coaxial line. A time convention of $e^{-i\omega t}$ is suppressed throughout. In region (I), TE and TM modes, in addtion to a TEM mode, will be used to represent the field. The total fields in region (I) $(z < 0,\ a_j < r_j < b_j)$ consist of the incident and reflected components as

$$\bar{E}^I_{tj} = (V^i_j e^{ik_j z} + V^r_j e^{-ik_j z})\bar{e}^I_{temj} + \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\left(A^{(j)}_{mn} e^{-i\kappa^{(j)}_{mn} z}\bar{e}^I_{mj} + B^{(j)}_{mn} e^{-i\kappa^{(j)}_{mn}{}' z}\bar{e}^I_{ej}\right) \quad (1)$$

$$\bar{H}^I_{tj} = (V^i_j e^{ik_j z} - V^r_j e^{-ik_j z})\bar{h}^I_{temj} - \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\left(A^{(j)}_{mn} e^{-i\kappa^{(j)}_{mn} z}\bar{h}^I_{mj} + B^{(j)}_{mn} e^{-i\kappa^{(j)}_{mn}{}' z}\bar{h}^I_{ej}\right) \quad (2)$$

where $k_j = \omega\sqrt{\epsilon_j \mu}$. $\kappa^{(j)}_{mn} = \sqrt{k_j^2 - \gamma^{(j)}_{mn}{}^2}$, and $\kappa^{(j)}_{mn}{}' = \sqrt{k_j^2 - \lambda^{(j)}_{mn}{}^2}$ are determined by $\Phi_m(\gamma^{(j)}_{mn} a_j) = 0$ and $\Psi'_m(\lambda^{(j)}_{mn} a_j) = 0$. Note that $\bar{e}^I_{temj}$, $\bar{e}^I_{mj}$, $\bar{e}^I_{ej}$, $\bar{h}^I_{temj}$, $\bar{h}^I_{mj}$, and $\bar{h}^I_{ej}$ are discrete eigen modes in a coaxial line. In region (II) $(0 < z)$, the fields are the sums of

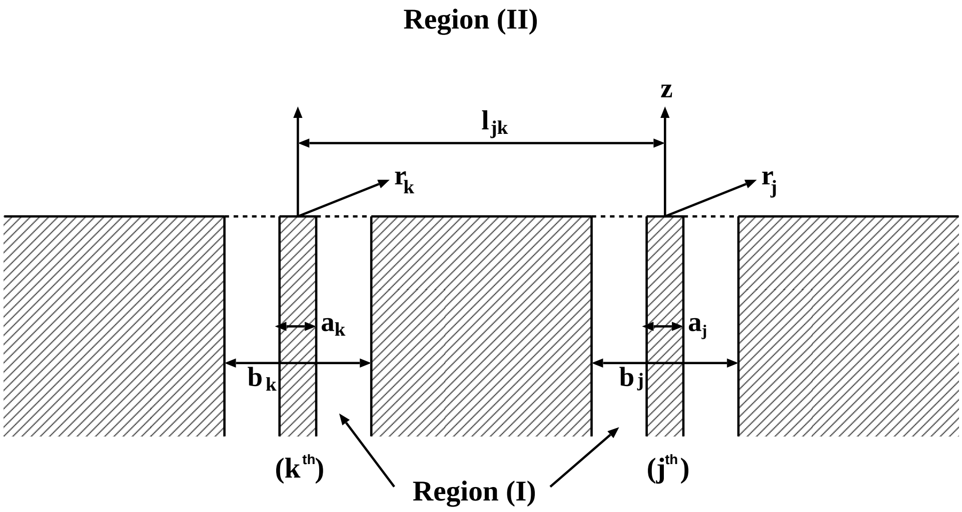

FIG. 1: Problem geometry

individual components based on the superposition principle.

$$\bar{E}_t^{II} = \sum_{j=1}^{N} \bar{E}_{tj}^{II}(r_j, \phi_j, z) \tag{3}$$

$$\bar{H}_t^{II} = \sum_{j=1}^{N} \bar{H}_{tj}^{II}(r_j, \phi_j, z) \tag{4}$$

$$\bar{E}_{tj}^{II}(r_j, \phi_j, z) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} \left[\tilde{\Pi}_{mj}(\zeta)\bar{e}_{mj} + \tilde{\Pi}_{ej}(\zeta)\bar{e}_{ej} \right] e^{i\kappa z} d\zeta \tag{5}$$

$$\bar{H}_{tj}^{II}(r_j, \phi_j, z) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} \left[\tilde{\Pi}_{mj}(\zeta)\bar{h}_{mj} + \tilde{\Pi}_{ej}(\zeta)\bar{h}_{ej} \right] e^{i\kappa z} d\zeta \tag{6}$$

where $\kappa = \sqrt{k^2 - \zeta^2}$ and $k = \omega\sqrt{\epsilon\mu}$. Note that $\bar{e}_{mj}^{II}$, $\bar{e}_{ej}^{II}$, $\bar{h}_{mj}^{II}$, and $\bar{h}_{ej}^{II}$ are continuous eigen modes in the open region. We enforce the boundary conditions on the field continuities. The tangential electric field continuities at $z = 0$ require, respectively,

$$\bar{E}_{tj}^{II}(r_j, \phi_j, 0) = \begin{cases} \bar{E}_{tj}^{I}(r_j, \phi_j, 0), & a_j < r_j < b_j \\ 0, & r_j < a_j \text{ or } r_j > b_j \end{cases} \tag{7}$$

It is convenient to utilize the power orthogonality for further simplification. The power orthogonality property is summarized in [5]. Applying the power orthogonality properties to (7) yields the series expression of continuous modal coefficients. The continuity of tangential magnetic field at $r_j < a_j$ and $z = 0$ is

$$\bar{H}_t^{II}(r_j, \phi_j, 0) = \bar{H}_{tj}^{I}(r_j, \phi_j, 0), \quad a_j < r_j < b_j \tag{8}$$

Applying the power orthogonality properties to (8), substituting the series expression of continuous modal coefficients into (8), and carrying out lengthy algebraic manipulations,

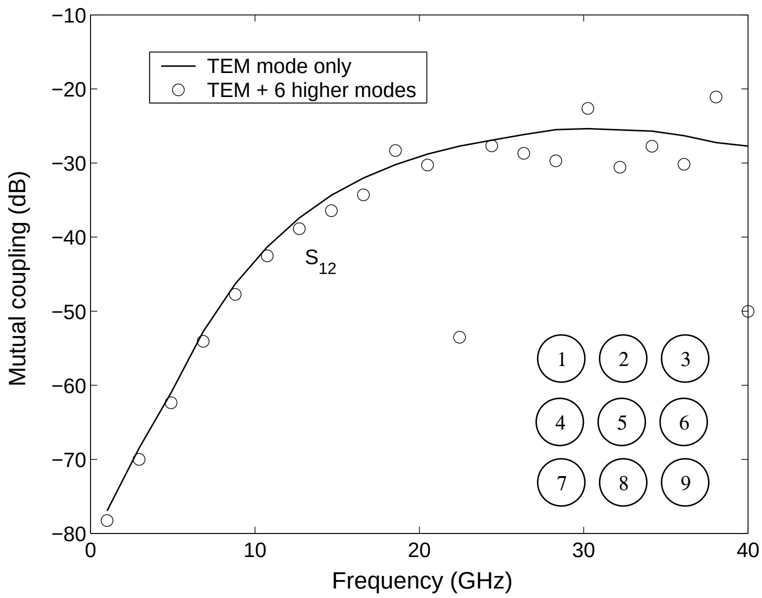

FIG. 2: Mutual coupling as a function of frequency (GHz)
($a = 1.062\ mm$, $b = 3.705\ mm$, $l = 12\ mm$)

we obtain a set of simultaneous equations for the modal coefficients V_j^r, $A_{mn}^{(j)}$, $B_{mn}^{(j)}$, $C_{mn}^{(j)}$, and $D_{mn}^{(j)}$ $(j = 1, 2, \cdots, N)$ for given V_j^i (incident TEM wave). To compare the full-wave analysis with the TEM mode approximation in [4], TEM mode mutual coupling is calculated. Consider nine RG-213/U coaxial lines ($a = 1.062\ mm$, $b = 3.705\ mm$, $\epsilon = 2.25\epsilon_0$) with a central displacement between adjacent coaxial lines $l = 12\ mm$ where the incident wave is assumed to excite coaxial line #1. Figs. 2 and 3 illustrate the mutual coupling S_{12} and S_{19}, respectively. Note that when the frequency is lower than about 20 GHz, the TEM mode solution generally shows a favorable agreement with the full-wave analysis that includes 6 higher modes in addition to TEM mode. This is because the cut-off frequency of TE_{11} mode in the coaxial cable is 20.6 GHz so that only TEM mode propagates in the cable. However, it is seen that the TEM mode solution deviates from the rigorous solution when frequency is over 20 GHz. The discrepancies may be attributed to the effect from higer order modes. Our theoretical formulation may be useful for the coupling computation in high-frequency region where higher modes become appreciable.

2.1 Conclusion

The boundary-value problem for electromagnetic coupling through a flanged coaxial line array is solved rigorously. The eigenfunction expansion, Hankel transform and superposition principle are used to represent the scattered fields in discrete and continuous modes. The boundary conditions are enforced to obtain the theoretical expression for mutual coupling between coaxial line modes. Computation is performed to compare the TEM mode approximation with the full-wave analysis. When the operating frequency is higher than

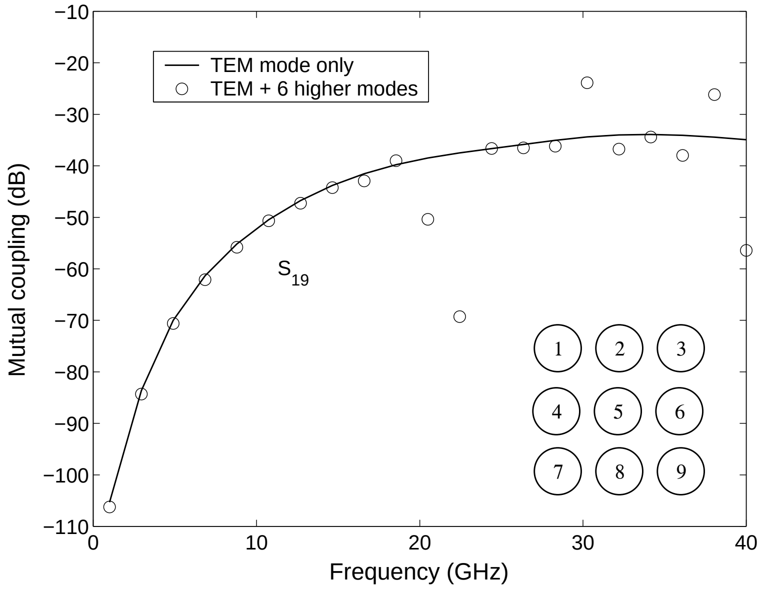

FIG. 3: Mutual coupling as a function of frequency (GHz)
($a = 1.062\ mm$, $b = 3.705\ mm$, $l = 12\ mm$)

the cut-off frequency of TE_{11} mode, the full-wave analysis is needed.

References

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